we can obtain the same results. With $\mathbf{2}$ and $\mathbf{3}$, the Hubble parameter is $(\Omega_m)^2 + (\Omega_r)^2 = H_0^2$. The Hubble parameter is the product of $H_0$, the Hubble constant, and the universe's density parameter, which is the ratio of the total energy density of the universe to the critical density of the universe.

In a contracting universe (with a positive cosmological constant) we have:

$$\Omega_m H^2 = 1.$$  

(1)  

$$\mathbf{y} = \mathbf{y}_{\mathbf{1}}, \quad \mathbf{y}_{\mathbf{1}} = \mathbf{y}, \quad \mathbf{y}_{\mathbf{2}} = \mathbf{y}$$

written

Then a solution to the universe's Hubble parameter can be obtained:

$$\mathbf{H} = H_0 \mathbf{y}.$$  

Therefore, the Hubble parameter of the universe is $H_0$. The Hubble constant, $H_0$, is the ratio of the universe's density parameter to the critical density of the universe, which is the ratio of the total energy density of the universe to the critical density of the universe.

**II. EXCITATION-NO-EXCITATION QUANTUM**

The Hubble parameter of the universe is $H_0$. The Hubble constant, $H_0$, is the ratio of the universe's density parameter to the critical density of the universe, which is the ratio of the total energy density of the universe to the critical density of the universe.

**III. INTRODUCTION**

Non-aberrable geometric quantization gates in semiconductor quantum dots...
three lasers in which change the intensity and the phase during the evolution.

For the two-qubit gate we have to exploit qubit-qubit interaction in order to construct non-trivial operators; then every system has different implementation of such gates. Since we work with semiconductor excitons we use exciton-exciton dipole interaction.

Let us consider two dots with exciton energy \( \omega_0/2 \) (the energy is rescaled in order to have \(- \omega_0/2 \) for the ground states). If the two dots are coupled the presence of an exciton in one of them causes a energetic shift \( (\delta) \) in the other because of the dipole-dipole interaction. States with a single exciton are not shifted. The energy levels are shown in figure 1. The Hamiltonian accounting for the biexcitonic shift is \( H_{\text{0}} = (\omega_0 + \delta)|EE\rangle\langle EE| - \omega_0|GG\rangle\langle GG| \).

The dipole interaction between dots can be used to construct non-trivial two-qubit gates both dynamical \( \mathbf{12} \) and geometrical \( \mathbf{10} \). In fact, if we use two lasers tuned to the two-exciton state transition \( \omega_1 = \omega_2 = (\omega_0 + \delta)/2 \), we can avoid single photon processes (which produce \( \left| EE \right\rangle \) and \( \left| GG \right\rangle \) states) and favour only two-photon processes (which produce \( \left| EE \right\rangle \)).

The effective interaction Hamiltonian for the two-photon process is:

\[
H_{\text{int}} = -\frac{2\hbar^2}{\delta} \frac{1}{\sqrt{2}} e^{-i(\omega_{L,1}+\omega_{L,2})} e^{-i(\phi_1+\phi_2)} |EE\rangle\langle GG| \right|^2 + \text{h.c.} \tag{4}
\]

where \( \omega_{L,i} e \phi_i \) are the frequency and the phase of the laser \( i \).

The total Hamiltonian is similar to the one in \( \mathbf{11} \) and then using a properly chosen sequence of synchronous pulses (so that the two-photon Rabi frequencies in \( \mathbf{11} \) simulate the one in \( \mathbf{10} \)), we can apply a phase gate similar to \( \mathbf{\mathbf{17}} \) and complete the universal set of quantum gates.

### III. EXCITON SPIN QUBIT

A further excitonic encoding can be obtained following the spin-based scheme presented in \( \mathbf{18} \). There a four-level system with three degenerate excited states (\( |E^\pm\rangle \)
and $|E^0\rangle$) and a ground state ($|G\rangle$) was used; the excitonic states were connected with $|G\rangle$ by three different lasers with circular ($\pm$) and linear (along $z$ axis) polarization and, modulating the phase and the frequency of the three lasers, we were able to construct adiabatic holonomic gates.

To obtain non-adiabatic geometrical gates in this system the basic idea is to encode logical information in two degenerate exciton states with different total angular momentum i.e. $|E^\pm\rangle$. The extension of the previous gating model is not completely straightforward; in fact the logical qubits $|E^+\rangle$ and $|E^-\rangle$, due to angular-momentum conservation in radiation-matter interaction, are not directly i.e., by a one-photon ladder operators, connected.

In order to circumvent this problem and to enact such a ladder operator one can resort to an off-resonant two-photon Raman process. This is a standard trick in quantum optics. Each quantum dot is shinied by a couple of lasers having polarizations + and − and a frequency with a detuning $\Delta$ with respect the excitonic transition energy. The level scheme with the associate transition is shown in Fig. B. Provided that $\Omega_\pm \ll \Delta$ (the $\Omega_\pm$’s are the laser Rabi frequencies) first order processes are then strongly suppressed; the dynamics is well-described by the following second-order effective Hamiltonian

$$H_{eff} = \frac{\Omega_+ \Omega_-}{\Delta} |E^+\rangle\langle E^-| + h.c.$$  \hspace{1cm} (5)

It should be now clear –since the above Hamiltonian has the same structure as (1) – that even for this kind of excitonic encoding using different polarizations one can realize all the required single-qubit operations.

Another single qubit gate that can be implemented easily is the phase shift gate. Our scheme has a priori separated sub-spaces because the different resepose to polarized laser. So if we want $|E^+\rangle$ to get a phase factor, we can just switch the + laser to resonant frequency, and then apply the pulse sequence that produce gate 2. Since we cannot neglect the phase accumulated by $|G\rangle$ and no phase is accumulated by $|E^-\rangle$ the gate operator will be $U = \exp(i\tilde{\gamma}|E^+\rangle\langle E^+|)$ where, as before, $\tilde{\gamma}$ is half the solid angle swept in the evolution. These two gates complete the single-qubit gate set.

Finally, to obtain a universal set of quantum logical gates we must construct a two qubit gates. The easiest to be implemented in our model is a selective phase gate. As shown before using lasers resonant with the two exciton with positive polarization we can select two-photon processes and couple only the $|E^+ E^+\rangle - |GG\rangle$ states. The effective Hamiltonian for these two-photon processes is similar to (1) with $|E^+\rangle$ instead of a generic exciton state $|E\rangle$.

The two lasers are polarized with + polarization and follow the pulse sequence for gate 1; the final geometric operator will be $U = \exp(i\tilde{\gamma}|E^+ E^+\rangle\langle E^+ E^+|)$, where $\tilde{\gamma}$ is half the angle swept on the Bloch sphere in the $|E^+ E^+\rangle - |GG\rangle$ space.

A few remarks are now in order regarding the different kind of excitonic polarization we have considered so far. In the second -polarization-based - encoding we need a more laser pulses (and then longer time for the
application of the gates) respect to the model with the first scheme with non-polarized excitons. This makes the set-up slightly more complicated but now the logical 1 and 0 states corresponds here to energetically degenerate states with the same orbital wave function structure. This facts should 1) make the qubit more robust against pure dephasing processes 2) set to zero the qubit self-Hamiltonian i.e., the σ_z component allowing for a simplified gate design and then no recoupling pulse are required.

On the other hand it should be be noted that in in the second scheme both the codewords correspond to unstable states, indeed excitons will eventually recombine through the semiconductor gap by emitting a photon. On the contrary in the first encoding scheme the logical 0 corresponds to the ground state $|G\rangle$ of the crystal, and it is therefore a stable state.

Exciton recombination corresponds in the first scheme to the amplitude-damping process $|1\rangle \rightarrow |0\rangle$. One can take care of this kind of environment-induced error by the both the techniques of quantum error correction or error avoiding depending on the spatial symmetry of the damping process. Using polarization encoding spontaneous decay gives rise to leakage to the computational subspace in the the ground state of the crystal $|G\rangle$ is no-longer a computational codeword. In this case one can resort to leakage-elimination strategies based of active intervention on the system.

IV. SIMULATIONS

To test our models we performed numerical simulations of the quantum gates solving the Schroedinger equation. For the first model (with no polarized excitons) we took $|E\rangle$ as starting state and then simulate the evolution when we apply the pulse sequences presented. In Fig. 4 the result of the simulation for gate 1 are shown; the parameters are chosen in order to obtain a NOT gate. In Fig. 4 (A) the curve traversed by the state in the Bloch space and (B) the population evolutions are presented. Once decided which gate to apply we can have an estimate of the gate time. For this NOT gate the laser frequency is not resonant and is constrained by the gate choice $\omega_L = \omega_0 - 2\Omega$; the time gate is fixed by the Rabi frequency of the laser. For realistic laser parameters ($\Omega^{-1} = 50 fs$) we have: $t_{gate} = 0.1$ ps.

In Fig. 5 we show (for gate 2) the loop in the Bloch space (A) the population evolutions and the phase accumulated during the evolution (inset) (B). The parameters are chosen in order to obtain $\gamma = \pi/4$ and the final state is $(1+i)/\sqrt{2}E$. The laser frequency is resonant with the transition $\omega_L = \omega_0$ and with the same Rabi frequency used before we have: $t_{gate2} = 0.15$ ps.

In the second model first we have to test the validity of the approximation used in 4. For this purpose we simulated the evolution of the three-level system showed in Fig. 4 and show the result in Fig. 6. We choose $\Delta/\Omega = 10$ ($\Omega_+ = \Omega_- = \Omega$) and, as we can see, this is sufficient to avoid population of $|G\rangle$ state and to have the standard Rabi oscillations between the logical states.

We note that, because of the perturbative request in the effective magnetic field $B$ has small x and y component, and then a sequence of two $\pi$-pulse is not sufficient to construct a generic superposition of logical qubits. Even if the geometrical phase accumulated during the loop is small it is sufficient to iterate the procedure to apply the desired geometrical operator. Using the same perturbation parameter as in 4 we simulate the evolution of $|E^\pm\rangle$. In Fig. 6 we show the population evolutions of the states $|E^+\rangle - |E^-\rangle$ when they are subjected to a $\pi$-pulse sequence in order to obtain a NOT gate. Of course the gate timing in this situation depends on which gate we want to apply and the parameter used the model.

V. CONCLUSIONS

In summary, we proposed two approaches to geometric non-adiabatic quantum information processing in semiconductor quantum dots. In both cases we have been able to construct a universal set of quantum gates using the Aharonov-Anandan phase. In the first scheme the qubit is realized by the presence or absence of a (ground) state
exciton. A coupling with an external laser field allows for the non-adiabatic realization of the geometrical-gates. The dipole-dipole coupling between excitons plays an essential role in action of the entangling two-qubit gate.

In the second approach we encode information in degenerate states using, as quantum degree of freedom, the polarization i.e., total spin, of the excitons ($|E^\pm\rangle$). The logical states are not directly connected but we showed, first how to avoid this problems with two-photon (Roman) transition and second how to implement in this way a selective phase gates (for one and two qubits). Numerical simulations with realistic parameters show that these gates can be in principle enacted within the decoherence time. The models for non-adiabatic (fast) QIP presented in this paper combine the features of geometrical gates with the ultra-fast gate control possible in semiconductor nanostructures; an experimental verification of these schemes seems under the reach of current technology.