Brane inflation and reheating

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ABSTRACT: We study inflation and reheating in a brane world model derived from Type IIA string theory. This particular setup is based on a model of string mediated supersymmetry breaking. The inflaton is one of the transverse scalars of a D4-brane which has one of its spatial dimensions stretched between two NS5-branes, so that it is effectively three-dimensional. This D4-brane is attracted to a D6-brane that is separated from the 5-branes by a fixed amount. The potential of the transverse scalar due to the D4/D6 interaction makes a good inflaton potential. As the D4-brane slides along the two 5-branes towards the 6-brane it begins to oscillate near the minimum of the potential. The inflaton field couples to the massless Standard Model fields through Yukawa couplings. In the brane picture these couplings are introduced by having another D6'-brane intersect the D4-brane such that the 4-6' strings, whose lowest lying modes are the Standard Model matter, couple to the scalar mode of the 4-4 strings, the inflaton. The inflaton can decay into scalar and spinor particles on the 4-6' strings, reheating the universe. Observational data is used to place constraints on the parameters of the model.

KEYWORDS: D-branes, Cosmology of Theories beyond the SM.

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1. Introduction

Recently there has been significant progress made in the field of string cosmology (see e.g. [1], [2]). Arguably, among the greatest triumphs are the stringy realizations of the inflationary universe paradigm. In this paper we construct an inflationary brane world scenario from Type IIA string theory. The novel feature of this model is the reheating mechanism. Our brane world inflates during its motion through a bulk spacetime. In this way, the setup shares some of the features of “Mirage Cosmology” [5]. The motion is caused by the exchange of massless closed strings (gravitons) between the D4-brane, whose transverse scalar plays the role of the inflaton, and a bulk D6-brane. Inflation ends when the D4-brane begins to rapidly oscillate in the bulk about the minimum of the inflaton potential. The inflaton field couples via Yukawa interactions to the massless Standard Model (SM) fields that live on strings stretched between the inflaton D4-brane and another D6'-brane. The inflaton also couples gravitationally to KK modes and massive string states. We find that the reheat temperature is too low to excite massive closed string states. Oscillations of the D4-brane excite the 4-6' strings of which the lightest modes are the Standard Model fields. The inflaton \( \phi \) decays into scalar \( \chi \) and spinor fields \( \psi \) on the 4-6' strings during the reheat process. The

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1For references to such models see [1, 2] and the references therein.
2Other inflationary models based on IIA string theory are [3] and [4].
3Inflation in Mirage Cosmology was studied in [5, 6].
interaction Lagrangian giving rise to these decays appears naturally in string theories. The inflationary phase is analyzed using the standard slow-roll approximations (see, e.g. [12]). Observational data is used to place constraints on the parameters of the model. Finally, we briefly comment on the nature of the dark matter in this scenario.

2. Brane dynamics

Our concrete starting point involves the branes of ten-dimensional Type IIA string theory. In [14], a similar configuration was considered within the context of “string mediated” supersymmetry breaking in a Hanany-Witten model with rotated D6-branes. This model allows us to break supersymmetry at the string scale in some very heavy non-MSSM messenger fields (the 4-6 strings) and then communicate the SUSY breaking to the inflaton field living on the D4-brane world at a lower scale. Although the original motivation for string mediation was to communicate supersymmetry breaking to the Standard Model fields, here we use it to generate a potential for the inflaton field as well.

Consider a D4-brane along the directions 1236, stretched in the 6-direction between two NS5-branes extended in 12345. We take the distance between the NS5-branes, \(L_6\), to be finite. Separated from the NS5-branes in the 7-direction is a D6-brane extended in 123689. The directions 456789 are compact so that we have 3 + 1 dimensional gravity coupled to the massless fields on the brane (i.e. the D4-brane appears 4-dimensional as long as we consider energies smaller than \(L_6^{-1}\)). This setup is depicted in Fig. 1. \(^4\)

Here we discuss the details of this brane configuration. The background metric and dilaton for the D6-brane are given by

\[
d s^2 = f^{-1/2} d\vec{x}_\parallel^2 + f^{1/2} d\vec{y}_\parallel^2 \\
 e^{-2\Phi} = f^{3/2}
\]

\[
f(r) = 1 + \frac{g_s \sqrt{\alpha'}}{2r} ,
\]

where \(x_\parallel\) are the coordinates parallel to the brane and \(y\) are the coordinates perpendicular to the brane. The fluctuations of the D4-brane are described by the Dirac-Born-Infeld action

\[
S = -\int e^{-\Phi} T_4 d^5x \sqrt{-\det(G_{ab} + B_{ab} + 2\pi \alpha' F_{ab})} ,
\]

where \(T_4 = \frac{M_5^5}{(2\pi)^3 g_s}\) is the tension of the D4-brane. Here \(G_{ab}\) and \(B_{ab}\) are the pull-back of the spacetime metric and anti-symmetric 2-form field to the brane and \(F_{ab}\) is the gauge field living on the brane \([13]\). The background values of the antisymmetric 2-form and the gauge field on the D4-brane are taken to be zero (\(F = B = 0\)). The metric \(G\) of the D4-brane is induced by the metric of the D6-brane

\[
G_{ab} = h_{ab} + d_a X^i d_b X^j g_{ij} = f^{-1/2} q_{ab} + f^{1/2} d_a X^i d_b X^j ,
\]

\(^4\)Note that in the complete setup we include orientifold planes, which will be discussed below and are not included in the figure.
where the $X^i$ are the coordinates transverse to the brane. The indices take on the values $i = 4, 5, 7, 8, 9$ and $a = 0, 1, 2, 3$. The determinant of the metric can be written
\[
\det G_{ab} = \det(h_{ac}\eta_{cb})\det(\eta + h^{ab}g_{ij}d_ad_bX^iX^j). \tag{2.6}
\]
For small fluctuations in the $X$ coordinates, we can write
\[
\det G = \det(h)\det(\eta)(\det\eta + Tr h^{-1}gdXdX + \cdots) = f^{-\frac{5}{2}}(-1 + f dXdX + \cdots) \tag{2.7}
\]
The Lagrangian for the fluctuations of the dimensionally reduced D4-brane in the background of a D6-brane is, therefore,
\[
S = \int d^4x \sqrt{HT}4L_6(-f^{-1/2} + \frac{1}{2}f^{1/2}H^{-1}d_aX^4d_aX^4 + \cdots), \tag{2.8}
\]
where $H$ is the (flat) metric seen by the low energy fields on the brane and we have dimensionally reduced the D4-brane in the $x^6$ direction by placing the D4-brane in the interval $(L_6)$ between two NS 5-branes. In the above, we focus on the $X^4$ coordinate along the NS5-brane because $\phi \equiv M_s^2X^4$ acts like an inflaton on the D4-brane. Let us define, $S \equiv M_s^2r$, where $r$ is the distance between the 6-brane and the 4-brane. We take the D4-brane to lie along the NS5-branes separated a distance $A\alpha'$ from the D6-brane where $A >> M_s$. $A$ here is nondynamical in the limit we are considering. Notice that $S = \sqrt{\phi^2 + A^2}$ never reaches zero for any value of $\phi$. Hence, the 4-6 string can never be shorter than the string scale and will not become tachyonic. Using these definitions and substituting the expression for the harmonic function $f (2.3)$, into the action (2.8) gives the Lagrangian of the brane in terms of $\phi$,
\[
\mathcal{L}_{brane} = T4L_6\left(\frac{1}{2}M_s^{-4}\sqrt{1 + \frac{g_sM_s}{2\sqrt{\phi^2 + A^2}}(\partial\phi)^2 - \frac{1}{\sqrt{1 + \frac{g_sM_s}{2\sqrt{\phi^2 + A^2}}}}}\right). \tag{2.9}
\]

3. Brane cosmology

In order to discuss cosmology on the brane we need the complete action on the brane. The full Type IIA bulk Lagrangian is
\[
S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\left\{e^{-2\phi}\left[R + 4|d\phi|^2 - \frac{1}{2}|H|^2\right] - \frac{1}{2}|G_2|^2 - \frac{1}{2}|\tilde{G}_4|^2\right\} + \frac{1}{4\kappa_{10}^2}\int \left\{\frac{1}{2}G_4G_4B - \frac{1}{2}G_2G_4B^2 + \frac{1}{6}G_2^2B^3\right\}, \tag{3.1}
\]
where the gauge invariant field strengths are given by

\[ H = dB, \]
\[ G_2 = dC_1, \]
\[ \tilde{G}_4 = dC_3 - C_1 \wedge H. \]  

(3.2)

In the above, \( \Phi \) is the dilaton, \( B \) is NS-NS antisymmetric tensor 2-form potential, \( C_1 \) and \( C_3 \) are the Ramond-Ramond 1-form and 3-form potential respectively. Dimensionally reducing to four-dimensions on a torus of radius \( R \) and focusing on the most relevant massless modes (the graviton and the dilaton), the bulk action becomes

\[
S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left\{ e^{-2\Phi} R + 4|d\Phi|^2 \right\} 
\]

(3.3)

where

\[
\frac{1}{2\kappa_4^2} = \frac{R^6}{2\kappa_{10}^2} 
\]

(3.4)

A conformal transformation of the metric takes us from the string frame to the Einstein frame.

\[
g^a_{\mu\nu} = g^E_{\mu\nu} e^{\phi/2} 
\]

(3.5)

In this frame Newton’s constant is independent of time and position. The bulk action becomes

\[
S_E = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\tilde{g}} \left\{ R + 2|d\Phi|^2 \right\}. 
\]

(3.6)

The full action becomes

\[
S = \int d^4x \sqrt{-g} \left( -\frac{R}{2\kappa_4^2} + \mathcal{L}_{D4-brane} + \mathcal{L}_{D6-brane} + \mathcal{L}_{NS5-brane} + \mathcal{L}_{O6-brane} + \mathcal{L}_{O5-brane} \right),
\]

(3.7)

where the first term is just the Einstein-Hilbert action and the second term is given by equation (2.9). We comment on the other contributions to the action below. Strictly speaking the D4-brane action given by equation (2.9) is in the string frame which is incompatible with the Einstein-Hilbert action. However, we will always be working in a regime where the dilaton is approximately constant, that is to say the large \( r \) limit of dilaton profile in equation (2.2).

In this limit there is only an order 1 difference between the D4-brane action in the string frame and the Einstein frame. 5

The D6-brane and the NS5-branes have scalar fields on their world volumes that can act as “inflaton” fields. However, these branes are heavier than the D4-branes and therefore do not move significantly on the time scales considered here. The three-dimensional energy density of the D6-brane compared to that of the D4-brane can be calculated using numerical values we find at the end of this paper. We find

\[
\frac{\rho_{D6}}{\rho_{D4}} = \frac{(M_7^7/g_s) R_6 R_8 R_9}{(M_5^5/g_s) L_6} = 10^2
\]

(3.8)

5The D4-brane in Einstein frame is \( S = \int d^4x \sqrt{H} T_4 T_6 \left( -f^{-19/8} + \frac{1}{2} f^{-5/8} H^{-3} dXdX + \ldots \right) \).
Since acceleration is two time derivatives of position we expect that the time scales on which the branes move will go like the square root of the mass density.

\[
\frac{t_{D6}}{t_{D4}} \simeq \sqrt{\frac{\rho_{D6}}{\rho_{D4}}} = 10
\]  

(3.9)

The D6-brane and the NS5-branes also carry tension which couples to gravity and can contribute to inflation. In order to eliminate their effects we have added orientifold planes which have negative tensions. Orientifolds do not fluctuate and so they do not introduce any new dynamical fields.

For the physics we are interested in, the only dynamically relevant scalar field in the brane configuration is the motion of the D4-brane. Note that in the above action we have made a critical assumption: the moduli of the compact space and the dilaton are stabilized by some unknown physics. 6 We will comment further on this point below.

We take \( \phi \) to be spatially homogeneous and time dependent and the metric on the brane is the Friedmann-Robertson-Walker (FRW) metric

\[
ds^2 = n(t)^2 dt^2 - a(t)^2 \left[ \frac{dr^2}{\sqrt{1 - kr^2}} + r^2 d\Omega^2 \right],
\]

(3.10)

where \( n \) is the lapse function, \( a \) is the scale-factor on the brane and \( d\Omega^2 \) is the metric on the unit sphere. For simplicity, we will assume a flat universe with \( k = 0 \). 7

The equations of motion on the brane are determined by varying the action (3.7) with respect to \( n, a \) and \( \phi \). Varying with respect to the lapse function (which we set to unity in the EOM) gives

\[
H^2 = \frac{\sqrt{2 \kappa_4^2 T_4 L_6 \left( 2(A^2 + \phi^2)(2M_s^4 + \dot{\phi}^2) + g_s M_s \sqrt{A^2 + \phi^2 \dot{\phi}^2} \right)}}{12M_s^4(A^2 + \phi^2)^{1/2} + \frac{g_s M_s}{\sqrt{A^2 + \phi^2}}},
\]

(3.11)

where \( H \equiv \dot{a}(t)/a(t) \) is Hubble parameter. It is easy to see from equation (3.11) that, in the slow-roll regime (large \( \phi \) and small \( \dot{\phi} \)), inflation will occur on the brane

\[
a(t) = \exp \mathcal{H} t,
\]

(3.12)

where \( \mathcal{H} = \kappa_4 \sqrt{T_4 L_6}/3 \).

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6After the completion of this paper we became aware of some recent progress made in solving the problem of moduli stabilization [10, 11].

7Note that during inflation the effects of non-zero \( k \) will be very quickly inflated away because the terms containing \( k \) in the Friedmann equation are diluted by \( a^{-2}(t) \).
3.1 Slow-roll inflation

The standard approximation technique for analyzing inflationary models is the slow-roll approximation. We assume that the inflaton $\phi$ is initially large. At this point, $\phi$ is on a relatively flat part of its potential, and therefore, $\dot{\phi}$ is small. A plot of the entire potential $V(\phi)$ is given in Fig. 2. In the slow-roll regime $\phi$ satisfies $g_s M_s / 2 S \ll 1$, ($\phi \simeq S$), and equation (2.9) becomes

$$L_{brane} \approx T_4 L_6 \left( \frac{1}{2} M_s^{-4} (\partial \phi)^2 - (1 - \frac{g_s M_s}{4 \phi}) \right).$$

(3.13)

By rescaling $\phi$ so that $M_s^2 \phi = \sqrt{T_4 L_6} \phi$, and defining $M \equiv g_s (T_4 L_6)^{3/2} / 4 M_s$, the full Lagrangian on the brane becomes

$$L_{tot} \simeq \sqrt{-g} \left( -\frac{R}{2 \kappa_4} + \frac{1}{2} (\partial_a \phi)^2 - V(\phi) \right),$$

(3.14)

where

$$V(\phi) = T_4 L_6 - \frac{M}{\phi}.$$  

(3.15)

From the potential (3.15), we can calculate the slow-roll parameters

$$\varepsilon \equiv \frac{1}{2} M_P^2 (V'/V)^2 \simeq \frac{1}{2} \left( \frac{M_P M}{L_6 T_4 \phi^2} \right)^2,$$

(3.16)

$$\eta \equiv M_P^2 (V''/V) \simeq -\frac{2 M M_P}{L_6 T_4 \phi^3},$$

(3.17)

where $M_P$ is the four-dimensional Planck mass and the prime denotes differentiation with respect to $\varphi$. In order for the slow-roll approximation to be valid, the inflaton must be on a region of $V$ which satisfies the flatness conditions $\varepsilon \ll 1$ and $|\eta| \ll 1$. Inflation ends when $\varphi = \varphi_{end}$ (when $|\eta|$ becomes of order one),

$$\varphi_{end}^3 = \left( \frac{2 M_P^2 M}{L_6 T_4} \right).$$

(3.18)

The amount of inflation is given by the ratio of the scale factor at the final time $t_{end}$ to its value at some initial time $t_i$. The total number of e-folds is

$$N \equiv \ln \frac{a(t_{end})}{a(t_i)} = \int_{t_i}^{t_{end}} H dt \simeq \int_{\varphi_{end}}^{\varphi_{end}} \frac{V}{V'} d\varphi \simeq \frac{2}{3} \left( \frac{\varphi}{\varphi_{end}} \right)^3 = \frac{L_6 T_4}{3 M P^2} \varphi^3.$$

(3.19)

We take the desired number of e-foldings to be $N = 58$. This implies the value of $\varphi$ when interesting scales cross outside the horizon is

$$\varphi \simeq 4.43 \varphi_{end}.$$  

(3.20)
The power spectrum measured by COBE at the scale \( k \simeq 7.5H_0 \) is \[ \delta_H \equiv \frac{2}{5}P_R^{1/2} = 1.91 \times 10^{-5}. \] (3.21)

In terms of the potential \( V \), we have

\[
\delta_H^2(k) = \frac{1}{75\pi^2M_P^6V'^2}V^3,
\]

where \( V \) and \( V' \) are evaluated at the epoch of horizon exit for the scale \( k = aH \). Using this expression it is possible to place constraints on the parameters of the potential,

\[
M_P^{-3}\frac{V^{3/2}}{V'} \simeq 5.3 \times 10^{-4}.
\]

(3.23)

Using (3.23) we find

\[
L_6 \propto \left( \frac{g_s}{N} \right)^2 \left( \frac{M_P}{M_s} \right)^5 \frac{1}{M_s}.
\]

(3.24)

The spectral index, defined by \( n - 1 = \frac{2}{\eta} - 6\epsilon \), is shifted slightly to the red (for \( N > 4/3 \), \( n \simeq 1 - 4/3N \)). For \( N = 58 \) the spectral index is \( n \simeq .98 \). The variation of \( n \) with respect to wave number \( k \) is

\[
\frac{dn}{d\ln k} = 2\xi^2 = 2M_P^3 V'V'' \simeq \frac{4}{3N^2} = .0004.
\]

(3.25)

### 3.2 Cosmological constant

When \( \phi = 0 \) the brane world reaches the bottom of its potential and we may approximate the Lagrange density (2.9) as

\[
\mathcal{L}_{brane} \simeq T_4L_6\left( \frac{1}{2}M_s^{-4}(c_1 - c_2\phi^2)(\partial\phi)^2 - \left( \frac{1}{c_1} + c_3\phi^2 \right) \right),
\]

(3.26)

\[ c_1 = \sqrt{1 + \frac{g_sM_s}{2A}} = \frac{g_sM_s}{8A^3 c_2^{-1}} = \left( \frac{g_sM_s}{8A^3 c_3} \right)^{1/3}. \]

(3.27)

Here we are confronted with a non-standard kinetic term, \( \mathcal{L}_{kin} \propto \phi^2(\partial\phi)^2 \). To place the Lagrangian into canonical form, we define

\[
\partial\Psi = M_s^{-2} \sqrt{T_4L_6c_1(1 - \frac{c_2}{c_1})} \partial\phi.
\]

(3.28)

In terms of the new inflaton variable \( \Psi \), equation (3.26) is

\[
\mathcal{L}_{brane} \simeq \frac{1}{2}(\partial\Psi)^2 - m^2\Psi^2 - \Lambda,
\]

(3.29)

where \( m^2 = M_s^2 \frac{T_4L_6}{c_1}c_3 \). Note the presence of a non-zero cosmological constant \( \Lambda = T_4L_6/c_1 \).

We do not attempt to address the cosmological constant problem in this paper. One way
to avoid this problem is to fine-tune away $\Lambda$ by adding a negative cosmological constant to equation (3.7). This might be achieved in string theory by embedding our brane set-up in $AdS_4 \times S^7$. A sketch of the entire inflaton potential is obtained by considering equations (3.15) and (3.29) (see Fig. 2).

4. The string scale

Empirical evidence (and assuming the MSSM is correct) seems to indicate that the gauge couplings of the Standard Model unify at the GUT scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV, $\alpha_{GUT} \simeq 1/25$ [15]. After compactification to four-dimensions, the Planck mass $M_P = (8\pi G N)^{-1/2} = 2.42 \times 10^{16}$ GeV, is determined by dimensional reduction (see e.g., [16]),

$$g_s^2 M_P^2 = \frac{M_s^8 V_{6d}}{(2\pi)^6 \pi},$$

(4.1)

where $V_{6d}$ is the 6d compactification volume. The one compact dimension of the D4-brane we live on has volume $2\pi R_6$. The factor of $(2\pi)^6 \pi$ in the above comes from $G_{10}$ in type I string theory [17],

$$8\pi G_{10} = \frac{g_s^2 (2\pi)^7}{2 M_s^8}.$$  

(4.2)

The string coupling $g_s = e^{\langle \Phi \rangle}$ is related to the gauge coupling by

$$g_s = 2 M_s (2\pi L_6)(2\pi)^{-1} \alpha_s.$$  

(4.3)

Using equations (3.23) and (4.3) it is possible to determine the relation between $M_s$ and $\delta_H$. At the perturbative level, string theory does not stabilize the dilaton. It is therefore likely that such stabilization will arise from non-perturbative effects (i.e when $g_s \gtrsim 1$). Taking $g_s = 1$, $\alpha \simeq 1/25$ and $N = 58$, we find

$$M_s \simeq 6.5 \times 10^{15} \text{ GeV},$$

(4.4)

which is within the allowed range, between 1 GeV and $10^{16}$ GeV [18, 19]. (Note that for $N \lesssim 20$, it is possible to have $M_s \simeq M_{GUT} \simeq 10^{16}$ GeV.) Using this value of $M_s$ and equation (4.3), we find $L_6 = (25/2) M_s^{-1}$.

The branes have a thickness of order the string scale. So that the branes have enough room to move freely in the extra dimensions we take $R_5, R_7, R_8$ and $R_9$ to be large (i.e., $R_i > M_s^{-1}$, where $i = 5, 7, 8, 9$). For simplicity we assume that all $R_i \simeq R$. We assume that the remaining compact dimensions ($R_4$ and $R_6$) are compactified with volume $(2\pi/M_s)$ (i.e.,
at the self-dual value) and $2\pi L_6$ respectively. Note that if the radius is less than the self-dual value ($R_4 < M_s^{-1}$), then the T-dual description is more appropriate. In the T-dual scenario,

$$gs \rightarrow g_s R_s^{-1} M_s^{-1}, \quad R_4 \rightarrow R_4^{-1} M_s^{-2}. \quad (4.5)$$

Here the T-dual torus has $R_4 \geq M_s^{-1}$. Therefore, it is possible to consider the cases where $R_4 \geq M_s^{-1}$, with $R_4 = M_s^{-1}$.

The entire six-dimensional compact space has volume

$$V_{6d} = (2\pi M_s^{-1})(2\pi L_6)(2\pi)^4(R_5 R_7 R_8 R_9). \quad (4.6)$$

From equation (4.1), we find

$$R_4 \simeq \left(\frac{M_P}{M_s}\right)^2 \frac{2\pi}{25M_s^4}, \quad (4.7)$$

from which $R_5^{-1}, R_7^{-1}, R_8^{-1}, R_9^{-1} \simeq 4.8 \times 10^{14}\text{ GeV}$. Note that in general, as long as all of the compactified dimensions are larger than the string scale by at least a factor of two, cosmologically dangerous defects will be too heavy to be produced.

5. Reheating the brane

There are now several different realizations of brane inflation. The details of reheating in these models has not been fully explored, although in general inflation ends due to tachyon condensation (either by brane-antibrane annihilation or brane-brane collisions). The process of tachyon condensation may lead to severe cosmological problems. Our model does not suffer from these difficulties since the D4 and D6 branes never get close enough for the 4-6 string to become tachyonic. Instead, the reheating period in this model begins when $\phi$ rolls to zero and oscillates about the minimum of its potential. Physically, this corresponds to the brane world bouncing up and down around the point $\phi = 0$ on the NS5 brane. In realistic models, chiral fields make up part of the brane modes and will couple to the gauge fields. During the reheat process the inflaton will decay into these chiral fields. Since, in this picture, the inflaton is a brane mode the reheating process should be quite efficient.

Consider the inflaton scalar $\phi$ interacting with a scalar field $\chi$ and a spinor field $\psi$ that live on the brane by the interaction

$$L_{\text{int}} = -\frac{1}{2}g^2 \phi^2 \chi^2 - h\bar{\psi}\psi\phi. \quad (5.1)$$

The total decay rate of $\phi$ particles into the scalar $\chi$ and spinor $\psi$ particles is

$$\Gamma = \Gamma(\phi \rightarrow \chi \chi) + \Gamma(\phi \rightarrow \psi \psi). \quad (5.2)$$

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8 See the models \cite{3, 24} for two exceptions.
9 For detailed discussions of the reheating process, see e.g. \cite{20, 21, 22, 23}.
Following the arguments of [23], the decay products of the inflaton are ultrarelativistic and their energy density decreases due to the expansion of the universe much faster than the energy of the oscillating field $\phi$. Therefore, the reheating phase ends when the Hubble constant $H \simeq 2 / 3t$, becomes smaller than $\Gamma$. This corresponds to a universe of age $t_{RH} \simeq 2 / 3 \Gamma^{-1}$. The age of the universe with energy density $\rho$ is $t = \sqrt{M_P/6\pi \rho}$. Hence, the energy density at the end of reheating is

$$\rho(t_{RH}) \simeq \frac{3M_P^2}{8\pi \Gamma^2}. \quad (5.3)$$

Assuming thermodynamic equilibrium sets in quickly after the decay of $\phi$, the matter on the brane reaches the “reheat temperature” defined by

$$T_{RH} \simeq \left(\frac{30\rho(t_{RH})}{\pi^2 n}\right)^{1/4} \simeq 0.2\sqrt{\Gamma M_P}, \quad (5.4)$$

where $n$, the number of relativistic degrees of freedom, is expected to be $n(T_{RH}) \sim 10^2 - 10^3$. In our model, there are two ways in which the inflation can decay: One is through gravitational, closed string modes which we’ll call $\Gamma_G$ and the other is via the Yukawa couplings which constitute open string modes which we’ll call $\Gamma_h$. $\Gamma_G$ can be estimated using a formula in [4]

$$\Gamma_G = \frac{G_\text{N}^2 M}{M_P^3 V_{6d}} \quad (5.5)$$

Plugging our numbers into this formula and using it to calculate the reheat temperature we find $T_{RH} \simeq 10^8$ GeV. There is also the decay channel via the Yukawa coupling $\Gamma_h$. We estimate this to be

$$\Gamma_h \simeq h M^{1/5} \quad (5.6)$$

Since $h$ is a tunable parameter in this model we can make this as small as we like. Namely, if we choose $h = 10^{-14}$, we can make $\Gamma_h$ an order of magnitude bigger than $\Gamma_G$ such that it is the preferred decay channel while still keeping the reheat temperature below $10^9$ GeV. In this way we avoid overproduction of gravitinos during reheating. \footnote{In this analysis we have neglected the possibility of parametric resonance effects which can enhance the rate of boson particle production, preheating [27]. We thank R. Brandenberger for pointing out this possibility.} Because the reheat temperature is not very large compared to $M_P$, the creation of undesirable defects seems unlikely.

Equation \footnote{In this analysis we have neglected the possibility of parametric resonance effects which can enhance the rate of boson particle production, preheating [27]. We thank R. Brandenberger for pointing out this possibility.} can appear naturally in our string model in the following way: Introduce a D6$'$-brane along directions 123(45)7(89) where the parenthesis means that the brane is at an angle $\theta_{48}$ in the 48 direction and an angle $\theta_{59}$ in the 59 direction. As long as the angle with the original D6-brane in the 123689 satisfies $\theta_{48} + \theta_{59} + \theta_{67} = 0$ then the D6-branes will be BPS \footnote{In this analysis we have neglected the possibility of parametric resonance effects which can enhance the rate of boson particle production, preheating [27]. We thank R. Brandenberger for pointing out this possibility.}. We can choose the position of the D6$'$-brane such that it intersects the D4-brane at $X^4 = 0$ and the 4-6$'$ strings are massless. The 4-6$'$ strings in this scenario will
play the role of the Standard Model matter with the Standard Model gauge fields living on the D4-branes. When the D4-brane begins to oscillate about its minimum, it will excite the 4-6' strings that couple to it. If however, \( \theta_{48} = \pi/2 \), that is to say the D6' is perpendicular to the D6-brane, then there is no part of the D6'-brane transverse to the D4-brane, and the D4-brane is not coupled to the 4-6' strings. Thus the angle \( \theta_{48} \) is proportional to the Yukawa coupling between the 4-4, 4-6', and the 6'-4 strings. In superspace notation there is a coupling of the form

\[
W = h \int d^2 \theta \Phi Q \bar{Q}
\]

(5.7)

where \( \theta_\alpha \) is the superspace coordinate, \( \Phi = \phi + \theta \psi + \theta^2 F_\phi \), is part of the supermultiplet living on the 4-4 string and \( Q = q + \theta \psi + \theta^2 F_q \) the supermultiplet living on the 4-6'. The F-terms are not dynamical and can be integrated out. The potential in components reduces to

\[
V = h \phi^2 q^* q + h \phi \psi \bar{\psi}
\]

(5.8)

where \( h = \sqrt{2} \cos \theta_{48} \). In this model the Standard Model fields will naturally get a mass of order the supersymmetry breaking scale which is the same as the inflaton mass. In such a situation energy conservation would forbid the inflaton from decaying into Standard Model fields via the Yukawa interaction proposed. However, one can fine tune the Standard Model mass to be light to avoid this problem. In the brane model, the mass of the Standard Model fields are generated by the bending of the D4-brane by the D6-brane. This has the effect of stretching the 4-6' strings giving the fields on them a mass proportional to the stretched distance. The fine tuning amounts to moving the D6'-branes in the \( x_7 \) direction such that the 4-6' strings are again light.

This suggests a way to have dark matter fields: Introduce another D6"-brane in the same direction 123(45)7(89) displaced from the D6'-brane in the \( x^6 \) direction. Now the D4-brane also couples to the 4-6" fields and so the inflaton field can decay into these fields. We can write an interaction

\[
V = t \phi^2 p^* p + t \phi \omega \bar{\omega} + h \phi^2 q^* q + h \phi \psi \bar{\psi}
\]

(5.9)

where \( p \) is a scalar field, \( \omega \) is a fermion, and \( t \) is the Yukawa coupling determined be the angle \( \theta_{48} \) between the D6"-brane and the D6-brane. We can have \( t/h \) very large by tuning the angles appropriately. In this way we can get more dark matter than visible matter. However, this dark matter will not be dark because it couples to the Standard Model gauge fields which we have taken to live on the D4-branes. To solve this problem we could put the Standard Model gauge fields on the D6'-branes, but this would modify our cosmological analysis. We leave the details of this model of dark matter for future work. The brane configuration for this model is shown in Fig. 3.
The inflaton can also decay into massive Kaluza-Klein modes gravitationally. These KK modes are heavy of a mass $10^{14}$ GeV and decay rapidly into light Standard Model fields. Massive closed string states are slightly heavier than the reheat temperature and so are not produced. In Fig. 4, we see the gravitational interaction of the inflaton field with KK modes.

6. Conclusions

In this paper we have studied a stringy realization of the inflationary universe model. The scenario is formulated within the context of Type IIA string theory and the separation between D-branes plays the role of the inflaton. Similar models were explored previously (see e.g., [2, 9] for an extensive list of references). The standard slow-roll approximation technique was used to study the inflationary dynamics. Using observational (COBE) data, we placed constraints on the parameters of the model. For a desirable 58 e-foldings of inflation (and taking $g_\ast \simeq 1$) the spectral index is shifted to the red (by an acceptable amount) $n \simeq .98$. The string scale is found to be $M_s \simeq 10^{15}$GeV. The “size” of the D4 brane in the $x_6$-direction (distance between the NS5 branes) is $L_6^{-1} \simeq 5.2 \times 10^{14}$ GeV. The scales of the six compactified dimensions are $R_4^{-1} \simeq 10^{15}$ GeV, $R_6^{-1} = L_6^{-1}$ and $R_5^{-1}, R_7^{-1}, R_8^{-1}, R_9^{-1} \simeq 4.8 \times 10^{14}$ GeV. We propose a novel mechanism for reheating within the brane world context. Reheating occurs when our brane world rapidly oscillates about the minimum of its potential. Massive string states can be created during the reheating period as well as KK modes in the $T^6$ which can then decay into the massless Standard Model fields that live on the brane. The reheat temperature is estimated to be $T_{RH} \lesssim 10^8$GeV avoiding overproduction of gravitinos during reheating. Compared to $M_P$, the reheat temperature is probably not large enough to create dangerous defects which would overclose the universe. In our model the cosmological constant problem remains and it must be fine-tuned away in the usual fashion.

Finally, we comment on the possible nature of dark matter in this model. It is possible to introduce an additional D6"-brane (the dark matter brane) that couples to the inflaton D4-brane through a Yukawa interaction in the same way as the inflaton couples to the Standard Model fields. By tuning the relative angle of the D6"-brane with the NS5-brane we can change the Yukawa couplings. In this way we can easily arrange to have the inflaton field decay into the appropriate ratio of dark matter to visible matter. By displacing the D6"-brane slightly from the D4-brane we can give the 4-6" strings a small mass. This mass can be fine tuned to
be much smaller than the string or supersymmetry breaking scale. To get the dark matter to be “dark”, we had to consider moving the Standard Model gauge fields from the D4-brane to the D6'-brane. If this was done and the gauge field on the D4-brane was taken to be $U(1)$, the $U(1)$ gauge field on the D6’-brane and the D4-brane will be free in the IR. Thus the 4-6” strings make for good dark matter because they are light massive particles that do not interact with the Standard Model fields or themselves at low energies.

Other dark matter candidates are the 6-6 strings that live on the displaced D6-brane. The inflaton field couples to these fields gravitationally. We can have $N_{D6}$ D6-branes giving us a $SU(N_{D6})$ gauge symmetry living on them. Supersymmetry breaking would give a mass to the gauginos. The inflaton could then decay into these fermionic fields. Since the D6-brane fields only couple gravitationally to the SM fields, they would be invisible to observers on our brane. A Higgs field would have to be introduced to break the gauge symmetry such that the gauginos do not interact strongly at low energies. The number of D6-branes $N_{D6}$ could be adjusted such that one gets a large enough amount of dark matter. We leave a more detailed discussion of this model to future work.

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