I review the current status of the application of effective field theory to nuclear physics, and its present implications for nuclear astrophysics.

1. Introduction

When we finally arrive at the complete effective field theory that faithfully reproduces QCD in the low-energy regime relevant for nuclear physics, we will be able to answer some fundamental questions. In addition, to being able to describe inelastic processes, such as $3\alpha \rightarrow ^{12}\text{C} + \gamma$, in the same framework as elastic processes and the framework with which the nuclear energy levels are computed to high accuracy, we will know how such processes depend upon the fundamental constants of nature, the strong interaction mass-scale, $\Lambda_{\text{QCD}}$, and the masses of the quarks, $m_q$. A small but significant step has been taken in this direction by a recent calculation of the behavior of the two nucleon systems as a function of $m_q$. There are three good reasons for understanding such behavior. The first reason is intellectual curiosity. If we believe that we completely understand the strong interaction we should be in a position to address this issue. The second reason is to put constraints on physics beyond the standard model. There are hints that the constants of the standard model, such as $\alpha_{\text{em}}$ and $m_q$, may be time-dependent [1]. The third and perhaps the most practical reason is that the $m_q$-dependence of nuclear properties and reaction rates needs to be known in order to extrapolate lattice-QCD calculations from the unphysical values that will be used in present simulations and in those of the near future, down to those of nature. For the foreseeable future lattice QCD calculations will be performed with unphysically large quark masses simply due to the fact that the time required to perform the simulations increases as one reduces the $m_q$. Therefore, effective field theory (EFT) will play a key role in any future lattice QCD program, as it will only be through matching onto the appropriate field theory that lattice QCD will be able to make predictions of physical observables. This is true, of course, only until such a time when such observables can be computed directly with the physical values of the quark masses. The link between EFT and data has been and continues to be very strong, with data constraining the finite number of counterterms that appear at any given order in the EFT expansion. However, EFT calculations are being performed at sufficiently high orders so that in some cases the number of counterterms is greater than the number of observables, and lattice-QCD will be the only way to further improve the calculation. At this time constraints are being placed on linear combinations of the Gasser–Leutwyler
coefficients, the $L_i$, through partially-quenched QCD (PQQCD) simulations [2].

For processes involving multiple nucleons it is convenient to describe different momentum regimes with different EFT's. If one is dealing with processes in which all momenta are less than the pion mass, $m_\pi$, it proves to be useful to use the pionless EFT, EFT($\pi$), while for processes involving momenta greater than $m_\pi$ one must use a pionful theory.

2. The $p \ll m_\pi$ Regime

In the kinematic regime where $|p| \ll m_\pi$ we can construct an EFT to describe the interactions of multiple nucleons and photons quite simply [3–5]. The fact that there is a bound state near threshold in the $^3S_1 - ^3D_1$ coupled-channels, and a pole on the second-sheet near threshold in the $^1S_0$ channel means that at least one operator in the EFT Lagrange density must be treated non-perturbatively. One is free to choose which operators are resummed, and this will be defined in the power-counting associated with the EFT. There are no explicit $\pi$'s, $\rho$'s or other hadrons that can be produced in the low-energy collisions. As EFT($\pi$) is only applicable in the very-low energy regime, chiral symmetry is not a good symmetry, however isospin is a good symmetry. The only input into the construction of EFT($\pi$) is Lorentz invariance, electromagnetic gauge invariance, baryon number conservation and the approximate isospin symmetry, the breaking of which is included perturbatively.

The strong interactions between two nucleons are described by a Lagrange density

$$\mathcal{L} = C_0 \left[ N^T P^j N \right] I N^T P^j N + \ldots,$$

where $P^j$ is a spin-isospin projector, and the ellipses denote operators involving more powers of the external energy. It is clear that there is a correspondence between the operators in Eq. (1) and the coefficients in the effective range (ER) expansion of the scattering amplitude. This Lagrange density is explicitly Galilean invariant and relativistic corrections can be included in perturbation theory straightforwardly [5]. Further, it is straightforward to include the interactions in higher angular momentum channels, such as in the $^3S_1 - ^3D_1$ coupled channels [5]. If the only thing that had been accomplished was to recover effective range theory from EFT($\pi$), nothing would have been achieved. However, EFT($\pi$) is more than effective range theory. In EFT($\pi$) one can systematically incorporate gauge fields, such as the photon and electroweak fields. In addition to interactions arising from gauging derivatives in the strong interactions, there are operators that are gauge-invariant by themselves. For instance, the lowest order (LO) gauge invariant operator that contributes to the deuteron quadrupole moment is

$$\mathcal{L}_Q = -e \left[ N^T P_i N \right]^I \left[ N^T P_j N \right] \left( \nabla^i \nabla^j - \frac{1}{n-1} \delta^{ij} \right) A_0,$$

where $A_0$ is the time-component of the electromagnetic field. A computation of the deuteron quadrupole moment up to next-to-leading order (NLO) allows one to fit the coefficient $C^{(Q)}$. Once this constant is determined other observables that receive contributions from this $E2$ operator can be computed up to NLO [5].

The cross section for $np \rightarrow d\gamma$ at energies relevant to big-bang nucleosynthesis provides an example of a high precision calculation in EFT($\pi$), as shown in Fig. 1. This is a
classic nuclear physics process in which meson-exchange currents play a significant role. From the EFT($\pi$) point of view, where mesons are not an explicit degree of freedom, such contributions are included via local gauge-invariant four-nucleon-one-photon interactions (e.g. with coefficient $L_{np}$ for $np \rightarrow d\gamma$). In this case the resonance saturation hypothesis works well, in so much as this local operator is saturated by one pion exchange (OPE), but in general this need not be the case.

![Figure 1](image-url)  

Figure 1. The left panel shows the cross section for $\gamma d \rightarrow np$. The curves correspond to $L_{np}$ determined by the cross section for cold $np \rightarrow d\gamma$ [6] and the dashed lines denote the $\sim 3\%$ theoretical uncertainty. Rupak has further reduced this uncertainty to below $1\%$ [6]. The right panel shows the total cross section for $n\alpha$ scattering as a function of the neutron kinetic energy [10]. The diamonds and black squares are data. The dashed and solid lines show the EFT results at LO and NLO, respectively.

Another place where EFT($\pi$) has had impact is in the analysis of SNO data, and the determination of solar neutrino fluxes. An integral part of the SNO analysis is the cross section for neutrino induced deuteron break-up, such as the charged current process $\nu_e d \rightarrow e^- pp$. In addition to the well-known one-body contributions to the break-up, there is also a contribution from two-body interactions. Butler, Chen and Kong [7] showed that only one constant, $L_{1,A}$, is required to describe the two-body contribution in order to calculate the cross-section at the percent level. Therefore, the question is where does one get $L_{1,A}$ from? So far there have been three independent determinations of $L_{1,A}$: from the $\beta$-decay of tritium in a hybrid EFT, from an analysis of reactor data on $\bar{\nu}d$ break-up, and from an analysis of SNO data [8,9]. They find $L_{1,A} = 6.5 \pm 2.4 \text{ fm}^3, 3.6 \pm 5.5 \text{ fm}^3, 4.0 \pm 6.3 \text{ fm}^3$, respectively, which are all consistent within errors.

Recently, Bertulani, Hammer and van Kolck [10] have examined the relatively simple $n\alpha$ “Halo-nuclear” system. They explored the cross section for $n\alpha \rightarrow n\alpha$ by treating the $n\alpha$ interaction as a sum of local interactions between a fundamental $n$-field and a fundamental $\alpha$-field. There are three partial waves that make a significant contributions in the low-energy regime, $J^\pi = 0^+, 1^-, 1^+$. The LO calculation requires the scattering lengths in each channel, while the NLO calculation requires both the scattering length and effective range in each. The experimental values $a_{0^+} = 2.4641 \pm 37 \text{ fm}$, $r_{0^+} = 1.385 \pm 41 \text{ fm}$, $a_{1^-} = -13.821 \pm 68 \text{ fm}^3$, $r_{1^-} = -0.419 \pm 16 \text{ fm}^{-1}$, $a_{1^+} = -62.951 \pm 3 \text{ fm}^3$, and $r_{1^+} = -0.8819 \pm 11 \text{ fm}^{-1}$ produce the curves shown in the right panel of Fig. 1.

There has been continued progress in the three-body sector, and I will discuss only a
small fraction of it here. Bedaque, Grießhammer, Hammer and Rupak [11] have reformulated the EFT construction in the triton channel and have produced results for the $nd \to nd$ phase-shift, as shown in Fig. 2. At LO and NLO there is a momentum independent three-body interaction, with coefficient $H_0$, that can only be determined in the three-body sector (or higher-body sectors), and the scattering length in the triton channel is sufficient. At NNLO there is a contribution from a momentum dependent three-body interaction, with coefficient $H_2$, and the triton binding energy is sufficient to determine it. One sees that the EFT calculation is converging to the data and at this order shows no serious deviation from a more traditional potential model calculation. One can also examine the behavior of the Phillips line order-by-order in the EFT expansion. If one takes an arbitrary model that describes to arbitrary precision the two-body sector and uses it to compute the scattering length in the $J = \frac{1}{2}$ channel and triton binding energy, one will generate values that lie (approximately) on a line in the plane formed by them. This is the Phillips line, and results from the fact that if three-body interactions are possible then, in general, they will be present. The Phillips line, as shown in Fig. 3, is generated by varying the coefficient of the momentum independent three-body force over all possible values.

There have also been some exciting results from Hammer regarding the hyper-triton in EFT(π) [15]. As the deuteron has only a small binding energy and the hyper-triton has a very small binding energy, $B_{Nd} = 2.35 \pm 0.05$ MeV, this particular three-body system can be described very well in EFT(π), and Hammer obtains $a_{Nd} = 16.8^{+4.4}_{-2.4}$ fm and $r_{Nd} = 2.3 \pm 0.3$ fm. I refer the reader to Ref. [15] for a detailed discussion. An improved experimental determination of $B_{Nd}$ would be welcome.

3. The $p \gg m_\pi$ Regime

It was Weinbergs’ pioneering efforts [16] in the early 1990’s that led to the field of EFT in nuclear physics. He was attempting to construct an EFT for nuclear processes and
nuclei involving momentum all the way up to the chiral symmetry breaking scale $\Lambda_\chi$, and necessarily included the pion as a dynamical degree of freedom. The power-counting that he developed, known as Weinberg power-counting (W), involved performing a chiral expansion of the nucleon-nucleon potential using the same power-counting rules that apply in the meson and single nucleon sectors. The chirally expanded potential is then inserted into the Schroedinger equation to determine observables, such as phase shifts. However, there is a formal problem with this power counting [17] in some of the scattering channels, particularly the $^1S_0$ channel. However, extensive phenomenological studies with W power-counting appear to be in good agreement with data [18,19], where such comparisons are possible, and the formal problem appears to have little impact when a massive regulator is used with a mass scale that is not radically different from a few hundred MeV. This problem led to KSW power-counting [4] in which the momentum independent four-nucleon operator is promoted to one lower order in the chiral expansion, and consequently pion exchanges are subleading order and treated in perturbation theory. This power-counting is formally consistent and gives renormalization group invariant amplitudes order-by-order in the power-counting. However, Fleming, Mehen and Stewart [20] (FMS) showed that the scattering amplitude in the $^3S_1 - ^3D_1$ coupled channels diverges at NNLO at relatively small momenta and KSW power-counting fails. FMS found that a contribution that remains large in the chiral limit destroys the convergence: it is the chiral limit of the tensor force that “does the damage”. Recently, it was suggested that one should expand observables about the chiral limit [21] (BBSvK power-counting). BBSvK power-counting has all the nice features of W and KSW counting: the chiral limit of the tensor force is resummed at LO along with the momentum and $m_q$ independent four-nucleon operator in the $^3S_1 - ^3D_1$ coupled channels, while pions are perturbative in the $^1S_0$ channel and in higher partial waves where analytic calculations are possible.

As I mentioned previously, the phenomenology of W counting has been explored extensively. Epelbaum et al [22] have performed an impressive analysis of the few nucleon systems with W power-counting and find the calculations in the NN sector to converge well. The S-wave phase shifts are shown in Fig. 4. One sees that higher order calculations move closer to the data in all partial waves and the uncertainty is reduced. This calculation uses a momentum space regulator, $\Lambda$, and one finds some sensitivity to the value chosen for $\Lambda$. An estimate of the uncertainty in this calculation can be made by varying $\Lambda$ between 500 and 600 MeV, however, it would appear that such variations in $\Lambda$ tend to

Figure 3. The Phillips line [11]. The dots correspond to the predictions of different models with the same two body scattering lengths and effective ranges [14]. The dotted and full line are the EFT results at LO and NLO, respectively. The cross is nature.
Figure 4. The S-wave phase-shifts for laboratory energies $E_{\text{lab}}$ below 200 MeV \[22\]. Left (right) panel is the result at NLO (NNLO*). The momentum-space cut-off is chosen between 500 and 600 MeV leading to the band. The filled circles correspond to the Nijmegen PSA results \[23\].

underestimate the true uncertainty, as the NNLO result does not lie entirely within the NLO band. The convergence of the theory has also been examined in detail by Entem and Machleidt \[24\]. In other work Epelbaum \textit{et al} \[25\] have performed a detailed study of the three-body sector with W power-counting. One finds that the uncertainty is reduced by going from NLO to NNLO as expected, and their results are very encouraging.

Figure 5. $\frac{d\sigma}{d\Omega}$'s for Compton scattering on deuterium \[26\]. The data are from Illinois \[27\] (circles), Lund \[29\] (diamonds) and SAL (squares) \[28\]. The solid line is the $O(Q^4)$ calculation with $\alpha_N = 9.0 \times 10^{-4}$ fm$^3$, $\beta_N = 1.7 \times 10^{-4}$ fm$^3$. The dashed line is the (parameter-free) $O(Q^3)$ calculation.

Beane \textit{et al} have obtained some exciting results for Compton scattering from the deuteron \[26\]. By working at order $Q^4$ in W power-counting the electric and magnetic isoscalar polarizabilities of the nucleon were determined directly from the Compton scattering cross section at various energies, as shown in Fig. 5 \[26\]. They find that
\[ \alpha_N = 9.0 \times 10^{-4} \text{ fm}^3, \quad \beta_N = 1.7 \times 10^{-4} \text{ fm}^3. \] A higher order calculation is required in order to reduce the uncertainties in \( \alpha_N \) and \( \beta_N \) and to ensure that the EFT calculation of this process is converging as expected. Further, it is clear that more data should be taken for this process, filling out the differential cross sections plots shown in Fig. 5.

\[ \alpha^+ \text{ vs } \alpha^- \text{ [30]. The light and dark shaded regions are from the experimental pionic-hydrogen width and shift, respectively, taken from Ref. [31]. The dotted line encompasses the constraints from } \pi\text{-}N \text{ phase shift data [32]. The dot is LO } \chi\text{PT. The two parallel bands are from } \pi d \text{ scattering [30] evaluated with the NLO wavefunction with a cutoff of 500 MeV (upper curve) and 600 MeV (lower curve).} \]

Recently, W power-counting has been used by Beane et al to examine \( \pi d \) scattering in EFT [30]. The EFT allows one to understand \( \pi d \) scattering in the same framework as \( \pi N \) scattering. Given the precise data from pionic hydrogen and from pionic deuterium one can extract both the isoscalar and isovector \( \pi N \) scattering lengths, \( a^+ \) and \( a^- \) respectively, with high precision. A plot of the constraints in the \( a^+ - a^- \) plane is shown in Fig. 6, from which it is found that \( a^- = 0.0918 \pm 0.0013 \text{ m}^{-1} \) and \( a^+ = -0.0034 \pm 0.0007 \text{ m}^{-1} \). These values are in very good agreement with the recent analysis of Ref. [33]. A higher order calculation is required in order to further reduce the uncertainty in these values.

An alternate approach to EFT calculations has been advocated by Rho and collaborators. They argue that performing a chiral expansion of the current operators (or whatever operator you are interested in) and using wavefunctions generated by the best modern potentials (“best” being defined as those with lowest \( \chi^2 \) in the two- and three-nucleon sectors) should be equivalent to the formal EFT expansion. While I do not presently see how to formally justify this approach it appears to numerically converge well, and its cut-off dependence is systematically reduced as one performs calculations to higher orders. With this method, Park et al [8] have computed the cross sections for \( p p \rightarrow d e^+ \nu_e \) and \( p ^3\text{He} \rightarrow ^4\text{He} e^+ \nu_e \). The wavefunctions are taken from the AV18/UIX potential while the current operator is expanded to a given order in W counting. To the order to which they work only one-body and two-body operators contribute. The one unknown coefficient appearing in the two-body contribution, \( L_{1A} \), is determined from the rate for tritium \( \beta \)-decay, which they compute in the same framework. While the coefficient of the two-body operator depends sensitively on the value of the cut-off, the complete two-body contribution to the rate is relatively insensitive to the cut-off. They find \( S_{\text{hep}}(0) = (8.6 \pm 1.3) \times 10^{-20} \text{ keV} \cdot \text{b} \). This is a very encouraging result, as the calculation of \( S_{\text{hep}}(0) \) suffers from significant cancellations. This method may provide a
bridge between the formal EFT constructions and the many-body methods developed in nuclear physics.

4. The $m_q$-Dependence of the Two-Nucleon Sector

In some recent papers by Beane and myself [34] and also by Epelbaum, Glockle and Meißner [35] EFT was used to determine the $m_q$-dependence of scattering in the two-nucleon sector. In the $^1S_0$-channel KSW power-counting can be used to derive an analytic expression for the scattering length,

$$
\frac{1}{a(^1S_0)} = \gamma + \frac{g_A^2 M_N}{8\pi f^2} \left[ m^2 \log \left( \frac{\mu}{m} \right) + (\gamma - m)^2 - (\gamma - \mu)^2 \right] - \frac{M_N m^2}{4\pi} (\gamma - \mu)^2 D_2 \ , \quad (3)
$$

where $\gamma$ is a LO constant and $D_2(\mu)$ is a combination of coefficients of operators with a single insertion of $m_q$, that is presently unknown. The best that one can do at this point in time is to use naive dimensional analysis (NDA) to estimate a range of reasonable values for $D_2$, defined by a parameter $\eta \ll 1$ [34]. The results of NDA are shown in Fig. 7. NDA suggests that the di-neutron remains unbound in the chiral limit, while a relatively small increase in $m_q$ could lead to a bound di-neutron.

In the $^3S_1-^3D_1$ coupled channels the situation is somewhat more complicated. At NLO in BBSvK counting not only does OPE contribute, but also the chiral limit of two-pion exchange. As a consequence, there are additional counterterms in the single nucleon sector that contribute in this channel but do not contribute to the $^1S_0$ channel, in particular $\tilde{d}_{16}$ and $\tilde{d}_{18}$ associated with the pion-nucleon interaction, and $\tilde{d}_4$ associated with $f_\pi$. This is in addition to the $D_2(\mu)$ contribution in the $^3S_1$ channel. The allowed regions for $\tilde{d}_{18}$ and $\tilde{d}_{16}$ are given in Ref.[36], and $\tilde{d}_4$ is known. Fig. 7 shows the presently allowed values of the scattering length in the $^3S_1$ channel where we again have used NDA to estimate the possible values for $D_2$. It is clear that for the range of parameters considered the deuteron could be bound or unbound in the chiral limit, and at present one cannot make a more definitive statement. This last statement disagrees with the conclusion of Ref. [35].
5. Conclusions

There has been rapid progress in the application of effective field theory to nuclear physics. A consistent power-counting has been established, improved model independent calculations of \( NN \rightarrow NN \), \( Nd \rightarrow Nd \), \( vd \rightarrow vd \), \( pp \rightarrow de \nu \), \( p \rightarrow ^{3}He \rightarrow ^{4}He \) and \( \pi d \rightarrow \pi d \) have been completed, and first efforts to describe halo-nuclei and hypernuclei have been undertaken. Further, efforts to understand fundamental questions about nuclear observables have been made with interesting results.

There are still many goals to be attained. It is clear that the calculations I have discussed here should be pushed to one higher order to convince ourselves about their precision. Moreover, it is clear that parameters in the single nucleon sector need to be known to higher precision than they are now in order to make precise statements in the multi-nucleon sectors. Ultimately, a much closer link between the lattice QCD community and the EFT community must be developed, as it is only with EFT that lattice calculations of complex hadronic systems will be possible, and further, it is only with lattice QCD that EFT will be able to make faithful calculations in multi-nucleon systems. This is clearly a symbiotic relationship that is yet to be realized.

I would like to thank the organizers of PaNIC02 for putting together a stimulating meeting and inviting me to share the recent developments in EFT. I would also like to thank those who have let me reproduce their work in these proceedings.

REFERENCES