Constraints on the binary evolution from chirp mass measurements

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ABSTRACT

We estimate the observed distribution of chirp masses of compact object binaries for the gravitational wave detectors. The stellar binary evolution is modeled using the StarTrack population synthesis code. The distribution of the predicted "observed" chirp masses vary with variation of different parameters describing stellar binary evolution. We estimate the sensitivity of the observed distribution to variation of these parameters and show which of the parameters can be constrained after observing 20, 100, and 500 compact object mergers. As a general feature of all our models we find that the population of observed binaries is dominated by the double black hole mergers.

Subject headings: binaries: close — gravitational waves

1. INTRODUCTION

Compact object mergers are one of the most promising sources of gravitational waves for the ground based interferometric detectors like LIGO (Abramovici et al 1992) and VIRGO (Bradaschia et al 1990). So far most of the theoretical papers on the properties of these sources related to the gravitational wave detections have been concentrated on calculation of the predicted rates (Narayan et al 1991, Phinney 1991, Kalogera et al. 2001). In this paper we wish to address another aspect of the gravitational wave detection, i.e. the distribution of observed masses of the compact objects.

Stellar mass compact object binaries shall be detected during the inspiral phase, while the consecutive merger and ringdown phases will most likely have lower signal to noise ratios. During the inspiral phase the motion of the binary components and also the wave form is
governed by the chirp mass $\mathcal{M} = (m_1 + m_2)^{2/5}(m_1 m_2)^{3/5}$ (Peters and Matthews, 1963). The waveform will depend on the individual masses of the binary components $m_1$ and $m_2$ when the post Newtonian effects are taken into account. However, the analysis of the inspiral phase alone shall not suffice to determine if a binary contained a neutron star or a black hole without the prior knowledge of the neutron star maximum mass. A careful modeling of the signal may yield the individual masses of the objects, however, the chirp mass will be the primary observable for the compact object mergers (Cutler & Flanagan 1994).

We use the StarTrack population synthesis code (Belczynski, Kalogera & Bulik 2002) to calculate the distributions of compact object binary masses, and we present these calculations in § 2. In § 3 we estimate the number of merger detections required to distinguish between different models of stellar binary evolution. Finally, § 4 contains the conclusions and discussion.

2. Distribution of the chirp masses

The StarTrack binary population synthesis code is described in detail in Belczynski, Kalogera & Bulik (2002). One of the important features of the code is the possibility to conduct parameter study of a given property of the population of binaries, i.e. to estimate the dependence of the result on each of the parameters used to describe the stellar and binary evolution. The models used in this paper are listed in Table 1. We first use the standard model A results to present the intrinsic distribution of the chirp masses. This is shown in Figure 1. The distribution shows a clear peak at low chirp masses $1M_\odot \mathcal{M} < 2M_\odot$ which is due to the double neutron star systems. The mixed (BH-NS) systems populate the intermediate region, while the chirp masses of the BH-BH binaries extend up to above $10M_\odot$.

In order to estimate the observed distribution of the chirp masses of compact objects one has to take into account the sensitivity of the gravitational wave detectors to signals from mergers of different binaries. The calculations of the signal to noise ratio (Finn and Chernoff 1993, Bonazzola and Marck 1994, Flanagan and Hughes 1997) show that the sampling distance in the first approximation is a function of the chirp mass only: $D \propto \mathcal{M}^{5/6}$. The additional corrections due to limited sensitivity window of the detectors have been calculated by Flanagan and Hughes (1997) and amount to approximately 10% for the binaries with the total mass below $18M_\odot$ for the initial LIGO, and less for the advanced LIGO. In this paper we neglect these corrections. The distribution of the expected observed chirp masses can be calculated using Monte Carlo method. We assume that the Universe is uniformly filled with merging binaries, and for each merger we estimate the signal to noise ratio
Table 1: Description of different population synthesis models for which the distributions of mass ratios have been found.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>standard model described in Belczynski, Kalogera, Bulik (2002), but with $T_{Hubble} = 15\text{Gyrs}$</td>
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<tr>
<td>B1,7,11</td>
<td>zero kicks, single Maxwellian with $\sigma = 50, 500, \text{km s}^{-1}$,</td>
</tr>
<tr>
<td>B13</td>
<td>Paczynski (1990) kick with $V_k = 600\text{km s}^{-1}$</td>
</tr>
<tr>
<td>C</td>
<td>no hyper–critical accretion onto NS/BH in CEs</td>
</tr>
<tr>
<td>E1–3</td>
<td>$\alpha_{CE} \times \lambda = 0.1, 0.5, 2$</td>
</tr>
<tr>
<td>F1–2</td>
<td>mass fraction accreted: $f_a = 0.1, 1$</td>
</tr>
<tr>
<td>G1–2</td>
<td>wind changed by $f_{\text{wind}} = 0.5, 2$</td>
</tr>
<tr>
<td>J</td>
<td>primary mass: $\propto M_1^{-2.35}$</td>
</tr>
<tr>
<td>L1–2</td>
<td>angular momentum of material lost in MT: $j = 0.5, 2.0$</td>
</tr>
<tr>
<td>M1–2</td>
<td>initial mass ratio distribution: $\Phi(q) \propto q^{-2.7}, q^3$</td>
</tr>
<tr>
<td>O</td>
<td>partial fall back for $5.0 &lt; M_{CO} &lt; 14.0 M_\odot$</td>
</tr>
<tr>
<td>S</td>
<td>all systems formed in circular orbits</td>
</tr>
<tr>
<td>Z1–2</td>
<td>metallicity: $Z = 0.01$, and $Z = 0.0001$</td>
</tr>
</tbody>
</table>
Fig. 1.— The intrinsic (galactic) distribution of the chirp masses in the framework of model A (left panel), and the distribution of the expected observations (right panel). The solid line corresponds to the NS-NS mergers, the short dashed line represents the NS-BH mergers and the dashed line stands for the BH-BH mergers. The sum of the three distribution in each panel is normalized to unity.

in the detector. We model the population of merging binaries assuming a continuous star formation rate. The result is shown in the right panel of Figure 1. One can note that these distribution could also be obtained analytically by multiplying the distributions of Figure 1 by the volume $\propto M^{5/2}$ and normalizing it. In this plot the BH-BH systems are now the dominant contribution of the distribution. This is due to the fact that the sampling volume for the BH-BH binaries more than 100 times larger than that for the NS-NS systems, which easily compensates for the lower merger rate of the BH-BH binaries.

3. Expected observations

Let us now address the following questions: are the distributions of observed chirp masses expected in the framework of alternative models different? If so, are these differences significant? We simulate the distributions of chirp masses in the expected observations with the binary populations obtained from the set of models of Table 1 similarly as we have done above for the model A. We present the results in Figure 2. Different stellar evolution models lead to drastically different distributions of the chirp masses in the expected observations. Various parameters describing stellar evolution affect the distribution of observed
chirp masses in several ways. Changing the kick velocity distribution (models B) alters the ratio between the number of the neutron star binaries and the black hole binaries. Other models change the maximal masses of the black holes produced. This is especially clear in the case of models G where the stellar winds are varied by a factor of two upwards (model G2) and downwards (model G1). We note that the shapes of these distributions do not depend on the sensitivity of a detector.

In order to verify if the differences between the distributions are significant we turn to a simulation of a finite number of merger observations. We assume that the true stellar evolution goes through one of the models of Table 1. We then simulate the observations of a given number of mergers (we use 20, 100 and 500 mergers) and for each such simulated observation we verify using the Kolmogorov Smirnov (KS) test if we can reject a hypothesis that the stellar evolution is described by model A. This allows to test the sensitivity of the shape of the distribution of expected chirp mass observations to the underlying model parameters describing stellar evolution. For each number of merger observations we repeat this test 10000 times to obtain a distribution of KS-test probabilities and find the lowest probability that appeared in the top one percentile of this distribution. We can now set a detection confidence level, say at $10^{-5}$ and compare each probability with this value: if it is higher we conclude that this particular model cannot be distinguished from model A with a given number of merger observations, while a smaller number means that this model can be distinguished, and that some constraints can be imposed on the particular parameter through which this model differs from model A. We present the results of the test in Figure 3.

Figure 3 presents a measure of sensitivity of the expected observed distribution of chirp masses to the parameters describing stellar evolution. One can see from Figure 3 that even observations of a small number of mergers (open circles correspond to 20 mergers) yield highly significant results for models E1, G2 and O. The reason for that can be is clear from Figure 2. These are the models for which the maximal chirp mass in the population is significantly lower than that for model A. Model G2 population (i.e. with increased stellar winds) contains hardly any black holes. In general we see that these observations are very sensitive to the value of maximum mass of stellar black holes in the population. Model G1 (with decreased stellar winds) which allows for formation of black holes binaries with chirp masses up to $16 M_\odot$ will stand out with less than a hundred merger observations.

With a larger number of merger observation (stars in Figure 3) correspond to 100 merger detections) more parameters can be constrained. Some constraints can be obtained for the value of common envelope efficiency $\alpha_{CE}\lambda$ (models E: model E2 is similar to A). Other parameters describing mass transfer events like the mass fraction accreted (models F), and the amount of angular momentum lost (models L) shall also be constrained. Moreover also
the metallicity of the progenitor stars may influence the observed distribution at a significant level (models Z).

Constraining the initial mass ratio distribution (models M) will require an even higher number of merger detections: only for the case of 500 observed mergers the differences become significant. Models C (no hypercritical accretion onto a compact object), J (initial mass function slope), and S (systems circular initially) lead to very small differences in the observed distribution of chirp masses.

Models B where the kick velocity distribution is varies begin to show significant differences only with a large number of observations. Changing the kick velocity distribution affects strongly the absolute rates (Lipunov Postnov Prokhorov 1997, Belczynski Bulik 1999), and the ratio of double neutron star mergers to the double black hole mergers (Belczynski, Kalogera and Bulik 2002).

4. CONCLUSIONS

We have applied the stellar population synthesis models to simulate the distribution of observed chirp masses in the gravitational wave detection of stellar mergers. We find that the population of observed mergers is dominated by the black hole - black hole binary mergers. In most models double black hole mergers constitute more than 90% of the observed events. The exception is model G2, in which the formation of black holes is suppressed because of increased stellar winds. The shapes of observed distributions of chirp masses vary considerably for different models of stellar binary evolution.

We simulate the observed distributions of chirp masses in the framework of various stellar evolution models and estimate the sensitivity with which these parameters can be estimated from a given sample of observed mergers. We find that there is a large number of parameters that can be constrained given a set of measured chirp masses. The main and immediate constraints come from the fact that the observed population seems to be dominated by the highest mass black hole binaries. Thus even a small set of observations yields constraints on the maximal mass of merging black hole binaries. A larger set of observations will lead to constraints on the evolution of high mass binaries.

In our simulation we use a simple statistical tool: the Kolmogorov Smirnov test. Given a set of real observations with some measurements of individual masses of coalescing stars, one could use a more sensitive tool like the maximum likelihood method. However, even with such simple statistic as used here we can show the general properties of the expected observations, and demonstrate the sensitivity of the observed distributions to different model
parameters.

we note that consideration of the distribution of observed masses will lead to stricter constraints than consideration of just the observed rates. The theoretical calculation of rates involves estimating a number selection effects and calibrating with other sources which leads to several uncertainties. The calculation of the observed distribution of chirp masses is free from such uncertainties because a distributions essentially equivalent to considering the ratios of the number of mergers of different type, and all the normaliztion factors that enter the rate estimates do cancel out.

Finally, we have to mention several effects not taken into account in this paper. A more detailed calculation must include the consideration of the effects of lifetimes of binaries of different type. Belczynski, Kalogera and Bulik (2002) have shown that the typical lifetimes of double neutron star binaries are much smaller than the black hole binaries. The effects due to changing of the star formation rate with the redshift will affect the observed population of merging binaries. When considering the advanced detectors sensitive to mergers at cosmological distances one also needs to take into account cosmological effects: the fact that the true measure quantity is the redshifted mass \((1 + z)M\), and also the change of the observed volume with the redshift (here we have assumed the Euclidean geometry). These issues will be considered in a separate paper.

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Fig. 2.— Distributions of the expected observed chirp masses in the framework of models listed in Table 1. For clarity each distribution is shifted up by a factor of ten.
Fig. 3.— The significance of rejecting model A. Open circles correspond to observations of just 20 mergers, stars to 100 mergers, and filled circles to observations of 500 mergers. The symbols with an arrow denote the case when significance is off the scale.