I. INTRODUCTION

Since their discovery in the 1970’s charm quarks have proved valuable probes of Quantum Chromodynamics (QCD) via the production of open- and hidden-charm states in hadronic collisions at high energy [1]. With the charm-quark mass, \( m_c \approx 1.5 \text{ GeV}/c^2 \), being on the borderline of perturbative and nonperturbative momentum scales, mechanisms associated with both regimes are likely to play a role in their elementary production. In addition, nonperturbative effects are inevitably involved in subsequent hadronization, being related to the wave functions of the final-state mesons or baryons.

The usual way of describing charmed hadron production is based on the factorization theorem [2], i.e., a perturbative treatment for the creation of a \( c \bar{c} \) pair through annihilation of partons within the incoming hadrons (folded over appropriate parton distribution functions), followed by individual hadronization of the \( c \)-quarks utilizing a (more or less) universal fragmentation function \( D(z) \) (where \( z \) is a normalized momentum fraction of the \( c \)-quark). Both stages in this description are not free of problems, or at least restricted to certain kinematic regimes. E.g., in leading-order (LO) perturbative QCD (pQCD), the elementary \( c \bar{c} \) production cross section underestimates open charm production in hadronic collisions by large factors of \( \sim 5 \) (the so-called \( K \)-factors). Next-to-leading order (NLO) estimates typically account for half of the deficit, with appreciable uncertainties inherent in underlying parameters such as the bare charm quark-mass, renormalization and factorization scales [1]. This obviously leaves room for additional contributions of nonperturbative origin.

Concerning the hadronization of \( c \) and \( \bar{c} \) quarks, some empirical observations, especially at low transverse momentum (\( p_t \)), cannot be captured by standard fragmentation functions. E.g., flavor systematics at forward rapidities exhibit the so-called leading-particle effect [3–12]; that is, charmed hadrons containing light quarks corresponding to projectile valence quarks are much enhanced over their antiparticles. A natural explanation is provided by a recombination [13–15] of projectile valence with forward-produced (anti-) charm quarks \([16–22]\), facilitated by small rapidity differences. More indirectly, it has been found [1] that \( x_F \)- and \( p_t \)-distributions of \( D \)-mesons are well reproduced by delta-function fragmentation functions, i.e., the produced charm quarks do not seem to lose momentum as one would expect for a (realistic) fragmentation process. This is suggestive for the importance of recombination mechanisms even at central rapidities. As opposed to high-\( p_t \) emission, charm quarks at low \( p_t \) move at low velocity relative to their environment, with little energy available for string breaking, thus facilitating a ”statistical” attachment to neighboring light quarks.

Finally, as has been pointed out in Ref. [1], available experimental data from both \( p-N \) and \( \pi-N \) collisions point at a significant enhancement of \( D^+ \)/\( D^- \) over \( D^0 \) total production (dominated by central rapidities and low \( p_t \)) over expectations from perturbation theory. This feature is also borne out when performing empirical fits using the PYTHIA event generator, which require separate \( K \)-factors to fit \( D^\pm \) and \( D^0 \)/\( \bar{D}^0 \), differing by a factor of \( \sim 2 \) [23].

In the present article we address the afore-mentioned flavor dependencies by evaluating a suitably formulated recombination model for charged and neutral \( D \)-mesons extended to the central \( x_F \) region. This allows to assess (”soft”) recombination processes involving sea-quarks from both projectile and target nucleons (or pions), and thus aims at a consistent treatment of forward and total flavor production asymmetries. In addition, one obtains estimates for \( D \) meson production within the same framework.

The following presentation is organized as follows. In Sect. II we recall the main features of recombination mechanisms as have been applied in the literature before, and generalize it for our purpose of central production. LO pQCD \( c\bar{c} \) cross sections are employed to obtain the primordial \( c \)-quark distribution, including a single (constant)
\( K \) factor adjusted to the measured total yields. The two key (nonperturbative) quantities to evaluate subsequent \( D \)-meson formation, namely a two-parton distribution function and a recombination function, are discussed in detail. The former provides the light quark for the "coalescence" process, whereas the latter characterizes the pertinent overlap probability to form a \( D \)-meson. In Sect. III we compare our results with available data from both \( \pi-N \) and \( p-N \) reactions, and in Sect. IV we quote our predictions for upcoming measurements at RHIC in the \( p-p \) mode at center-of-mass (CM) energy \( \sqrt{s}=200-500 \) GeV. We finish with conclusions and an outlook for future work in Sect. V.

II. CHARM-QUARK RECOMBINATION INTO \( D \)-MESONS

A. General Outline of the Approach

The basic idea of the recombination approach is that quarks produced in hadronic collisions hadronize by "coalescing" with (anti-) quarks pre-existing in the wave function of projectile (and/or target). This mechanism is quite different from the usual fragmentation, where produced quarks hadronize individually by string breaking, independent of their environment. Fragmentation functions therefore ought to be universal objects, which, in principle, can be extracted from \( e^+e^- \) collisions, where fragmentation is expected to be the sole source of hadron production. However, as already pointed out in the introduction, the application to hadronic collisions is limited. The first clear deviations have been identified in forward production of low-\( p_t \) kaons and pions in \( p-p \) collisions at ISR/Fermilab fixed target energies (\( E_{\text{lab}} \geq 100 \text{ GeV} \)) [24], which showed strong enhancement of the \( \pi^+/\pi^- \) ratio at large \( x_F \) (up to a factor \( \sim 5 \)), and even more pronounced for \( K^+/K^- \). This lead Das and Hwa [13] to propose, within the parton model framework, the recombination model: forward produced quarks preferentially "pick up" valence up-quarks from the projectile, thus favoring the "leading" hadrons (\( \pi^+ = d\bar{u} \) and \( K^+ = s\bar{u} \)) over their "non-leading" antiparticles. In addition, as shown in Ref. [14], recombination is also capable of nicely describing the observed increase of \( \pi^-/K^- \) and \( K^+/\pi^+ \) for \( x_F \rightarrow 1 \).

From a theoretical point of view, (anti-) charm quarks are a cleaner probe of recombination dynamics due to a negligible probability of producing them in secondary reactions. Indeed, the recombination approach has been successfully applied [16–22] (see also [25,26]) to forward production of charmed hadrons, which similarly exhibit leading particle effects. However, little attention has so far been paid to light-flavor asymmetries in the bulk production of charmed hadrons, which necessarily requires substantial contributions in the central region (\( x_F \) around 0) at low \( p_t \) where most of the yield is concentrated.

Here we generalize the recombination approach to incorporate \( D \)-meson formation at small \( |x_F| \). As the low (Bjorken-) \( x \) region is predominantly populated by sea-quarks, the main extension concerns the evaluation of their recombination with \( c \)-quarks. One of our ideas here is, that, besides the valence content, the proton sea possesses a well established flavor asymmetry which could reflect itself in the flavor composition of \( D \)-mesons.

To well begin with, the evaluation of the production of \( c\bar{c} \) quarks has to be specified. For forward \( D \)-meson production it has been suggested that an "intrinsic" charm component [27,28] in the projectile wave functions could be responsible for significant recombination contributions [29,20]. Such a "hard" charm component seemed to be required to account for the leading \( D \)-meson distributions towards large \( x_F \) at ISR energies. On the other hand, more recent data [9] indicate that the shape of inclusive \( D \)-meson \( x_F \) distributions agrees well with pQCD predictions for bare (anti-) charm quarks (i.e., without momentum loss due fragmentation). We take this as a motivation for the following picture [16,17] that will be employed below: \( c\bar{c} \) quarks are assumed to be exclusively created in (primordial) hard parton-parton collisions, evaluated in LO pQCD upscaled by a single empirical \( K \) factor (\( \gtrsim 5-6 \) for typical choices of parton distribution functions (PDF’s) and charm-quark mass \( m_c = 1.35 \) GeV) to match the experimental yields. The such generated charm-quark \( (x_F-\text{-}) \) distributions are then subjected to recombination processes which necessarily involves nonperturbative information 1. For comparison with data, charm quarks that do not recombine are hadronized via isospin symmetric fragmentation with the usual polarization weights (i.e., 3:1 for \( D^*/D \)), and, for simplicity, in \( \delta \)-function approximation for the \( x_F \) dependence.

An important part of the inclusive charged and neutral pseudoscalar \( D \)-meson yields (as reported by most experiments) stems from feeddown contributions from \( D \)-meson resonances, most notably the vector-mesons \( D^*(2010) \). The

\[ 1 \] Note that in Ref. [22], a hard-scattered (projectile-) parton itself (after radiating off a gluon that fuses into \( c\bar{c} \) with a target gluon) participates in subsequent recombination, whereas in the present analysis spectator anti-\( /q \)-quarks recombine. Formally, the former process is suppressed by one power in \( \alpha_s \), which, for very forward production, is compensated by a large kinematic enhancement. Consequently, the results of Ref. [22] exhibit essentially no asymmetry for central \( x_F \lesssim 0.2 \).
two charge states of the latter exhibit rather asymmetric decay characteristics: the $D^*(2007)^0$ decays to $\sim 100\%$ in $D^0(1865)$ (with accompanying $\pi^0$ or $\gamma$), whereas the $D^*(2010)^+$ has a branching ratio of $68\%$ into $D^0(1865)\pi^+$ and only $32\%$ into $D^+ (1869)$ final states (charge conjugate modes for anti-$D$ mesons implied). As a consequence, the flavor composition of inclusive $D$-mesons depends on the relative abundance of directly produced $D$ and $D^*$ mesons (higher resonances are neglected). For flavor-symmetric direct production, and defining $\epsilon = N_{D^*}/(N_{D^*}+N_D)$, one finds for the inclusive charged-over-neutral $D$-meson ratio

$$R_{c/n} = \frac{(D^+ + D^-)}{(D^0 + D^{0*})} = \frac{1 - \frac{2}{3} \epsilon}{1 + \frac{2}{3} \epsilon},$$

whereas the inclusive particle-over-antiparticle ratios, $R_{+/+}$ and $R_{0/0}$, are independent of $\epsilon$. For $c$-quark fragmentation, we employ the usual partitioning according to spin-isospin polarization counting implying a 3:1 ratio for $D^*: D$, i.e., $\epsilon = 3/4$ and thus $R_{c/n} = 0.33$. On the other hand, the recombination contribution is essentially a coalescence process and therefore expected to be sensitive to the mass of the product. Taking as a guideline effective thermal weights as extracted from light hadron production in $pp$ collisions [31], with $T_H = 170-175$ MeV, results in a ratio $D^*/D \approx 3(m_{D^*}/m_D)^{3/2}\exp[-(m_{D^*} - m_D)/T_H] \approx 3/2$, i.e., $\epsilon = 0.6$, which will be adopted below. In the absence of any flavor asymmetry this value for $\epsilon$ implies $R_{c/n} = 0.43$. The $x_q$-dependence of particle/antiparticle asymmetries is often displayed in terms of the so-called asymmetry function, defined via the production ratios as

$$A_{-/+}(x_F) = \frac{N_{D^-} - N_{D^+}}{N_{D^-} + N_{D^+}} = \frac{R_{-/+} - 1}{R_{-/+} + 1},$$

$$R_{-/+} = \frac{1 + A_{-/+}}{1 - A_{-/+}},$$

and likewise for $D^0$ vs. $D^0$ (for baryon projectiles, anti-$D$ mesons are the "leading" particles).

For the remainder of this section we focus on the recombination contribution to $D$-meson formation. In the above specified approach, the pertinent cross section in hadronic collisions at high energies takes the form [16,17]

$$x^* \frac{d\sigma_{rc}}{dx_F} = \int \frac{dx_q}{x_q} \int \frac{dz}{z} \left( x_q z^* \frac{d^2\sigma}{dx_q dz} \right) \mathcal{R}(x_q, z; x_F),$$

and a similar expression for $\bar{D}$'s upon replacing $c \leftrightarrow \bar{c}$ and $\bar{q} \leftrightarrow q$. Here, $x^* = E/\sqrt{s}/2$ and $x = p_t/\sqrt{s}/2$ denote the $D$-meson energy and longitudinal momentum, respectively, normalized to the incoming proton (carrying $CM$ energy and momentum $\sqrt{s}/2$), $z^*$ and $z$ those of the charm-quark, and $x_q$ the momentum fraction of the (projectile) light antiquark participating in the recombination (note that, for nucleons, the antiquark necessarily arises from the sea). The double differential cross section for the simultaneous production of $c$ and $\bar{q}$ quarks is given by

$$x_q z^* \frac{d^2\sigma}{dx_q dz} = \int \frac{d\xi}{m_c^2} \sum_{i,j} \int_{x_1^{\min}}^{x_1^{\max}} \int_{x_2^{\min}}^{x_2^{\max}} \frac{x_q x_1 f_{qi}^{(2)}(x_{qi}, x_1) x_2 f_{j}^{(2)}(x_2)}{(x_1 - z_+)} \frac{d\delta_{ij}}{dt},$$

with kinematic boundaries $x_1^{\min} = z_+/(1 - z_-)$, $x_1^{\max} = 1 - x_q$ (defining $z_+ = \frac{1}{2}(z^* + z)$), and $W = s(1 - z - x_q)(1 - x_\bar{q})(1 + z)/(2 - x_q)^2$. $d\delta_{ij}/dt(s, m_{c,\perp})$ denotes the standard LO pQCD parton fusion cross section [32,27] (including the $K$-factor). The summation over $i, j$ accounts for the possible parton combinations from target and projectile hadrons, with $f_{j}$ the distribution function for the target (nucleon). An analogous expression describes recombination processes with target (light-) quarks, upon replacing $f_{qi}^{(2)}(x_1) \rightarrow f_i(x_1)$ and $f_{j}(x_2) \rightarrow f_{\bar{q}i}^{(2)}(x_2)$. The two crucial ingredients in evaluating the cross section, Eq. (4), are the recombination function $\mathcal{R}$ and the two-parton distribution function $f_{\bar{q}i}^{(2)}$. In the following we discuss both entities in more detail.

### B. Recombination Function

The recombination function $\mathcal{R}$ essentially represents the (absolute value squared) wave function of the nascent charm-meson in terms of its quark constituents. For forward production one can rather straightforwardly employ the parton model in the collision (CM) according to [13,18]

$$\mathcal{R}_D = [B(a,b)]^{-1} \xi_a^\alpha \xi_b^\beta \delta(1 - \xi_{\bar{q}} - \zeta)$$

3
rapidity variables, as first suggested in Ref. [15] via a (normalized) Gaussian distribution,

$$1 = \int_0^1 \frac{d\xi}{\xi} \frac{d\zeta}{\zeta} R_D .$$

(7)

The actual values of the exponents $a$ and $b$ have been motivated in two ways. On the one hand, in Ref. [18], the requirements that the average momentum fractions of the two constituents scales with their masses, and that $G_D \equiv R_D/(\xi \zeta)$ stays finite for $\xi \zeta \to 0$ (in the spirit of a constituent quark distribution function), are met by the choice $a=1$ and $b=5$. On the other hand, in Ref. [16], it has been argued on the basis of leading Regge trajectories that $a = 1 - \alpha_v$ with $\alpha_v = 0.5$ for $v = u, d$, and $b = 1 - \alpha_c$ with $\alpha_c \approx -2.2$. Our results shown below turn out to have little sensitivity to the difference between these two cases.

A more severe complication arises when applying the parton model wave function for small $D$-meson momenta, i.e., for $x_F$ close to zero (central production); here, light-cone momentum fractions become ill-defined, involving even contributions from backward moving charm or light (anti-) quarks (w.r.t. the collision CM). Due to the, in principle, invariant nature of the parton-model we can resolve this problem by evaluating the recombination function in a boosted frame, which we choose as the projectile rest frame. In practice this procedure is, however, beset with a residual frame dependence owing to the finite transverse masses of charm- and, more sensitively, light-quarks. We will use the bare-charm quark mass together with typical light quark transverse masses of $m_{q,t} \simeq 0.3 - 0.5$ GeV.

Another approach which avoids this complication is based on the formulation of the wave function in terms of rapidity variables, as first suggested in Ref. [15] via a (normalized) Gaussian distribution,

$$R_D(y_1, y_c, y_q) = \frac{1}{\sqrt{2\pi} \sigma_y} \exp \left( \frac{\Delta y^2}{2\sigma_y^2} \right) .$$

(8)

The latter is solely characterized by its width $\sigma_y$ and the rapidity difference $\Delta y = y_c - y_q$ with the charm- and light-quark rapidities given by

$$y_i = \frac{1}{2} \ln \left( \frac{E_i + p_{i,z}}{E_i - p_{i,z}} \right)$$

(9)

Again one is sensitive to the transverse mass, $m_{q,t}$ of the light quark within the incoming hadron, which here, however, carries a physical meaning: it is a nonperturbative quantity (indirectly) accessible by various experimental information (e.g., transverse momentum spectra of Drell-Yan pairs).

In Sect. III we will discuss both approaches of evaluating the recombination function, i.e., parton-model vs. rapidity-space wave functions. The pertinent results presented turn out to be rather robust. This, after all, can be understood from the notion that also the parton model wave functions imply the main contribution from a rather limited relative rapidity interval of $\langle \Delta y \rangle = 1 - 2$ [15,16].

C. Two-Parton Distributions

1. Kinematic Correlations

The two-parton distribution function (2-PDF) $f_{\bar{q}i}^{(2)}$, which importantly figures into the double-differential cross section, Eq. (5), characterizes the probability to simultaneously find parton $i$ at $x_1$ (participating in the hard fusion process into $c\bar{c}$) and the antiquark $\bar{q}$ with momentum fraction $x_{\bar{q}}$ (participating in the recombination process). In general it can be cast into the form

$$x_{\bar{q}}x_1 f_{\bar{q}i}^{(2)}(x_{\bar{q}}, x_1) = x_{\bar{q}} f_\bar{q}(x_{\bar{q}}) x_1 f_i(x_1) \rho(x_{\bar{q}}, x_1) ,$$

(10)

where $f_q(x_{\bar{q}})$ represents the constrained probability for parton $\bar{q}$ at momentum $x_{\bar{q}}$ under the condition that parton $i$ is at $x_1$, and $\rho(x_{\bar{q}}, x_1)$ encodes all other correlations which are usually assumed to be governed by phase space.

As a simple ansatz the following factorized form has been proposed [13],

$$f_{\bar{q}i}^{(2)}(x_{\bar{q}}, x_1) = C_2 \ f_\bar{q}(x_{\bar{q}}) \ f_i(x_1) \ (1 - x_{\bar{q}} - x_1)^p ,$$

(11)
where $C_2$ is a normalization constant. The last factor in Eq. (11) implements a phase space dependence for large momentum fractions $x_\eta + x_1 \to 1$ (for a collinear 3-body phase where, in the CM system, target and forward $[\bar{q}q]$ state recede from each other with the projectile at rest, one has $p = 1$ [13]). Some confidence in this ansatz was drawn from phenomenological successes in describing, e.g., (semi-) inclusive $\pi$- and $K$-production in the fragmentation region [13,14]. In subsequent work [18] the 2-PDF has been elaborated on a more microscopic basis within the so-called valon model of (low-$p_t$) hadronic structure, where light (anti-) quark distributions are generated from composite valence quarks at low momentum scales, see, e.g., Ref. [30] for a recent update. Within this approach, also $c\bar{c}$ production arises from Fock components of the valon distributions, similar to the "intrinsic-charm" model [27,28]. Their contribution can be significant at large $x$, but cannot account for inclusive charm yields. However, since our primary objective here is the evaluation of $D$-meson flavor dependencies in bulk production, we retain the $c\bar{c}$ cross section as a hard process in factorized form, cf. Eq. (5). Thus, results to be discussed below will refer to the simplified ansatz, Eq. (11), for the 2-PDF's.

A more explicit treatment anti-charm quark recombination with leading (valence) light quarks within a factorized scheme has been undertaken in Refs. [16,17]. At the price of more schematic underlying 1-PDF's, a 2-PDF of the form

$$f_{vi}^{(2)} = A \eta x^{-\alpha_v} x^{\gamma} (1 - x_1 - x_v)^m (1 - x_1)^k$$

(12)

has been proposed. Here, $x_v$ denotes the momentum fraction of the (recombining) valence quark in the nucleon, and the exponents $\alpha_v$, $\gamma$, $k$ and $m$ reflect the appropriate limiting behaviors for $x_{1,v} \to 0,1$ (e.g., $\alpha_v = 0.5$ for valence quarks, $\gamma = 1$ for sea quarks and gluons). The ansatz, Eq. (12), is furthermore constrained by the normalization condition

$$\int_0^{1-x_1} dx_v f_{vi}^{(2)}(x_1, x_v) = f_1(x_1),$$

(13)

which can be used to fix the numerical constant $A$ as well as $k$ and $m$ according to the parton type $i$ (note that this can only be applied for the case of valence quarks $v$). In particular, it has been argued [16] that for fixed $x_v$, $f_{vi}^{(2)}(\eta_1)$ with $\eta_1 = x_1/(1-x_v)$ can be interpreted as the single-parton distribution of the quark $i$ in the hadron remnant $\tilde{h}_1$ (with the valence quark $v$ removed), and that for $x \to 1$ and $z \to x$ an approximate factorization into two single-parton distributions holds, i.e.,

$$f_{vi}^{(2)} \approx C x^{1-\alpha_i} (1-x_v)^{-\gamma} f_i(\eta_1) \propto f_v(x_v) (1-x_v)^{-\gamma} f_i(\eta_1).$$

(14)

In an alternative view, one might consider $f_{vi}^{(2)}$ at fixed $x_1$ (being determined on a shorter timescale – hard $c\bar{c}$ production – than $x_v$) as the single-parton distribution of quark $v$ in the hadron remnant without parton $i$. In this case, one straightforwardly obtains from Eq. (12) the factorized form

$$f_{vi}^{(2)} = \tilde{A} f_v(\eta_v) f_i(x_1) (1-x_1)^{-1}$$

(15)

with $\eta_v = x_1/(1-x_1)$ and $\tilde{A} = 1$ upon imposing Eq. (13). This result suggests an immediate generalization for sea quarks according to

$$f_{qi}^{(2)} = f_q(\eta_q) f_i(x_1) (1-x_1)^{-1}$$

(16)

with $\eta_q = x_\bar{q}/(1-x_1)$.

To illustrate the differences between the simple factorization ansatz with phase space correction, Eq. (11), and the newly suggested ansatz "remnant" 2-PDF, Eq. (16), we show in Fig. 1 partice-antiparticle asymmetries in $D$-meson production for fixed target $p\bar{p}$ collisions. The results include the recombination contribution only, and are based on a rapidity-space recombination function, Eq. (8), with $\sigma_y = 0.5$ (using GRV94-LO [33] for the underlying 1-PDF's). In the central region, where both $x_\bar{q}$ and $x_1$ are small, mutual (anti-) correlations are suppressed, and the two ansätze indeed approximately coincide. However, the "remnant"-2-PDF, Eq. (16), develops a somewhat steeper increase of the leading-particle asymmetry with increasing $x_F$. Note also, that recombination with valence quarks is the prevalent process even for central $x_F$ (indicated by nonzero values at $x_F = 0$, as well as the larger asymmetry in the neutral ratio involving $u$-quarks). The $d/\bar{u}$ asymmetry in the proton sea, which favors $D^+$ formation, does not significantly suppress the charged ratio. One should also point out that the absolute magnitude of the recombination cross section for the individual $D$-meson states is about 20% larger with the 2-PDF of Eq. (16) (which leads to larger total asymmetries once combined with isospin-symmetric fragmentation contributions).
FIG. 1. Asymmetry function $A(x_F)$, Eq. (2), for $\bar{D}/D$ production from the recombination part of the cross section only in $p$-$p$ collisions at $E_{lab} = 250$ GeV. The dashed lines are obtained by using "remnant"-2-PDF’s, Eq. (16) for both valence and sea-quarks, whereas the full lines correspond to the factorized 2-PDF, Eq. (11) with $C_2 = p = 1$. The respective upper (lower) lines are for the $\bar{D}^0/D^0$ ($D^-/D^+$) asymmetries.

2. Flavor Correlations

An obvious (global) flavor correlation that will be imposed throughout consists of "vetoing" a light (anti-) quark flavor $i$ for the recombination process if this type has been used in the hard process of creating a $c\bar{c}$ pair, i.e., $\bar{q} \neq i$. The significance of this anti-correlation mainly establishes itself for leading $D$-mesons ($c\bar{c}$) in forward production with meson beams [34], where the valence anti-quark provides a large part of the $c\bar{c}$ yield.

A more delicate issue is the one of local correlations, relating to anisotropies of the quark and gluon distributions within a hadron. To illustrate their possible effects we will investigate one rather extreme scenario, which can be motivated, e.g., by the importance of instantons in hadron structure. The latter provide a qualitative explanation for the observed $\bar{d}/\bar{u}$ excess in the proton [35,36] (cf. Ref. [37] for a recent overview), as follows. Due to the specific properties of the instanton-vertex, a valence $u$-quark coupling to it has to be accompanied by an in- and outgoing $d$-quark. The $d$-quark line can either be provided by another valence-quark (which generates a strong $ud$-diquark binding in the nucleon), or be closed off by a condensate insertion (which, after all, generates the constituent mass of the $u$-quark). The second case implies that $u$-valence quarks are essentially accompanied by a $d\bar{d}$ "cloud", and $d$-valence quarks by a $u\bar{u}$ one, thus generating a 2:1 asymmetry in $\bar{d}/\bar{u}$, which roughly corresponds to the experimental value in the $x \leq 10^{-1}$ region. At the same time, one expects the gluon cloud to be concentrated around the valence quarks. Imposing such a flavor correlation on the 2-PDF (e.g., for gluon fusion, in $2/3$ of the cases $c$ and $\bar{c}$ can only recombine with either $u$ valence or $d$ and $\bar{d}$ sea-quarks), will be referred to as "local flavor correlations" below.

III. COMPARISON WITH EXPERIMENT

A. Total $c\bar{c}$ Yields and $K$-Factors

It is well-known that LO pQCD calculations for $\bar{q}q, gg \rightarrow c\bar{c}$, coupled with LO PDF’s for the colliding hadrons, underestimate total charm production by an appreciable magnitude characterized by a typical factor $K \simeq 5$. The latter depends rather little on the specific parton distribution function, but bears some sensitivity to the value of the charm-quark mass. NLO results can in principle account for a substantial part of the discrepancy, but large uncertainties due to the choice of renormalization and factorization scales persist [1] (see also Ref. [41] for a recent update). We here adopt the LO results for $d\bar{\sigma}_{ij\rightarrow c\bar{c}}/d\hat{t}$ in Eq. (5), with the (bare) charm-quark mass fixed at $m_c = 1.35$ GeV. For

\[ p-p \rightarrow DX \]

\[ s^{1/2} = 21.7 \text{GeV} \]
comparison we will allow for two different sets of PDF’s in our calculations, i.e., the GRV-94 LO [33] and MRST01 [42] sets for nucleons, as well as GRS-99 LO [43] for pions. Fig. 2 shows the fit to fixed target $pN$ (left panel) and $\pi N$ (right panel) reactions, which are reasonably well reproduced, albeit with significantly different $K$-factors (which might be related to the fact that for pion projectiles valence-antiquarks contribute appreciably through annihilation on target valence-quarks).

Based on these total $c\bar{c}$ cross sections we will in the following address the magnitude and flavor composition of $D$-meson production from recombination based on Eqs. (4) and (5). As detailed in Sect. II, charm quarks that do not recombine will be hadronized with "standard" fragmentation assuming no longitudinal momentum loss. We will mainly focus on $x_F$ distributions and excitation functions (i.e., inclusive yields for $x_F > 0$ as a function of collision energy, $\sqrt{s}$) for the ratios $(D^+ + D^-)/(D^0 + \bar{D}^0)$, $D^-/D^+$ and $\bar{D}^0/D^0$.

### B. Flavor Asymmetries I: Proton-Nucleon Collisions

#### 1. Inclusive $x_F$ Distributions

Let us first address the typical features of recombination cross sections as a function of $x_F$. Fig. 3 confronts E769 data [9] for inclusive $D+D_s$ mesons with our calculations for recombination with different wave functions $R_D$ according to Eqs. (6), (8), and a common two-parton distribution function of factorized form, Eq.(11), with $C_N = p_N = 1$ (at this point there is no sensitivity to more detailed properties of the 2-PDF). Also shown is (80% of) the pQCD yield for $(c$ and $\bar{c}$ quarks (solid curve), assumed to convert into $D$-mesons without longitudinal momentum loss ($\delta(x-z)$-)fragmentation). This distribution (for the MRST01 set with $K=5$ as fixed above) reproduces the data reasonably well. In general, the fraction due to recombination is sizable in the central region (where it makes up 30-60% of the total cross section), and starts to dominate at forward $x_F \geq 0.4$, where it can even exceed the primordial $c$-quark distribution by "pick-up" of a comoving light quark.

In more detail, we find that using the parton-model recombination function with $a=1$, $b=5$ and $m_t = 0.3$ GeV gives a $\sim50\%$ contribution to the total $D$-meson yield. This is reduced to $\sim30\%$ upon using $m_t = 0.5$ GeV (not shown), with most of the reduction occurring at central $x_F \leq 0.2$. On the contrary, with the rapidity space recombination function the sensitivity to the transverse mass is essentially absent, giving identical yields within $2\%$ for the two values of

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$^2$the MRST01 parametrization is performed to higher order (HO) in $\alpha_s$, which is, strictly speaking, not consistent with the LO evaluation of the $c\bar{c}$ cross section; on the other hand, nonperturbative effects introduced via the recombination mechanism evade any expansion in $\alpha_s$ for the $D$-meson production cross section of Eq. (5).
FIG. 3. Comparison of E769 p-A data for inclusive production of D and D_s mesons to the calculated recombination contributions using the parton-model wave function (dashed-dotted line), Eq. (6) with \(a=1\) and \(b=5\), as well as the rapidity-space ones, Eq. (8) with \(\sigma_y=0.5\) (solid line) and 1 (dotted line). The long-dashed line is obtained from the solid one upon replacing the MRST01 PDF’s by the GRV94-LO set.

...of \(m_t\). When increasing \(\sigma_y\) from, e.g., 0.5 to 1, the \(x_F\) distribution broadens slightly with the integrated yield again being stable (within 1%). Also, the use of rapidity wave functions increases the recombination fraction of \(c\) and \(\bar{c}\) quarks to almost 70%. Due to these rather robust features of \(R_D(\Delta y)\), (and the absence of boost ambiguities for central \(x_F\) as opposed to the parton model wave function, cf. Sect. II B) we will employ this form from now on (fixing \(\sigma_y = 0.5\)) unless otherwise stated.

FIG. 4. Comparison of E743 p-p data for inclusive production of nonstrange D mesons to the recombination contributions calculated with the rapidity wave function, Eq. (8), with \(\sigma_y=0.5\) (lower solid line). The upper solid line corresponds to 67% of the underlying \(c\) and \(\bar{c}\) quark distributions (MRST01 with \(K=5\)). The dashed line indicates the recombination cross section into \(D_s\)-mesons.
A similar comparison at higher energy is performed in Fig. 4 with nonstrange $D$-mesons from $p$-$p$ collisions measured by E743 [3]. At $\sqrt{s}=39$ GeV, the fraction of the recombination cross section to the expected integrated yield (which corresponds to $2/3$ of the total $c\bar{c}$ cross section) amounts to almost 80\%, as compared to about 70 \% for the corresponding calculation at $\sqrt{s}=21.7$ GeV, cf. Fig. 3. This is due to the fact that at higher energies smaller $x$ values of the parton distributions are probed where the occupancy is higher. Of course, for recombination fractions approaching 1 interference effects will play an increasingly important role, which have been neglected here. We also note that the recombination cross section into strange $D_s$-mesons, displayed by the dashed line in Fig. 4, constitutes about 25\% of the nonstrange one, very similar to what has been inferred for total yields [40]. This fraction is somewhat smaller when using the GRV94-LO distributions.

2. Excitation Function of Flavor Ratios

From here on all our results for $D$-meson recombination cross sections are supplemented with (isospin-symmetric) $\delta$-function fragmentation of the remaining $c$ and $\bar{c}$ quarks (except at large $x_F$ where the recombination part may even exceed the $c$-quark distribution), with appropriate feeddown systematics from $D^*$ resonances, as elaborated in Sect. II A. Throughout, the two parameters of the nucleon 2-PDF, Eq. (11), are fixed at $C_2=0.8$ and $p=1$, together with rapidity space recombination functions ($\sigma_y=0.5$). For the underlying 1-parton distribution functions we choose the GRV94-LO set (in order to be consistent with the GRS99 [43] pion sets in subsequent sections). For nuclear targets, the appropriate neutron fraction has been employed.

![FIG. 5. Collision energy dependence of various $D$-meson flavor ratios in $pp$ and $pA$ collisions compared to model calculations accounting for both recombination and isospin-symmetric fragmentation contributions (including feeddown from $D^*$ mesons using recent branching ratios). Rapidity space recombination functions (with $\sigma_y=0.5$) have been employed in connection with the factorized 2-PDF, Eq. (11), with $C_2=0.8$ and $p=1$. The results for $pN$ collisions ($Z/N=0.8$ for target nucleons) are given by the solid and dashed lines, where the latter include an additional assumption of local flavor correlations, cf. Sect. II C 2. The dashed-dotted lines are for $pp$ collisions including local flavor correlations as well. The dotted lines represent the isospin-symmetric fragmentation mechanism alone (assuming a $D^*/D$ ratio of 3).]
Fig. 5 shows the pertinent $\sqrt{s}$-dependence of $x_F$-integrated nonstrange $D$-meson flavor ratios in $pp$ and $pN$ collisions, indicating reasonable agreement with available measurements for charged and neutral ratios (middle and lower panel, respectively), whereas the asymmetry in the charged over neutral ratio (upper panel) appears to be underpredicted. A marginal increase of the latter can be achieved by introducing local flavor correlations within a valence-quark type picture (as outlined in Sect. II C 2), but not enough to significantly improve the description of the data. For pure proton targets, an additional small increase for the charged over neutral ratio is found (dashed-dotted lines). In fact, for proton targets the increase in the individual recombination with both valence $u$-quarks to form $\bar{D}^0$ mesons and with sea $d$-quarks to form $D^+$ mesons is very moderate, as indicated by the $D^-/D^+$ and $\bar{D}^0/D^0$ ratios on proton targets (cf. dashed dotted lines in the middle and lower panels).

C. Flavor Asymmetries II: Pion-Nucleon Collisions

1. $x_F$ Dependencies

Flavor asymmetries in forward $D$-meson production from $\pi N$ collisions are experimentally well-established [6–8,10]. In Fig. 6 we compare results of our approach with the $x_F$-dependence of $D^-/D^+$ and $D^0/\bar{D}^0$ ratios as measured by WA92 [10] in 350 GeV $\pi^- N$ reactions.

The simple factorization for the 2-PDF, i.e., Eq. (11) with $C_2 = p = 1$ for pions, leads to a $x_F$-dependence of the charged ratio (upper dashed-dotted line) that is somewhat flatter than the data. The description is much improved (upper solid curve) when employing a weaker phase space suppression (reducing the exponent to, e.g., $p_{\pi} = 1/4$) in connection with a smaller normalization constant (e.g., $C_2^{\pi} = 0.4$). The latter suppresses the asymmetry in the central region, whereas the former entails a stronger increase towards forward $x_F$. On the other hand, the neutral $D$-meson ratio is insensitive to this modification. The main feature of these data is the near absence of a leading particle asymmetry, even at very forward $x_F$. The model calculations essentially reproduce this behavior, which arises from the fact that the $\bar{u}$-valence quark in the pion, which on average carries a large projectile momentum fraction, contributes significantly to the (hard) $c\bar{c}$ production process (via annihilation on $u$-valence quarks in the nucleon), and thus is not at disposal for subsequent recombination. For small $x_F$ the measured asymmetry seems to become even negative, which in the calculations is due to significant contributions from recombination processes of $\bar{c}$ quarks with $u$ quarks from target nucleons. This effect becomes more pronounced when introducing local flavor correlations within the nucleon (lower dashed line), which increases (decreases) recombination with $u$ (\bar{u}) into $\bar{D}^0$ ($D^0$) mesons; at

![Fig. 6. WA92 data [10] for the inclusive $D^-/D^+$ (circles) and $D^0/\bar{D}^0$ (squares) asymmetries versus Feynman-$x_F$ in 350 GeV $\pi^- A$ collisions, compared to recombination model calculations supplemented with isospin-symmetric $\delta$-function fragmentation contributions.](image-url)
At a higher projectile energy (500 GeV), the E791 collaboration measured the $x_F$-dependence of the inclusive $D^-/D^+$ ratio. As before, the modified 2-PDF with $C_\pi^2 = 0.4$ and $p_\pi = 1/4$ accounts much better for the data than with $C_\pi^2 = p_\pi = 1$, cf. Fig. 7. In general, over the range of fixed target energies discussed here, the projectile-energy variation in the $x_F$-dependencies of the flavor asymmetries is rather weak.

FIG. 7. E791 data [8] for the $D^-/D^+$ asymmetry versus $x_F$ in 500 GeV $\pi^-A$ collisions, compared to recombination model calculations supplemented with $\delta$-function fragmentation contributions.

FIG. 8. $x_F$-differential cross sections from WA92 [10] for inclusive yields of light $D$-mesons in 350 GeV $\pi^-A$ reactions, compared to recombination model calculations supplemented with fragmentation contributions.
In Fig. 8 differential $x_F$-spectra for the absolutely normalized recombination + fragmentation model calculations (with $K=3$ in accordance with Fig. 2) for the individual $D$-meson states (underlying the solid lines in Figs. 6 and 7) are compared to data from WA92 [10]. Reasonable agreement in all channels over the entire $x_F$-range is observed.

2. $\sqrt{s}$ Dependencies

We finally turn to the excitation function of the flavor asymmetries in $\pi N$ collisions. Again, our combined fragmentation + recombination approach gives a rather satisfactory description of the observed ratios. In contrast to the $pN$ case (where, however, the data accuracy is rather limited), now also the charged over neutral ratio is in line with the data. Overall, the flavor asymmetries introduced by recombination processes improve the agreement as compared to contributions from (isospin-symmetric) fragmentation alone (represented by the dotted lines) in all channels.

![Graph](image_url)

FIG. 9. Collision energy dependence of various $D$-meson flavor ratios in $\pi^- A$ collisions.

It should also be noted that overall charm conservation has not been enforced explicitly. However, the slight excess of anti-charm quarks induced by the $D^-/D^+$ asymmetry is conceivably compensated by a positive $\Lambda_c/\bar{\Lambda}_c$ asymmetry, which is also observed experimentally [12] (but which we did not address here). The same argument applies for proton projectiles, where both $D^-/D^+$ and $D^0/D^+$ are larger than one, but at the same time the $\Lambda_c$ asymmetry is more pronounced than for pion beams [12].

We close this section with a note on $D_s$ mesons. For the recombination contribution alone, the ratio for (inclusive) strange over nonstrange $D$-mesons at $\sqrt{s} = 26$ GeV turns out to be 12% for forward production (13% for all $x_F$), with almost no energy dependence. This ratio increases slightly to 14-15% when replacing the GRV-LO with the GRV-HO PDF’s. From a compilation in Ref. [10], the experimentally measured ratios can be found to be $15.8 \pm 2.9\%$ for 230 GeV $\pi^-$ beams [39], $16.7 \pm 3.3\%$ at 250 GeV [9], and $11.6 \pm 1.4\%$ at 350 GeV, with a combined average of $R_{D_s/D} = 12.9 \pm 1.2\%$. This value is in surprising agreement with the recombination approach. In fact, for the remaining charm quarks, fragmentation should apply. The prediction of the LUND event generator [44] (where hadronization is essentially performed by string fragmentation) turns out to be $\sim 8\%$, somewhat below the data. However, the combined recombination + fragmentation result would still be close to the observed value.
An important part of the physics program at the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven consists of (polarized) $p$-$p$ collisions up to a maximum CM of 500 GeV. Collisions at 200 GeV have been performed in the year-2 (2002) and year-3 running periods, although direct open charm measurements are not expected before detector upgrades have been performed. We here quote our predictions for $D$-meson flavor ratios by extrapolating our recombination + fragmentation approach according to Sect. III B to 200 GeV CM energy. We find:

\[ R_{(D^+ + D^-)/(D^0 + D_s)} = 0.40 ; \quad R_{D^-/D^+} = 1.24 ; \quad R_{D^0/D^s} = 1.35 ; \quad R_{D_s/D} = 0.23 . \]

These values are all rather close to the ones at fixed target energies.

V. CONCLUSIONS

Charm-meson production in hadronic collisions at fixed target energies has been investigated with emphasis on asymmetries in the light-flavor sector. The calculations have been performed within a recombination model for produced anti-/charm quarks with surrounding light quarks in projectile and target, supplemented with a standard fragmentation treatment for the remaining $c$ and $\bar{c}$ quarks. More precisely, the recombination component has been evaluated assuming a factorization of the perturbatively calculated (hard) $c\bar{c}$ production vertex and the subsequent (nonperturbative) "coalescence" process. On the one hand, this requires the use of a 2-parton distribution function, for which we used a simple factorization ansatz (as first suggested 25 years ago). On the other hand, the recombination function, as compared to previous analyses, has been generalized to incorporate coalescence with sea-quarks within the colliding hadrons, necessary to address bulk production dominated by the central region ($x_F \approx 0$). Color correlations have been neglected throughout.

Employing recent parameterizations of parton distribution functions for nucleon target as well as pion and proton projectiles, we have found that (i) recombination processes contribute significantly to total $D$-meson yields and, (ii) the description of observed flavor asymmetries \(- (D^+ + D^-)/(D^0 + D^s), D^-/D^+ \text{ and } D^0/D^s \)- in bulk production is improved throughout as compared to isospin-symmetric fragmentation alone. The nontrivial isospin asymmetries (i.e., beyond known feeddown corrections from $D^*$ states) are driven by the valence- and sea-quark content of the colliding hadrons. No significant sensitivity was found with respect to local flavor correlations, as one might expect from valence-quark or quark-diquark substructures in the nucleon. Except for the charged-over-neutral asymmetry (which is, however, beset with large uncertainties), the magnitude and (not very pronounced) collision energy dependence of available fixed target data is rather well reproduced.

As has been noted before, the same framework is also able to explain the differential $x_F$ dependencies of the flavor asymmetries as measured in $\pi N$ collisions. These become more pronounced at forward $x_F$, and are, within the model calculations, more sensitive to phase space correlations in the 2-parton distribution functions.

Future data on identified charm mesons at collider energies, such as expected from $p$-$p$ runs at RHIC, will be most valuable to further test the relevance of recombination mechanisms in the hadronization process, and thus contribute to a better understanding of the latter.

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3We recall that the recombination part of the charm cross section increases with collision energy; at 200 GeV, the calculated fraction of $D+D_s$-mesons amounts to 67% of the total $c\bar{c}$ yield (or 83% of the expected total $D+D_s$ yield; note that these numbers include the coefficient $C_2 = 0.8$ for the nucleon 2-PDF used in all results starting with Fig. 5); our estimates for the ratios at RHIC energy do not account for potentially sizable destructive interference effects.