Renormalization Group Approach to the SUSY Flavor Problem

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Abstract

Renormalization group approach to the SUSY flavor problem is elucidated.

1 Introduction

1.1 Why do we like supersymmetry?

You certainly often have heard about what supersymmetry (SUSY) is good for. Let me mention few reasons for SUSY [1].

The first one is the no-go theorem, which was conjectured by Coleman and Mandula [2]. The theorem states: No other bosonic symmetries of the S-matrix than the Poincaré invariance and internal symmetries are possible in relativistic massive quantum field theories. SUSY is the unique, nontrivial extension of the Poincaré invariance. This is why we would like that the super Poincaré invariance is realized in nature. SUSY is a symmetry between bosons and fermions, and predicts the existence of superpartners with different spins. Since however no superpartners have been observed yet, the super Poincaré invariance should be badly broken at energies presently accessible to us, while the Poincaré invariance is exact so far. [3]
The second reason for SUSY is that SUSY can stabilize the different mass scales. In the Standard Model (SM), for instance, there is a typical scale, which is the mass of the Z boson. In the SM, there exists an yet undiscovered particle, the Higgs particle, whose mass is expected of the order of the weak scale, too.

It is also bounded from above due to the triviality constraint, which was estimated to be about 0.6 TeV. Then, assume that the SM is embedded to a more fundamental theory whose typical scale \( \Lambda \) is higher than the weak scale. In the fundamental theory, there will be heavy particles with mass of \( O(\Lambda) \), and they will contribute to the correction of the SM Higgs mass. The correction \( \delta m^2_H \) will be typically proportional to \( (\alpha/\pi)\Lambda^2 \), where \( \alpha \) is a generic coupling of the fundamental theory. It would be “unnatural”, if the correction \( \delta m^2_H \) is larger than the mass itself \( m^2_H \) \[4, 5\]. Since \( m_H \) is bounded from above, we obtain the maximal natural scale of the SM, which is about few TeV, where we assumed \( \alpha \) could be at most \( O(1) \).

This means, there will be no natural extension of the SM for energies larger than few TeV, and consequently, there are no natural grand unified theories (GUTs) \[4\].

SUSY can help a lot in this regard, thanks to the non-renormalization theorem \[6\]. The theorem states: There are only wave function renormalizations. So, there is no extra renormalization for mass parameters. In other words, there are no quadratic divergences. Therefore, the correction \( \delta m^2_H \) will be proportional to \( (\alpha/\pi)m^2_H \ln \Lambda/m_H \) in a SUSY extension of the SM. The requirement \( \delta m^2_H < m^2_H \) is satisfied, even if \( \Lambda \approx M_{\text{PL}} \) (the Planck scale).

The third reason for SUSY is experimental. If the SM is unified to a SUSY GUT, it predicts the unification of gauge couplings of the SM. According to the report of Raby \[7\], the gauge unification in SUSY GUTS fits to within 3\( \sigma \) of the present precise experimental data. There are other experimental indications: Jens Erler and Wolgang Hollik reported in this meeting that the experimental values of the W mass and \( g-2 \) of the muon are in favor of SUSY. There are other reasons for SUSY, but they are often more subjective.

1.2 How do we like SUSY to be broken?

There are various mechanisms to break SUSY. \(^3\) Although we do not know which mechanism is realized in nature, we would like SUSY to be broken in such a way that it still can allow a natural extension of the SM to a higher energy. It has been studied how this requirement can be met, and found that SUSY has to be only softly broken \[8\]. The soft-supersymmetry breaking (SSB) terms are those which do not change the infinity structure of the parameters of the symmetric theory, and do not produce quadratic divergences. So they are additional terms in the Lagrangian that do not change the RG functions such as the \( \beta \) and \( \gamma \) functions of the symmetric theory. There exist four types of such terms \[8\]: \( (m^2_j)\phi_j\phi^* \) (soft scalar mass terms), \( B^{ij}\phi_i\phi_j \) (B-terms), \( M\lambda \) (gaugino mass terms), \( h^{ijk}\phi_i\phi_j\phi_k \) (trilinear scalar couplings), where \( \phi_j \) and \( \lambda \) denote the scalar component in a chiral supermultiplet and the gaugino in a gauge supermultiplet, respectively.

The SSB parameters are massive parameters. Although it depends on the model, we expect that the SSB parameters are more than less in the same order, which we call

\(^3\)See for instance \[1\].
the SUSY breaking scale $M_{\text{SUSY}}$. To make compatible supersymmetry breaking with the naturalness notion of 't Hooft and Veltman, we have to impose the constraint on $M_{\text{SUSY}}$. A simple calculation yields that $M_{\text{SUSY}}$ should be less than few TeV.

1.3 What is the SUSY flavor problem?

If we do not specify the SUSY breaking mechanism, because we do not know which one is realized, and rely only on renormalizability of the minimal extension of the SM, the minimal supersymmetric SM (MSSM), it is possible to introduce ciento cinco new parameters [9].

Therefore, all together we have ciento veinticuatro parameters! These 105 new parameters, of which 43 parameters are CP violating phases, produce a lot of FCNC and CP violating processes [9].

For instance, there exist diagrams due to the SSB term $(m_{RL}^2)^{e\mu} \bar{e}_R \bar{\mu}_L$ or $(m_{LL}^2)^{e\mu} (\bar{\epsilon}_L)^* \bar{\mu}_L$, which allow the decay $\mu \to e \gamma$ [10], where $\bar{\epsilon} (\bar{\mu})$ denotes the scalar partner of the electron (muon). The decay has not been observed yet. A similar FCNC process is also possible in the hadronic sector, which allows the decay $b \to s \gamma$ [11]. Its branching ratio $(= (3.3 \pm 0.4) \times 10^{-4})$ has been measured at CLEO and BELLE [12], which agrees well with the SM calculation $(= (3.29 \pm 0.33) \times 10^{-4})$ [13]. Renormalizability cannot explain why these SSB parameters should be so small. One of the most stringent constraint on the CP-violation phases is the electric dipole moment (EDM) of the neutron: $d_N/e \leq 0(10^{-26})$ cm [7]. The SSB term of the form $(m_{LR}^2)^{uu} \bar{u}_L \bar{u}_R$ can induce $d_N$, for instance, if $(m_{LR}^2)^{uu}$ is a complex parameter [14]. There exist a number of FCNC and CP-violating processes, which give severe constraints on the SSB parameters [10, 11, 14, 15, 16].

Fortunately, the experimental constraints are not random: They suggest that the soft scalar masses $(m^2)_{ij}$ are diagonal in the space of generations and the trilinear couplings $h_{ijk}$ are proportional to the Yukawa couplings $Y_{ijk}$. Although these experimental constraints hint a flavor symmetry in the SSB sector, flavor symmetry is broken in nature and is not good enough to overcome the SUSY flavor problem. (Non-Abelian discrete symmetries such as $A_4$ could be used to overcome the problem[17].)

2 What the RG approaches to the SUSY Flavor Problem are based on.

There exist various approaches to the SUSY Flavor Problem. The popular one is the hidden sector scenario. In the so-called minimal supergravity model [1] it is simply assumed that the SSB parameters have a universal form, say, at the GUT scale. In this model, supersymmetry breaking occurs in a sector that is hidden to the MSSM sector, and supersymmetry breaking is mediated to the MSSM sector by gravity. There exist other ideas of mediation: gauge mediation [18], anomaly mediation [19] and gaugino mediation [20].

Another possibility is that certain superparticles are so heavy that FCNC and CP-violating processes may be sufficiently suppressed [21]. But this scenario could lead to problems, because, as we heard from María Herrero and María Krawczyk

\[4\text{The mass and mixing of neutrinos are not taken into account.}\]
in this meeting, the heavy superparticles in the MSSM do not necessarily decouple at low energies. The approach that I am going to discuss below is based on the renormalization group (RG).

2.1 Renormalizability and Infrared Attractiveness

The main idea to solve the SUSY flavor problem in the RG approach is to use the infrared (IR) attractive behavior of the SSB terms, which can restrict the huge number of the degrees of the freedom and then to increase the predictive power. The basic idea goes back to Wilson [22], who succeeded to relate IR and ultraviolet (UV) physics to understand universality of critical phenomena. Let me illustrate in a simple example how renormalizability and IR attractiveness are closely related in the Wilsonian RG. The example is the scalar theory in three Euclidean dimension. In the Wilsonian RG, a set of infinite number of independent couplings are allowed. A RG flow is a trajectory in this infinite dimensional space of couplings. However, all the trajectories converge to a single trajectory, called the renormalized trajectory (RT), in the IR limit for this example [22]. The RT trajectory intersects the critical surface at the Wilson-Fisher fixed point. The RT, on one hand, defines the single coupling that survives in the IR limit, which is identified with temperature in statistical physics. The critical exponent expresses how fast a point on the RT goes way from the fixed point. To define a non-trivial relativistic field theory, that is, nonperturbative renormalizability, the couplings, on the other hand, have to lie on the RT. So, all the couplings have to be functions of this single independent coupling. Of infinite number of couplings, there exists only one independent coupling, the same one in the IR as well as in the UV limit.

2.2 Perturbative Renormalizability and Infrared Attractiveness

The next question is: Can one understand perturbative renormalizability in terms of the Wilsonian RG? This question was addressed by Polchinski [23], who investigated RG trajectories in the scalar theory in four dimensions. His observation, assuming that the mass parameter can be neglected, is that there exists a trajectory, which is attractive in the IR limit. That is, whatever the initial values of the unrenormalizable couplings at some UV scale are, they become definite functions of renormalizable couplings in the IR limit. This is then interpreted as perturbative renormalizability. Therefore, in the class of theories, in which Polchinski’s criterion [23] on perturbative renormalizability can be applied, the UV cutoff should be sufficiently large so that physics in the infrared regime has less dependence of their initial values in the UV regime.

2.3 Pendleton-Ross fixed point and Zimmermann’s reduction of couplings

There exists a further extension; extension to relations among renormalizable couplings. The Pendleton-Ross IR fixed points [24] are IR fixed points that one finds in various theory models in the lower order in perturbation theory. The IR fixed points have been used in phenomenological approaches to various problems in particle physics [25]. The approach
emphasizes the IR attractiveness of certain couplings. The reduction of couplings of Zimmermann [26] is to actively reduce the number of renormalizable couplings so that it does not destroy perturbative renormalizability. So, this approach emphasizes the UV behavior of a theory. But as I have shown you in the Wilsonian RG, UV and IR physics are closely related. In fact, they give similar results.

The well known example is the SM. If we neglect all the couplings except the QCD gauge coupling $g_3$, the top Yukawa coupling $Y_t$ and the Higgs self coupling $\lambda_H$, one finds at the one-loop level that the ratios of the couplings $(Y_t/g_3)^2$ and $(\lambda_H/g_3)^2$ approach to $2/9$ and $(\sqrt{689} - 25)/18$ in the IR limit, respectively [24]. In fact these numbers are exactly the first coefficients in the power series expansions $(Y_t/g_3)^2 = 2/9 + \sum_{n=1}^{\infty} \rho_{t,n}^2 (g_3)^{2n}$ and $(\lambda_H/g_3)^2 = (\sqrt{689} - 25)/18 + \sum_{n=1}^{\infty} \rho_{H,n}^2 (g_3)^{2n}$, which are the power series solutions to the so-called reduction equations, and ensure perturbative renormalizability to all orders in perturbation theory in the reduced system of the SM [27].

2.4 Problems

So, it is the obvious thing to try to apply these approaches to softly broken SUSY theories [28]. The question is whether one can reduce the huge number of the independent SSB parameters of the MSSM in such a way that they do not cause problems with the strong experimental constraints. The IR attractiveness results from asymptotically free gauge interactions, and fortunately, gauge interactions of the MSSM are flavor independent. It was found [28] in fact that they are attracted to certain points in the IR limit, and the points do not depend very much on the flavor. But it turned out that the IR attractiveness is not very strong.

To summarize, the IR attractiveness of the SSB parameters in 4D dimensional softly broken Yang-Mills (YM) theories is not strong enough that we can satisfactory overcome the SUSY flavor problem. The weakness of the IR attractiveness of the SSB parameters in four dimension originates from the fact that the scaling violation in four dimension are generally only logarithmic. So, to achieve a stronger IR attractiveness of the SSB parameters, we have to have a situation in which the dimension is effectively or really more than four. There are so far two ideas as far as I know. The first one is based on a strongly coupled super YM theory which has an IR fixed point. The second one is to simply go to higher dimensions.

3 Two viable solutions

3.1 Coupling to a superconformal gauge theory

The first idea, proposed by Nelson and Strassler [30], is to use a superconformal force in a supersymmetric YM theory and rely on the Seiberg conjecture [31]. The conjecture states that a SUSY QCD-like YM with has an IR fixed point in the so-called conformal window [32, 31]. As a result, the anomalous dimensions for the matter supermultiplets become large of $O(1)$ near the IR fixed point. Consequently, the parameters near the IR

\footnote{There exist also works based on the reduction of couplings [29].}
fixed point run according to a power-law, rather than a logarithmic law. In a QCD-like theory we can not introduce the MSSM matter multiplets. Fortunately, the conjecture says that the QCD-like theory can be described by another YM theory, which contains gauge singlet mesons. These mesons couples to the quark-like matter multiplets though the Yukawa coupling. The conjecture implies that in the plane of the Yukawa and gauge couplings there exists a nontrivial IR fixed point. Nelson and Strassler [30] identified the gauge singlet mesons with the MSSM matter multiplets. In this way, a large (in fact negative) anomalous dimensions are transferred to the MSSM sector. The hierarchy of the generations at low energies could be achieved in this way.

What happens now with the SSB parameters? In [33], it was shown that the SSB parameters approach zero in the IR limit both in the original and dual theories. What about the SSB parameters of the MSSM? Since the matter and Higgs supermultiplets directly couple to the (dual) superconformal sector, the corresponding SSB parameters, i.e., $m^2$’s and $h$’s, will approach zero, while the gaugino masses remain as they are. Then the scenario to solve the SUSY flavor problem is: There are two scales $M_>$ and $M_<$ in addition to $M_{SUSY}$. Above $M_>$ (which may be between $10^{14}$ to $10^{19}$ GeV) the strong sector is perturbative. Between $M_>$ and $M_<$, the theory is in the conformal regime, and the SSB parameters should obey the power law running, and become almost zero at $M_<$. Below $M_<$ (which may be between $10^{10}$ to $10^{16}$ GeV), the superconformal sector escapes by some mechanism, that is, the MSSM sector decouples from the superconformal sector, the MSSM interactions, especially the gauge interactions, generate $m^2$’s and $h$’s of the MSSM at $M_{SUSY}$. Remember, the MSSM gauge interactions are flavor independent. Therefore, they can generate flavor independent SSB parameters at low energies [30], although the flavor independence is not perfect and it could be a problem [34]. There are further extensions and applications of the idea of Nelson and Strassler [35, 36].

### 3.2 Going to higher dimensions

The next idea is to really go to higher dimensions [37, 38]. In higher dimensions, due to the power-law running of couplings [39, 40, 41, 42], stronger infrared attractiveness [43] of the SSB parameters is expected [44, 45, 46, 47]. In [46] we considered the simplest case in which only the gauge supermultiplet propagates in the $(4 + \delta)$-dimensional bulk and the supermultiplets containing the matter and Higgs fields are localized at our 3-brane [37, 38, 40]. The gaugino mass $M$, which is assumed to be generated at the fundamental scale $M_{PL}$ by some SUSY breaking mechanism, receives a correction proportional to $(M_{PL}/M_{GUT})^\delta$ at the grand unification scale $M_{GUT}$, and more importantly it can induce flavor-blind corrections to other SSB parameters. We found [46] that the soft scalar masses $(m^2)_j^i$ and the soft-trilinear couplings $h^{ijk}$ become so aligned at $M_{GUT}$ that FCNC processes and dangerous CP-violating phases are sufficiently suppressed (see also [47]):

$$h^{ijk}/M^{Y^{ijk}} = -\eta^{ijk}_{Y} + O(10^{-6}), \quad (m^2)_j^i/|M|^2 = C(i)/C(G)\delta_j^i + O(10^{-3}),$$

at $M_{GUT}$, if we take into account only power law corrections, where $Y^{ijk}$ are Yukawa couplings, $C$’s and $\eta$’s are grouptheoretic constants, and we assumed that $M_{PL}/M_{GUT} = 10^2$ and $\delta = 2$. 
As a concrete example, we [46] considered the minimal supersymmetric SU(5) GUT model in six dimensions including logarithmic corrections, and found that all the A-parameters $h$'s, B-parameter $B$ and soft-scalar masses $m^2$'s for the MSSM are fixed once the unified gaugino mass $M$ is given. For instance, $\tan \beta \simeq 19.5$ for $M = 0.5$ TeV.

Nevertheless, the stringent constraints coming from the $K_S - K_L$ mass difference $\Delta m_{K}$, $\epsilon'/\epsilon$ in the $K^0 - \bar{K}^0$ mixing, the decay $\mu \to e\gamma$, and the electric dipole moments (EDMs) of the neutron and the electron [16] are satisfied in this model.

The suppression mechanism of the FCNC and CP-phases presented above does not work in four dimensions. Therefore, the smallness of FCNC as well as of EDM is a possible hint of the existence of extra dimensions.

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References


