observable for BHD, where $\hat{a}_1$ and $\hat{a}_2$ are annihilation operators for the LO field and the signal field, respectively. If the signal field $|\psi\rangle_s$ satisfies $r \langle \psi| H_0 |\psi\rangle_s > |\psi| H_0 |\psi\rangle_s$, which holds when the intensity of the LO field is extremely larger than that of the signal field, this observable satisfies

$$
\langle a^\dagger_1 a_2 + a_1 a^\dagger_2 |e^{i\hat{\phi}}| \psi\rangle_s \sim r \langle X_s(\theta)| e^{i\hat{\phi}}| \psi\rangle_s,
$$

(1)

where $X_s(\theta) \equiv \hat{a}_1 e^{i\hat{\phi}} + \hat{a}_2 e^{-i\hat{\phi}}$ and $|e^{i\hat{\phi}}|$ is the coherent state in polar coordinates. According to the standard interpretation of the quantum theory, Eq. (1) implies if we obtain the measurement outcome $x$ in one trial of BHD with the prior knowledge of $r$, $|\psi\rangle_s$ instantaneously reduces to $|\theta\rangle_s$ satisfying $X_s(\theta)|\theta\rangle_s = x|\theta\rangle_s$.

Since $x$ of the laser field is measurable beforehand, we may define the measurement operator $\hat{r}$ for BHD as

$$
M(x, r, \theta) = \pi^{-\frac{1}{2}} |x| e^{-\frac{x^2}{2}} \exp\left(|x| e^{i\hat{\phi}} a_1 - \frac{1}{2} |x|^2 e^{i\hat{\phi}} a_1^\dagger \right) |0\rangle,
$$

(2)

where $|x, \theta\rangle$ is the quadrature eigenstate written as

$$
|x, \theta\rangle = \left(2\pi\right)^{-\frac{1}{2}} e^{-\frac{x^2}{2}} \exp\left(x e^{i\hat{\phi}} a_1 - \frac{1}{2} x^2 e^{i\hat{\phi}} a_1^\dagger \right) |0\rangle.
$$

(3)

Then, Eq. (3) satisfies the orthonormalization condition $\langle x_1, \theta| x_2, \theta \rangle = \delta(x_1 - x_2)$ and the completeness relation $\sum_{x = -\infty}^{+\infty} dx \langle x, \theta| x, \theta \rangle = 1$ on $x$. Since the coherent state also satisfies the completeness relation $\sum_{x = -\infty}^{+\infty} dx \langle x, \theta| x, \theta \rangle = 1$ on $x$, the probability of obtaining the measurement outcome $x = x$ with the prior knowledge of $r$ is

$$
P(x) = \int_0^{2\pi} d\theta \int_0^{\infty} dr M(x, r, \theta) \rho_s M^\dagger(x, r, \theta),
$$

(4)

and the density operator after the measurement is

$$
\rho_s = P(x)^{-1} \int_0^{2\pi} d\theta \int_0^{\infty} dr M(x, r, \theta) \rho_s M^\dagger(x, r, \theta),
$$

(5)

where $\rho_s$ is the density operator before the measurement.

We will denote the procedure described above the observable-based projection method (OBPM) in the rest of this Letter. Note that above discussion is not based on the assumption that the laser field is the coherent state (“partition ensemble fallacy”). It is the property of the observable for BHD that approximately projects the strong laser field of the LO mode onto the coherent state after the measurement. On the contrary, the number states in the LO mode cannot be eigenstates of the observable for BHD, because $|n\rangle \neq |n - 1\rangle$ even in the limit $n \rightarrow +\infty$ due to their rigid orthogonality.

As an example of BHD, we will calculate $P(x)$ in the squeezed light generation scheme by OBPM. In the scheme, the same laser source is used for supplying the LO field, and pumping the nonlinear medium to generate the squeezed state. The density operator of the system before the measurement is

$$
\rho_0 = \int_0^{2\pi} d\phi \left| r_s e^{i(\phi + \psi)} \right\rangle \langle 0, s | e^{i2\phi} \right\rangle \langle 0, s | e^{i2\phi} \right\rangle |r_s e^{i(\phi + \psi)}\rangle_0,
$$

(6)

where $\phi$ is the unknown phase of the pump field, $\psi$ is the phase delay by a controllable phase shifter, and $|0, s\rangle \equiv |S(0)|0\rangle$ is the squeezed vacuum state. The unknown phase of the squeezed state is $2\phi$ instead of $\phi$, because of frequency of the pump field is doubled by second harmonic generation before the field enters an optical parametric oscillator. By using Eqs. (1) and orthogonality approximation of the coherent state $|e^{i\hat{\phi}}|^2 \approx (\pi/\hbar) \delta(r - r_s) \delta(\theta - \theta_s)$ in the limit $r_s \rightarrow +\infty$ derived from $\lim_{r_s \rightarrow +\infty} \exp[-|x|^2/(4r)]/(2\sqrt{\pi}) = \delta(t)$, and the relation

$$
\langle x, \theta|0, s\rangle e^{i2(\phi - \psi)} = \sum_{n = 0}^{+\infty} \langle x, \theta| n\rangle |n, 0\rangle e^{i2(\phi - \psi)},
$$

(7)

where $H_n(x)$ are Hermite polynomials, we find $P(x) = |\langle x, \phi |0, s\rangle|^2$, which agrees with the experimental result of Ref. [16].

Next, we will apply OBPM to CVQT with a laser [19]. In the measurement step by Alice, the probability of obtaining $x_1$ in BHD1 and $x_2$ in BHD2 is $P(x_1, x_2) \equiv \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \int_0^{2\pi} d\theta_3 \int_0^{2\pi} d\theta_4 \int_0^{2\pi} d\theta_5 \text{Tr}\{|M_{22} M_1 M_1 M_2\}\}$ and the density operator after the measurement is $\rho_{12} \equiv \int_0^{2\pi} d\phi \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{2\pi} d\phi_3 \int_0^{2\pi} d\phi_4 \text{Tr}\{|M_{22} M_1 M_1 M_2\}$, where $M_{ij} \equiv \text{exp}[-|x_j|^2/(4r)]_i_j |x_j|^2 |x_j\rangle \langle x_j|_i_j$. $\rho_{12}$ is the density operator of the total system before the measurement written as

$$
\rho_1 = \int_0^{2\pi} d\phi [r_s e^{i(\phi + \psi)}]_1 \int_0^{2\pi} d\phi_2 [r_s e^{i(\phi + \psi)}]_2 \langle \psi | r_s e^{i(\phi + \psi)} \rangle_1 \langle \psi | r_s e^{i(\phi + \psi)} \rangle_2.
$$

(8)

where the modes $1, 2$ are for LOs of BHD1,2 in Alice, $13$ to $15$ in Bob, $\phi$ is the unknown phase of the pump field, $\rho_0$ is an arbitrary density operator supplied by a third party “Victor” to Alice, and $|\psi|^2_{12} + |\psi|^2_{13} + |\psi|^2_{14} + |\psi|^2_{15} = 1$. The modes $1, 2$ is a two-mode squeezed state $|\psi\rangle$ as the FPR state. Again, the unknown phase in the modes $1, 2$ is $2\phi$ instead of $\phi$. (See Fig. 1 in Ref. [11]).

By using Eq. (8) and $a_{s1} = (a_{s1} - a_1)/\sqrt{2}$, $a_{s2} = (a_{s1} + a_1)/\sqrt{2}$ where the modes $s1, s2$ are for the signal field of BHD1,2, the quadrature eigenstates of the modes $s1, s2$ are written in the modes $in, 1$ as
\[
\langle \vec{x}_1, \phi | \gamma_0^2 \rangle \sim \exp(-\frac{\gamma_0^2}{2}) \exp[\gamma \alpha_0^2 - \gamma \alpha_0^2] e^{i\alpha_0 \delta} |\phi\rangle_0 |\psi\rangle_0 |\phi\rangle_0 |\psi\rangle_0,
\]
where \(\gamma_0^2 = \frac{1}{\gamma^2} \exp(-\frac{\gamma_0^2}{2}) \exp[\gamma \alpha_0^2 - \gamma \alpha_0^2] e^{i\alpha_0 \delta} |\phi\rangle_0 |\psi\rangle_0 |\phi\rangle_0 |\psi\rangle_0\). We will subsequently discuss generation of a strongly phase-correlated quantum state necessary in CVQT by measuring two independent laser fields.

In the case of BHD, since the observable satisfies \(\langle a| \hat{A} |\psi\rangle \sim \frac{1}{\gamma} \exp(-\frac{\gamma_0^2}{2}) \exp[\gamma \alpha_0^2 - \gamma \alpha_0^2] e^{i\alpha_0 \delta} |\phi\rangle_0 |\psi\rangle_0 |\phi\rangle_0 |\psi\rangle_0\), because \(\gamma_0^2 \leq \gamma\), the density operator after the measurement becomes

\[
\hat{\rho} \sim \frac{1}{\gamma} \exp(-\frac{\gamma_0^2}{2}) \exp[\gamma \alpha_0^2 - \gamma \alpha_0^2] e^{i\alpha_0 \delta} |\phi\rangle_0 |\psi\rangle_0 |\phi\rangle_0 |\psi\rangle_0 + \frac{1}{\gamma} \exp(-\frac{\gamma_0^2}{2}) \exp[\gamma \alpha_0^2 - \gamma \alpha_0^2] e^{i\alpha_0 \delta} |\phi\rangle_0 |\psi\rangle_0 |\phi\rangle_0 |\psi\rangle_0,\]

and the density operator becomes

\[
\hat{\rho} \sim \frac{1}{\gamma} \exp(-\frac{\gamma_0^2}{2}) \exp[\gamma \alpha_0^2 - \gamma \alpha_0^2] e^{i\alpha_0 \delta} |\phi\rangle_0 |\psi\rangle_0 |\phi\rangle_0 |\psi\rangle_0 + \frac{1}{\gamma} \exp(-\frac{\gamma_0^2}{2}) \exp[\gamma \alpha_0^2 - \gamma \alpha_0^2] e^{i\alpha_0 \delta} |\phi\rangle_0 |\psi\rangle_0 |\phi\rangle_0 |\psi\rangle_0,\]

which corresponds to the transfer operator in Ref. 10 from the mode \(2\) to the mode 2.

Eqs. 10 and 11 clearly show that in the special case \(\eta = 1 \frac{1}{T_{2,\phi}}\) is independent of the unknown phase \(\psi\), where ideal quantum teleportation is realized, while in the usual case \(0 \leq \eta < 1 \frac{1}{T_{2,\phi}}\), it is dependent on the unknown phase \(\psi\), where the reconstructed density operator in the mode 2 is distorted from \(\hat{\rho}_{\phi}\).

We will subsequently discuss generation of a strongly phase-correlated quantum state necessary in CVQT by measuring two independent laser fields.
phase correlation between the fields unchanged. The absorption rate is assumed to be quite high, where $t$ is much smaller than the dynamical time scale of an individual laser.

For $s \gg 1$, the distribution of the phase difference of states in the integrand of Eq. 18 has a peak at $|\phi_d - \phi_b| = \pi$ when $p = s$, or at $|\phi_d - \phi_b| = 0$ when $p = 0$. Since Eq. 19 has peaks at $p = 0, s$, the probability of obtaining Eq. 19 with $p = 0, s$ is not negligible.

The photon number distribution of the mode $c$, $P_c(m) \equiv \langle m | \text{Tr}_d [\rho(t; p, q)] | m \rangle_c$, is found to be

$$P_c(m) = e^{-2r_s^2 (2r_s^2)^n} \frac{B(m + p + \frac{p}{2}, q + \frac{q}{2})}{m!} \frac{B(p + \frac{p}{2}, q + \frac{q}{2})}{(2r_s^2)^{m+n}} \times F_1 \left( q + \frac{q}{2}, m + p + q + 1; 2r_s^2 \right),$$

(14)

where $F_1(\alpha; \beta; z)$ is the confluent hypergeometric function of the first kind. $P_d(n)$ is easily obtained by replacing $m$ with $n$ and interchanging $p \leftrightarrow q$ in Eq. 15. Fig. 7 is for $P_d(m)$, $P_d(n)$.

When $p = 0, s$ with $s \gg 1$, the generated quantum state is applicable to CVQT as a means to share the unknown phase of the laser field between Alice and Bob, though the phase correlation formed after the continuous measurement will slowly be broken by the phase diffusion effect of lasers.

The famous experiment for interference of two independent lasers by Pfeifer and Mandel 19, where weak laser fields were mixed by beamsplitters and all the output fields were continuously measured by photomultipliers, should carefully be reviewed in terms of phase-correlated quantum state generation by measurement.

In conclusion, we have pointed out that the field state outside the laser cavity is not equivalent to the expression in terms of noncontinuous operators given in Ref. 2. We have presented OBP exemplified for BHDF to analyze CVQT with a conventional laser whose phase is completely unknown.

CVQT is found to be possible only if the unknown phase of the laser field is shared among Alice's LOs, the EPR state, and Bob's LO by a certain means. The demonstrated experiment for CVQT 19 is valid, but needs an optical path other than the EPR channel and a classical channel allowed to use in the teleportation protocols 12 to share the unknown phase of the same laser field between Alice and Bob. We have proposed a method to probabilistically generate a strongly phase-correlated quantum state via continuous measurement of independent lasers, which is applicable to realizing CVQT without the additional optical path.

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