Stated succinctly, the original version of the Campbell-Magaard theorem says that it is always possible to locally embed any solution of 4-dimensional general relativity in a 5-dimensional Ricci-flat manifold. We discuss the proof of this theorem (and its variants) in $n$ dimensions, and its application to current theories that postulate that our universe is a 4-dimensional hypersurface $\Sigma_0$ within a 5-dimensional manifold, such as Space-Time-Matter (STM) theory and the Randall & Sundrum (RS) braneworld scenario. In particular, we determine whether or not arbitrary spacetimes may be embedded in such theories, and demonstrate how these seemingly disparate models are interconnected. Special attention is given to the motion of test observers in 5 dimensions, and the circumstances under which they are confined to $\Sigma_0$. For each 5-dimensional scenario considered, the requirement that observers be confined to the embedded spacetime places restrictions on the 4-geometry. For example, we find that observers in the thin braneworld scenario can be localized around the brane if its total stress-energy tensor obeys the 5-dimensional strong energy condition. As a concrete example of some of our technical results, we discuss a $Z_2$ symmetric embedding of the standard radiation-dominated cosmology in a 5-dimensional vacuum.