ABSTRACT

Lensing probabilities of quasars with image separations greater than $\Delta \theta$ and flux density ratios less than $q_{\ell}$ are calculated by foreground dark matter halos in a flat, cosmological constant dominated ($\Lambda$CDM) universe. The mass density of the lenses is taken to be the Navarro-Frenk-White (NFW) profile on all mass scales, plus a central point mass for low-mass halos with $M < M_c = 5 \times 10^{13} h^{-1} M_\odot$. We introduce a quantity $M_{\text{eff}}$, which is a point mass ranging from 1 to 1000 times the mass $M_\bullet$ of a supermassive black hole (SMBH) inhabiting the center of each galaxy, to describe the contributions of galactic central SMBHs and galactic bulges to lensing probabilities. The lensing cross section and thus the lensing probability are quite sensitive to the flux density ratio $q_{\ell}$ of multiple images in our calculations. It is shown that, to reproduce the lensing survey results of JVAS/CLASS for $q_{\ell} < 10$, about 20% of the bulge mass is needed as a point mass for each galaxy. Since there is still considerable uncertainty regarding the value of the spectrum normalization parameter $\sigma_8$, we investigate the effect of varying this parameter within its entire observational range (from 0.7 to 1.1), and find that low $\sigma_8$ values ($\leq 0.7$) are ruled out, and the best fit value is $\sigma_8 \simeq 1.0$.

Subject headings: cosmology: theory - cosmology: observations - galaxies: bulges - galaxy: center - gravitational lensing

1. Introduction

Gravitational lensing provides us a powerful probe of the mass distributions of the universe. By comparing the lensing probabilities predicted by various cosmological models and the density profiles of lenses with observations, we are able to test the mass distribution of dark matter (CDM) halos and in particular, the inner density slope in the sense that the JVAS/CLASS radio survey (Browne & Meyers 2000; Helbig 2000; Browne et al. 2002; Myers et al. 2002) has provided us the observed lensing probabilities at small image separations ($0.3'' < \Delta \theta < 3''$).
It is well known that, Cold Dark Matter (CDM) model has become the standard theory of cosmological structure formation. The ΛCDM variant of CDM with $\Omega_m = 1 - \Omega_\Lambda \approx 0.3$ appears to be in good agreement with the available data on large scales (Primack 2002). On smaller (sub-galactic) scales, however, there seems to be various discrepancies. Issues that have arisen on smaller scales have prompted people to propose a wide variety of alternatives to the standard CDM model, such as warm dark matter (WDM) and self-interacting dark matter (SIDM). Now that problems arise from galaxy-size halos and the centers of all dark matter halos, high-resolution simulations and observations are the final criterion. Recent highest-resolution simulations appear to be consistent with NFW (Klypin, Zhao, & Somerville 2002; Power, Navarro, & Jenkins 2002) down to scales smaller than about 1 kpc. Meanwhile, a large set of high-resolution optical rotation curves has recently been analyzed for low surface brightness (LSB) galaxies, which suggests that the NFW profile is a good fit down to about 1 kpc. Although further simulations and observations, including the observations of CO rotation curves (Bolatto et al. 2002), may help to clarify the issue, it is likely that both WDM and SIDM are probably ruled out, while the predictions of ΛCDM at small scales may be in better agreement with the latest data.

Unfortunately, if dark halos are modelled with NFW profile on all mass scales, they will produce too few small angular separation images in ΛCDM model as compared with the results of JVAS/CLASS (Li & Ostriker 2002). A possible solution to the problem is to modify the inner structure of dark halos by introducing baryonic cooling and compression (Porciani & Madau 2000; Kochanek & White 2001; Keeton 2001; Sarbu, Rusin, & Ma 2001; Li & Ostriker 2002; Oguri 2002). Namely, there exist two types of dark halos: small mass halos (galactic size) with a steep inner density slope (singular isothermal sphere, SIS) and very massive halos (e.g. galaxy clusters) with a shallow inner slope (NFW). This can reproduce the observational data of JVAS/CLASS (Li & Ostriker 2002; Sarbu, Rusin, & Ma 2001). One may also consider a unified model in which dark halos are composed of an NFW-like component and a bulge component for galaxies as lenses (Chen 2003). If the NFW profile is believed to be universal, this model allow us to constrain the structure of the galactic centers using the strong lensing surveys.

In this paper, we investigate the plausibility of the NFW+Bulge model by fitting the observational data of JVAS/CLASS in a much more accurate way to improve our previous work (Chen 2003). Furthermore, we emphasize the importance of the flux density ratio $q_r$ of the two images produced by a central point mass in each galaxy for the predicted results.

In our model, there are two important issues when galactic bulges are involved: First, the presence of supermassive black holes (SMBHs) at the centers of most galaxies appears by now firmly established (Melia & Falcke 2001, and references therein). SMBH masses are
estimated to be in the range $10^6 - 10^9 M_\odot$, and are correlated with the masses and luminosities of the host spheroids and, more tightly, with the stellar velocity dispersions (Magorrian et al. 1998; Ferrarese & Merritt 2000; Ravindranath, Ho, & Filippenko 2001; Merritt & Ferrarese 2001a,b; Wandel 2002; Sarzi et al. 2002). Recent high-resolution observational data give $M_*/M_{\text{bulge}} \approx 10^{-3}$ (Merritt & Ferrarese 2001c). Furthermore, Ferrarese (2002) finds a relation between masses $M_*$ of SMBHs and the total gravitational masses of the dark matter halos as $(M_*/10^8 M_\odot) \sim 0.046(M_{\text{DM}}/10^{12} M_\odot)^{1.57}$. Since these correlations extend well beyond the direct dynamical influence of the SMBH it seems likely that there is a close link between the formation of both the SMBH and its galaxy (Silk & Rees 1998; Adams, Graff, & Richstone 2001; Haehnelt & Kauffmann 2001; Islam, Taylor, & Silk 2002; Madau & Rees 2001; Menou, Haiman, & Narayanan 2001; Schneider et al. 2002). So we can simply add galactic SMBHs as point masses to the NFW density profile when we calculate lensing probability.

Second, galactic bulges, in which multiple black holes may form and inhabit (e.g., Haehnelt & Kauffmann 2002), can also contribute to the lensing probabilities at small image separations. Since light rays are affected by the mass within the sphere of their impact distances, we can attribute the light deflections induced by such a mass to an effective point mass, which is referred to as $M_{\text{eff}}$ in this paper. We thus avoid the complexity of gas compression and mass distributions of galactic bulges. Of course, we are unable to reveal the detail structures of the inner part of galaxies at the same time. We argue that this may be an adequate way if we want to compare our predictions with the results of strong gravitational lensing survey. So, the total central pointlike masses by including central SMBHs and bulges range from $1 - 1000M_\bullet$. The upper limit of $M_{\text{eff}}$ is $1000M_\bullet$ because the bulge mass correlates linearly with SMBH mass as $M_*/M_{\text{bulge}} \approx 10^{-3}$.

2. Lensing equation and probabilities

We choose the most generally accepted values of the parameters for the $\Lambda$CDM cosmology, in which, with usual symbols, the matter density parameter, vacuum energy density parameter and Hubble constant are respectively: $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.75$. The NFW density profile is $\rho_{\text{NFW}} = \rho_s r_s^3/[r(r + r_s)^2]$, where $\rho_s$ and $r_s$ are constants. We can define the mass of a halo to be the mass within the virial radius of the halo $r_{\text{vir}}$: $M_{\text{DM}} = 4\pi \rho_s r_s^3 f(c_1)$, where $f(c_1) = \ln(1 + c_1) - c_1/(1 + c_1)$, and $c_1 = r_{\text{vir}}/r_s = 9(1 + z)^{-1}(M/1.5 \times 10^{13} h^{-1} M_\odot)^{-1}$ is the concentration parameter, for which we have used the fitting formula given by Bullock et al. (2001).

The surface mass density for a halo as lens is

$$\Sigma(\vec{x}) = M_{\text{eff}} \delta^2(\vec{x}) + \Sigma_{\text{NFW}}(\vec{x}),$$

(1)
where $x = |\vec{x}|$ and $\vec{x} = \vec{\xi}/r_s$, $\vec{\xi}$ is the position vector in the lens plane. $\delta^2(\vec{x})$ is the two dimensional Dirac-delta function, and $\Sigma_{\text{NFW}}(\vec{x})$ is the surface mass density for an NFW profile. $M_{\text{eff}}$ is a point mass ranging from 1 to 1000 times the mass $M_\odot$ of a supermassive black hole (SMBH) inhabiting the center of each galaxy, to describe the contributions of galactic central SMBHs and galactic bulges to lensing probabilities. For galaxy cluster, $M_{\text{eff}} = 0$. The lensing equation for this model is then

$$y = x - \mu_s f_{\text{eff}} + g(x),$$

where $y = |\vec{y}|$, $\vec{\eta} = \vec{y} D_5^A/D_L^A$ is the position vector in the source plane, in which $D_5^A$ and $D_L^A$ are angular-diameter distances from the observer to the source and to the lens, respectively. It should be pointed out that, since the surface mass density is circularly symmetric, we can extend both $x$ and $y$ to their opposite values in Eq.(2) for convenience. The parameter $\mu_s = 4\rho_c r_s/\Sigma_{\text{cr}}$ is independent of $x$, in which $\Sigma_{\text{cr}} = (c^2/4\pi G)(D_5^A/D_L^A D_{LS}^A)$ is critical surface mass density, with $c$ being the speed of light, $G$ the gravitational constant and $D_{LS}^A$ the angular-diameter distance from the lens to the source. The term $f_{\text{eff}} = 2.78 \times 10^{-4} f(c_1) M_{15}^{0.57} (M_{\text{eff}}/M_\odot)$, where $M_{15}$ is the reduced mass of an NFW halo defined as $M_{15} = M_{\text{DM}}/(10^{15}h^{-1}M_\odot)$, stands for the contribution of a point mass $M_{\text{eff}}$, and, of course, $f_{\text{eff}} = 0$ for cluster-size lenses. The function $g(x)$ stands for the contribution of the NFW halo, and it has an analytical expression originally given by Bartelmann (1996).

When the quasars at redshift $z_s = 1.5$ are lensed by foreground CDM halos of galaxies and clusters of galaxies, the lensing probability with image separations larger than $\Delta \theta$ and flux density ratio less than $q_\ell$ is (Schneider, Ehlers, & Falco 1992)

$$P(> \Delta \theta, < q_\ell) = \int_0^{z_s} \frac{dD_L(z)}{dz} dz \int_0^\infty \tilde{n}(M,z)\sigma(M,z)B(M,z)dM,$$

where $D_L(z)$ is the proper distance from the observer to the lens located at redshift $z$. The physical number density $\tilde{n}(M,z)$ of virialized dark halos of masses between $M$ and $M + dM$ is related to the comoving number density $n(M,z)$ by $\tilde{n}(M,z) = n(M,z)(1+z)^3$, the latter is originally given by Press & Schechter (1974), and the improved version is $n(M,z)dM = (\rho_0/M)f(M,z)dM$, where $\rho_0$ is the current mean mass density of the universe, and $f = (0.301/M)(d\ln \Delta_z/d\ln M)\exp(-|\ln(\Delta_z/1.68) + 0.64|^{3.88})$ is the mass function for which we use the expression given by Jenkins et al. (2001). In this expression, $\Delta_z = \delta_c(z)/\Delta(M)$, in which $\delta_c(z)$ is the overdensity threshold for spherical collapse by redshift $z$, and $\Delta(M)$ is the rms of present variance of the fluctuations in a sphere containing a mean mass $M$. The overdensity threshold is given by $\delta_c(z) = 1.68/D(z)$ for the ΛCDM cosmology (Navarro, Frenk, & White 1997), where $D(z) = g[\Omega(z)]/[g(\Omega_m)(1+z)]$ is the linear growth function of the density perturbation (Carroll, Press, & Turner 1992), in which $g(x) = 0.5x(1/70 +$
209x/140 - x^2/140 + x^{4/7})^{-1} and \( \Omega(z) = \Omega_m(1+z)^3/[1 - \Omega_m + \Omega_m(1+z)^3] \). When we calculate the variance of the fluctuations \( \Delta^2(M) \), we use the fitting formulae for CDM power spectrum \( P(k) = A k^2 T^2(k) \) given by Eisenstein & Hu (1999), where \( A \) is the amplitude normalized to \( \sigma_8 = \Delta(r_m = 8 h^{-1} \text{Mpc}) \) given by observations. Note that the mass of an NFW halo is taken to be the virial mass \( M_{\text{DM}} = 4\pi \delta_{\text{vir}} \bar{\rho} r_{\text{vir}}^3 / 3 \), where \( \delta_{\text{vir}} \bar{\rho} \) is the average density within \( r_{\text{vir}} \). For the standard CDM model with \( \Omega_m = 1 \), \( \delta_{\text{vir}} \) is given by the familiar value \( \delta_{\text{vir}} \approx 178 \); for flat \( \Lambda \)CDM cosmology, \( \delta_{\text{vir}} \) can be approximated by \( \delta_{\text{vir}} \approx (18\pi^2 + 82w - 39w^2)/\Omega_m(z) \) with \( w = \Omega_m(z) - 1 \) (Bryan & Norman 1998). Jenkins et al. have specifically stated that their formula gives better fit to \( \frac{\text{d}n}{\text{d}M} \) with \( \delta_{\text{vir}} = 200 \), and thus \( c_1 = r_{\text{vir}}/r_s = r_{200}/r_s \) in our actual calculations, as was originally used by NFW.

The key step in working out the final results of lensing probabilities is how to calculate the lensing cross section \( \sigma(M, z) \) in Eq. (3). Since we are interested in the lensing probabilities with image separations larger than a certain value \( \Delta \theta \) (ranging from 0 ~ 10 arcseconds, for example) and flux density ratio less than \( q_t \), the cross section should be defined under two conditions. The first condition is that multiple images can be created, this condition can be used to define the cross section of cluster-size NFW lenses (i.e., with no central point masses), for which, multiple images can be produced only if \( |y| \leq y_{\text{cr}} \), where \( y_{\text{cr}} \) is the maximum value of \( y \) when \( x < 0 \), which is determined by \( dy/dx = 0 \) when \( f_{\text{eff}} = 0 \) in Eq.(2). So, for cluster-size lenses, the cross section in lens plane is \( \sigma(M, z) = \pi (r_s y_{\text{cr}})^2 \). On the other hand, because of the existence of the central point mass, theoretically, every galaxy-size lens will always produce two images, with the fainter one near the center of the lens, and the other outside the Einstein ring, no matter what the value \( |\bar{y}| \) is. So the first condition fails in this case and we need the second condition to define the cross section of galaxy-size lenses, which is the allowed upper limit of flux density ratio of lensing images in any lensing survey experiments. The flux density ratio \( q_t \) for the two images is just the ratio of the corresponding absolute values of magnifications (Schneider, Ehlers, & Falco 1992; Wu 1996), \( q_t = |\mu_+ / \mu_-| \), where \( \mu_+ [y(x)] = (\frac{\text{d}y}{\text{d}x})_{x>0} \) and \( \mu_- [y(x)] = (\frac{\text{d}y}{\text{d}x})_{x<0} \). So \( y_{\text{cr}} \) is determined by \( |\mu_+ (y_{\text{cr}})| = q_t |\mu_- (y_{\text{cr}})| \). The cross section for images with a separation greater than \( \Delta \theta \) and a flux density ratio less than \( q_t \) is (Schneider, Ehlers, & Falco 1992)

\[
\sigma(M, z) = \pi r_s^2 \vartheta(M - M_{\text{min}}) \times \begin{cases} 
\frac{y_{\text{cr}}^2}{y_{\text{cr}}^2 - y_{\Delta \theta}^2}, & \text{for } \Delta \theta \leq \Delta \theta_0; \\
0, & \text{for } \Delta \theta_0 \leq \Delta \theta < \Delta \theta_{y_{\text{cr}}}; \\
\frac{y_{\text{cr}}^2}{y_{\text{cr}}^2 - y_{\Delta \theta}^2}, & \text{for } \Delta \theta \geq \Delta \theta_{y_{\text{cr}}},
\end{cases}
\]

where \( \vartheta(x) \) is a step function, and \( M_{\text{min}} \) is the minimum mass of halos above which lenses can produce images with separations greater than \( \Delta \theta \). From Eq.(2), an image separation for any \( y \) can be expressed as \( \Delta \theta(y) = r_s \Delta x(y)/D_A \), where \( \Delta x(y) \) is the image separation in lens plane for a given \( y \). So in Eq.(4), the source position \( y_{\Delta \theta} \), at which a lens produces the image
separation $\Delta \theta$, is the reverse of this expression. And $\Delta \theta_0 = \Delta \theta(0)$ is the separation of the two images which are just on the Einstein ring; $\Delta \theta_{y_{cr}} = \Delta \theta(y_{cr})$ is the upper-limit of the separation above which the flux ratio of the two images will be greater than $q_r$. Note that since $M_{DM}(M_{15})$ is related to $\Delta \theta$ through $r_s = (1.626/c_1)(M_{15}^{1/3}/[\Omega_m(1+z)^3 + \Omega_\Lambda]^{1/3})h^{-1}\text{Mpc}$ (Li & Ostriker 2002), we can formally write $M_{DM} = M_{DM}(\Delta \theta(y))$, and determine $M_{min}$ for galaxy-size lenses by $M_{min} = M_{DM}(\Delta \theta(y_{cr}))$, and for cluster-size lenses by $M_{min} = M_{DM}(\Delta \theta(0))$.

In the latter case we have used the fact that the separation of the outermost images is insensitive to the value of $y$ in cluster-size NFW lenses.

To compare the predicted lensing probabilities with the combined data from JVAS/CLASS, magnification bias must be considered. For the JVAS/CLASS sample, we use the result given by Li & Ostriker (2002): $B(M, z) = 2.22A_m^1(M, z)$, where $A_m(M, z) = \Delta x(y = 0)/y_{cr}$.

One major uncertainty in the estimate of $P(\Delta \theta, < q_r)$ by the NFW halo arises from the assignment of the concentration parameter $c_1$ to each halo of mass $M$. There exist several empirical fitting formulae or analytic models to fulfill the task. However, for a given halo mass and redshift, there is a scatter in $c_1 = r_{vir}/r_s$ value that is consistent with a log-normal distribution with standard deviation $\sigma_c = \Delta (\log c) \approx 0.18$ (Jing 2000; Bullock et al. 2001):

$$p(\log c_1|M, z) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp \left\{ -\frac{[\log[c_1/c_{\text{med}}(M, z)]]^2}{2\sigma_c^2} \right\},$$

(5)

where $c_{\text{med}} = r_{\text{vir}}/r_s = 9(1+z)^{-1}(M/1.5 \times 10^{13}h^{-1}M_{\odot})^{-1}$ is the median concentration parameter which has been used earlier. We take into account the scatter in $c_1$ by averaging the lensing probability with the log-normal distribution (the right panel of figure 1 and all the panels of figure 2)

$$\bar{P}(\Delta \theta, < q_r) = \int_0^\infty p(\log c_1|M, z)dc_1 \int_0^{z_s} \frac{dD_L(z)}{dz}dz \int_0^\infty \bar{n}(M, z)\sigma(M, z)B(M, z)dM.$$  

(6)

Another Major uncertainty in predicting $P(\Delta \theta, < q_r)$ arises from the considerable uncertainty regarding the value of the CDM power spectrum normalization parameter $\sigma_8$, so it would be useful to consider the effect of varying this parameter within its entire observational range, roughly $\sigma_8 = 0.7 \sim 1.1$ (see Fig. 2).

3. Discussion and conclusions

The lensing probabilities predicted by Eq. (3) and Eq. (6) and calculated from the combined JVAS/CLASS data are compared in Fig.1. Since we are interested in the degeneracy
Fig. 1.— Predicted lensing probability with image separations $\Delta \theta$ and flux density ratios $q_r$ in $\Lambda$CDM cosmology. The cluster-size lens halos are modelled by the NFW profile, and galaxy-size lens halos by NFW+BULGE. Instead of SIS, we treat the bulge as a point mass, its value $M_{\text{eff}}$ is so selected for each $q_r$ that the predicted lensing probability can match the results of JVAS/CLASS represented by histogram. In the left panel, the value of the concentration parameter $c_1$ is taken to be its median for any given halo mass and redshift. In the right panel, the scatter in $c_1$ is considered by averaging the probability with the well known log-normal distribution. In both panels, $\sigma_8 = 0.95$, and the null result for lenses with $6'' \leq \Delta \theta \leq 15''$ of JVAS/CLASS is shown with the thick dashed horizontal line indicating the upper limit.
between $q_r$ and $M_{\text{eff}}$ in matching the predicted results to observational data, we have calculated the lensing probabilities for four different values of $q_r$ and their corresponding values of $M_{\text{eff}}$, as indicated in Fig. 1. We have assumed a “cooling mass” of $M_c \approx 5 \times 10^{13} M_\odot$, above which the lenses are assigned the NFW profile and below which the lenses are the NFW + point mass. The predicted lensing probabilities are quite sensitive to $M_c$, the value of $M_c$ used here is higher than those preferred by other authors (Kochanek & White 2001; Li & Ostriker 2002; Sarbu, Rusin, & Ma 2001). A lower value (e.g., $M_c \approx 10^{13} M_\odot$) requires a larger $M_{\text{eff}}$ for each $q_r$, however, this will not affect our conclusions, since our main goal is not to “measure” the exact value of $M_{\text{eff}}$. The combined JVAS/CLASS survey is now completed and the VLA observations of 16,503 sources have been carried out, resulting in the largest sample of arcsec-scale lens systems available. Contained within the 16,503 sources is a complete sample of 11,685 sources. A subset of 8,958 sources form a well-defined statistical sample containing 13 multiply-imaged sources (lens systems) suitable for analysis of the lens statistics. One of the four observational selection criteria for this “well-defined” sample is: the image components in lens systems must have separations $\geq 0.3$ arcsec and the ratio of the flux densities of the brighter to the fainter component in double-image systems must be $q_r \leq 10$ (Chae et al. 2002). The observed lensing probabilities can be easily calculated: $P_{\text{obs}}(> \Delta \theta) = N(> \Delta \theta)/8958$, where $N(> \Delta \theta)$ is the number of lenses with separation greater than $\Delta \theta$ in 13 lenses. $P_{\text{obs}}(> \Delta \theta)$ is plotted as a histogram in both panels of Fig. 1.

First of all, as shown in Fig. 1, when averaged over concentration parameter $c_1$ with the log-normal distribution (right panel), the probabilities are increased considerably at larger image separations and only slightly increased at smaller separations for all cases, and the “scatter” among the four cases is reduced. So the scatter in $c_1$ should be considered when one uses the NFW profile to constrain some related parameters.

It is also shown in Fig. 1 that, for low flux ratios ($q_r \leq 10$, as for the JVAS/CLASS survey), the NFW plus a single SMBH model produces far too few small separation lenses (the dash-dot-dot-dot line in the figure). This is confirmed by the fact that, among the 22 confirmed lenses in JVAS/CLASS, none has a fainter image very close to the center of the lens (Browne et al. 2002). In other words, up to date strong gravitational lensing effect of a single SMBH has not been found.

A larger flux ratio requires a smaller fraction of the bulge mass as a point mass. It’s interesting to note that, when $q_r \approx 10^4$ and $M_{\text{eff}} \approx 30 M_\odot$, and when no scatter in $c_1$ is included (the dotted line in the left panel of Fig. 1), the predicted lensing probability fit quite well the JVAS/CLASS results. As pointed out earlier, this conclusion can be equivalently obtained with the model of two populations of halos, the combination of SIS and NFW, when no constraints on flux density ratio $q_r$ are taken into account (i.e., $q_r \sim \infty$). Note that, the
flux density ratios with values as high as \( q_r \sim 10^4 \) and infinity have approximately the same effect on the predicted probabilities. However, since the flux density ratio for the well-defined JVAS/CLASS sample is limited to \( q_r \leq 10 \), the above mentioned good fit cannot be regarded as reflecting the real nature of CDM halos. In fact, to match the JVAS/CLASS results, for \( q_r \leq 10 \), about 20\% of the bulge mass is required as a point mass (i.e., \( M_{\text{eff}} \approx 200M_\bullet \), the solid lines in Fig. 1), while for \( q_r \leq 10^4 \), only 3\% of bulge mass is needed. This difference is very important when one attempts to use the observational results of JVAS/CLASS to constrain the density profile of galactic bulge and/or dark matter halos. It’s well known that, the flux density ratio will reduce the lensing cross section considerably. All the models without considering this would have overestimated the lensing probabilities. In our NFW + Point mass model, we can adjust the value of the point mass to the corresponding flux density ratio to match the observational results, and will never fail. In the NFW+SIS model, however, the contributions of SIS to lensing probabilities cannot be changed. Put in another way, after considering the flux density ratio limited by JVAS/CLASS, NFW+SIS model cannot reproduce the observational results, not even marginally, if some other parameters like \( M_c \) are the same as used here.

We have noted that, the solid lines in both panels of Fig. 1 can only marginally fit the observations of JVAS/CLASS at larger image separations. But it is still acceptable, in the sense that, the curve is lower than the upper limit provided by the JVAS/CLASS survey for \( 6'' \leq \Delta \theta \leq 15'' \) (Phillips et al. 2001). We know that among the total of 22 confirmed lens systems of the JVAS/CLASS survey, 21 of them have image separations between 0.3'' and 3'', and one of them (CLASS B2108+213) has an image separation of 4.6'' (Browne et al. 2002). The null result of the JVAS/CLASS survey for \( 6'' \leq \Delta \theta \leq 15'' \) means that the NFW profile is suitable for all mass sizes of halos, when a certain fraction of the mass of each galactic bulge is considered.

Although we believe that the reasonable match to the observations of JVAS/CLASS is \( q_r \leq 10 \) and \( M_{\text{eff}}/M_\bullet = 200 \) (the solid line in each panel of Fig. 1), it is helpful if we treat the four cases (four upper lines matching the histogram) as a whole to constrain the sensitive parameter \( \sigma_8 \). We plot in each panel of Fig. 2 the averaged lensing probability as a function of image separation \( \Delta \theta \). All the parameters are the same as those in the right panel of Fig. 1 except for \( \sigma_8 \), for which five different values within the entire observational range (from 0.7 to 1.1, as explicitly indicated) have been investigated to see their effect on the predicted lensing probabilities. Clearly, the larger values of \( \sigma_8 \) will produce the higher probabilities. In the case of \( \sigma_8 = 0.7 \), too few lenses are produced at small image separations, so a low value of \( \sigma_8 \) (\( \leq 0.7 \)) is unlikely. The best fit is \( \sigma_8 \sim 1.0 \). This result is very close to that obtained most recently by Bahcall & Bode (2002), in which \( \sigma_8 \) is determined from the abundance of massive clusters at redshifts \( z = 0.5 - 0.8 \). Our result is also in excellent agreement with
Fig. 2.— The same as the right panel of Fig. 1 except $\sigma_8$ for each panel here. From left to right, $\sigma_8$ is 0.7, 0.8, 0.9, 1.0 and 1.1, respectively.
that of Komatsu & Seljak (2002) ($\sigma_8 = 1.04 \pm 0.12$ at 95%) suggested by the excess cosmic microwave background fluctuations detected on small scales by the CBI (Mason et al. 2002) and the BIMA (Dawson et al. 2002) experiments.

In summary, strong gravitational lensing probability is quite sensitive to the cooling mass $M_c$, concentration parameter $c_1$, flux density ratio $q_r$ and the spectrum normalization parameter $\sigma_8$. Including the scatter in $c_1$ in the calculations would increase the lensing probabilities considerably at larger image separations, while neglecting $q_r$ would overestimate the lensing probabilities. In our NFW + point mass model, 20% of the galactic bulge mass is required as a point mass to match the JVAS/CLASS results; the low values of $\sigma_8$ ($\leq 0.7$) are ruled out, and the preferred value in our model is $\sigma_8 \approx 1.0$.

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