The boundary condition is defined by the Fierz-Perelomov equation

\[ \phi(x) \in \{ \phi \in \mathbb{R} \mid \partial^a \phi = \partial^b \phi \} \]

where \( \phi \) is the field, \( \partial^a \) and \( \partial^b \) are the boundaries of the region where the field is defined.

We have also defined the projection operator

\[ (x^a x^b) \phi \in \{ \phi \in \mathbb{R} \mid \partial^a \phi = \partial^b \phi \} \]

and used it to define the stress tensor.

\[ \tau^{\alpha \beta} = \partial^\alpha \phi - \partial^\beta \phi \]

where \( \alpha \) and \( \beta \) are the boundary conditions.

\[ \tau^{\alpha \beta} \in \{ \phi \in \mathbb{R} \mid \partial^a \phi = \partial^b \phi \} \]

The stress tensor is a measure of the force per unit area acting on a surface.

\[ \sigma = \partial^a \phi - \partial^a \phi \]

where \( \sigma \) is the stress.

\[ \sigma \in \{ \phi \in \mathbb{R} \mid \partial^a \phi = \partial^a \phi \} \]

The stress tensor is a measure of the force per unit area acting on a surface.

\[ \tau^{\alpha \beta} \in \{ \phi \in \mathbb{R} \mid \partial^a \phi = \partial^a \phi \} \]

The stress tensor is a measure of the force per unit area acting on a surface.
charge $e$ and the magnetic flux $\Phi$ should be identified with the longitudinal linear momentum $\nu$ and $2\pi\kappa$, respectively.

The renormalized propagator is given by

$$D^{(\alpha,\kappa)}(x, x') = \frac{i}{2\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} d\tau \frac{\tau^2 + \tau^2 - (2\pi\alpha n - \Delta \varphi)^2}{\left\{ \left( \pi(2\alpha n + 1) - \Delta \varphi \right)^2 + \tau^2 \right\}^{1/2}} \left[ \left( \pi(2\alpha n - 1) - \Delta \varphi \right)^2 + \tau^2 \right]^{-1}, \quad (7)$$

where $\Delta t := t - t'$, likewise for $\varphi$ and $Z$. As $\kappa \to 0$ the dominant contribution in Eq. (6) is the renormalized scalar propagator in an ordinary conical background. Therefore when $\kappa/r \to 0$, Eq. (6) yields for the diagonal components essentially the expressions long known in the literature for the vacuum fluctuations around an ordinary cosmic string ($\kappa = 0$). Regarding the remaining components, the prescription in Eq. (6) kills off the dominant contribution in Eq. (4), resulting that the subleading contribution yields two non vanishing off-diagonal components,

$$\langle T^{\nu^2} \rangle = \frac{i}{r^2} \lim_{x' \to x} \partial_{x'} \partial_{x} D^{(\alpha,\kappa)}(x, x') = \frac{\kappa}{r^2} B(\alpha), \quad (8)$$

and

$$\langle T^{x^2} \rangle = \frac{\kappa}{r^2} B(\alpha), \quad (9)$$

where

$$B(\alpha) := \frac{1}{32\pi^2} \int_0^{\infty} d\tau \alpha \sin(\pi/\alpha) \left[ \cos(\pi/\alpha) - \cosh(\tau) + \tau \sinh(\tau) \right] - \pi \left\{ \cos(\pi/\alpha) \cosh(\tau) - 1 \right\}$$

$$\left[ \cosh(\tau) - \cos(\pi/\alpha) \right]^{1/2} \cosh^2(\alpha \tau/2). \quad (10)$$

It is worth remarking that, unlike the diagonal components, $\langle T^{\nu^2} \rangle$ and $\langle T^{x^2} \rangle$ do not depend on the coupling parameter $\xi$.

The plot of $B(\alpha)$ against the disclination parameter $\alpha$ is shown in Fig. 4. When $\alpha = 1$, the integration in Eq. (11) can be analytically evaluated, resulting $B = 1/60\pi^2$, which corresponds approximately to the value of $\alpha$ suggested by the physics of formation of ordinary cosmic strings.

![FIG. 1: Plot $B(\alpha)$ versus $\alpha$.](image)

It is instructive to display both disclination and screw dislocation effects in a same array. When $\xi = 1/6$ (conformal coupling), for example, $\langle T^{\nu^2} \rangle$ with respect to the local inertial frame [cf. Eq. (4)] can be cast into the form

$$\langle T^{\nu^2} \rangle = \frac{1}{r^2} \begin{pmatrix} -A & 0 & 0 & 0 \\ 0 & -A & 0 & 0 \\ 0 & 0 & 3A & \kappa B/r^2 \\ 0 & 0 & \kappa B & -A \end{pmatrix}, \quad (11)$$

where $A(\alpha) := (\alpha^{-4} - 1)/1440\pi^2$, and which holds far away from the defect (and for $\alpha \neq 1$, when $\kappa \neq 0$). When $\kappa \neq 0$, by setting $\alpha = 1$ in Eq. (11), $A$ vanishes and subleading contributions depending on $\kappa$ take over.

Before closing this note, let us interpret the polarization effect displayed in Eq. (6) in the light of the analogy with the Aharonov-Bohm effect following Eq. (2). Observing Eq. (6) we can say that a disappear (more precisely, a
screw dislocation) polarizes the vacuum of a scalar field, inducing a flux of longitudinal linear momentum around the defect. Such a flux depends on the direction of the screw dislocation (i.e., on the sign of $\kappa$) in the same way that vacuum currents around a needle solenoid depend on the direction of the magnetic flux.

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