We show how to derive the equations of light propagation in the gravitational field of uniformly moving mass monopoles without formulating and integrating the differential equations of light propagation in that field. The well-known equations of light propagation in the gravitational field of a motionless mass monopole are combined with a suitable Lorentz transformation. The possibility to generalize this technique for the more complicated case of uniformly moving body of arbitrary multipole structure is discussed.

Astrometry – Reference systems – Relativity – Gravitational Lensing

Light Propagation in the Gravitational Fields of Moving Bodies S.A. Klioner

Introduction

Within the next decade the accuracy of space-based astrometric positional observations is expected to attain an accuracy of 1 microarcsecond (µas). This technical progress is expected to be achieved due to a number of space astrometry projects (e.g., GAIA GAIA:2000, Perryman:et:al:2001, Bienayme:Turon:2002 and SIM SIM:1998 approved by ESA and NASA). Modeling of the observed data with such an accuracy is a highly non-trivial task that in any case requires the use of general relativity. The whole reduction scheme should be consequently formulated within the framework of general relativity. Recently, a practical relativistic model for positional observations of microarcsecond accuracy performed from space has been formulated by Klioner:2003. In particular, many subtle relativistic effects in light propagation should be accounted for to attain the goal accuracy of 1 µas. One of the most intricate point of the whole relativistic model is to compute the effects of the translational motion of gravitating bodies on the light propagation.

For the first time this problem was treated probably by Hellings:1986 who recommended to use the standard post-Newtonian formulas for the light propagation in the gravitational field of a motionless body and substitute in those formulas the position of each gravitating body at the moment of closest approach of that body and the photon. Next step has been done by Klioner:1989 where the problem has been solved completely for the bodies moving with a constant velocity in the first post-Newtonian approximation. The effects of accelerations of the bodies have been further treated by Klioner:Kopeikin:1992 where it was shown that if the coordinates and velocities of the bodies are computed at the moments of closest approach of the corresponding body and the photon, the residual terms of the solution are in some sense minimized. The complete solution of the problem for arbitrarily moving bodies in the first post-Minkowskian approximation was found by Kopeikin:Schaefer:1999 who succeeded to integrate analytically the post-Minkowskian equations of light propagation in the field of arbitrarily moving mass monopoles (see Appendix appendix:Kopeikin:Schaefer for an explicit form of the Kopeikin-Schäfer solution). These authors used the representation of the metric tensor through the Lienard-Wiechert potentials with retarded argument. The Kopeikin-Schäfer scheme has been generalized by Kopeikin:Mashhoon:2002 onto the case of arbitrarily moving bodies possessing mass monopoles and spin dipoles.

Extensive numerical simulations of light propagation in the time-dependent gravitational field of the solar system have been recently done by Klioner:Peip:2003. That publication contains also detailed information of all the approaches mentioned above. The aim of the numerical simulations was to check the practical accuracy of various approximate analytical solutions for the light propagation in the field of moving bodies and verify the practical recommendations formulated by Klioner:2003.

Up to now all theoretical results concerning light propagation in the field of moving bodies concerned moving mass monopoles (i.e. “point masses”). It is well known, however, that for GAIA and other space missions not only the mass monopoles, but also the gravitational fields produced by higher multipoles
(especially, by mass quadrupoles) are important. This paper is the first one in a series of papers where the light propagation in the gravitational field of moving bodies with full multipole structure will be investigated. The aim of this paper is to show that the Lorentz transformation can be used to derive the laws of light propagation in the gravitational field of a system of uniformly moving bodies in the first post-Minkowskian (or post-Newtonian) approximation of general relativity.

Let us clarify here that the post-Minkowskian approximation scheme deals with expansions in powers of the gravitational constant $G$. The first post-Minkowskian approximation implies that all terms of order $O(G^2)$ are neglected. The post-Newtonian approximation scheme operates with expansions in powers of $c^{-1}$. In the first post-Newtonian approximation terms of order $O(c^{-4})$ are neglected in the equations of light propagation. One can prove that in the case of light propagation the formulas of the first post-Newtonian approximation are linear in $G$ and, therefore, contained in those of the first post-Minkowskian approximation.

It is well known that the differential equations of light propagation in both the post-Newtonian and post-Minkowskian approximation are linear with respect to the non-Galilean components of the metric tensor $h_{\alpha\beta}$:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta},$$