On Winding Branes and Cosmological Evolution of Extra Dimensions in String Theory

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We consider evolution of compact extra dimensions in cosmology and try to see whether wrapped branes can prevent the expansion of the internal space. Some difficulties of Brandenberger and Vafa mechanism for decompactification are pointed out. In both pure Einstein and dilaton gravities, we study cosmology of winding brane gases in a continuum approximation. The energy momentum tensor is obtained by coupling the brane action to the gravity action and we present several exact solutions for various brane configurations. T-duality invariance of the solutions are established in dilaton gravity. Our results indicate that phenomenologically the most viable scenario can be realized when there is only one brane wrapping over all extra dimensions.

I. INTRODUCTION

String/M theory has a number of fascinating features as a unified theory of all interactions. However, consistency of the theory requires existence of extra dimensions. Observationally, the sizes of these dimensions are much smaller than the size of our perceived universe. On the other hand, standard cosmology tells us that the universe started out very small (possibly close to Planck length) and after various cosmological eras grew to its size today. Therefore, one of the main problems of string/M theory applied to cosmology is to determine why the extra dimensions remained comparatively small during this cosmological evolution?

In string theory, a remarkable mechanism to answer this question was proposed in [1]. In that paper, Brandenberger and Vafa (BV) first assumed that all dimensions were compact and started out very small. Then, they noted that the winding states of the string would like to prevent expansion since increasing the volume increases the energy of these states. In thermal equilibrium, one expects a decrease in the number of winding modes during expansion. Since winding number is topologically conserved, only the strings having opposite orientations can cause unwinding. However, such an interaction takes place when the world-sheets of two such strings intersect and this happens most efficiently in three spatial dimensions. Therefore in a three dimensional subspace winding strings can be annihilated keeping them in thermal equilibrium with the rest of the string modes and this results a decompactification. On the other hand, strings winding other directions fail to annihilate each other, fall out of equilibrium and stop the expansion.

In [2] (see also related work on the subject of brane gas cosmology [3]-[19]), this idea has been generalized to include higher dimensional extended objects. Generically, two $p$-branes most efficiently intersect in at most $2p+1$ spatial dimensions. Therefore, in nine dimensions (required by string theory) all winding modes of $p$-branes with $p > 3$ are annihilated. Branes having smaller dimensions dictate a hierarchy of sizes during expansion. First, 3-branes allow seven dimensions to become large. Then, a five dimensional subspace is chosen by membranes. And finally strings permit only a three dimensional subspace to grow.

It is possible to point out three difficulties with the above scenario. The first one is that the question

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of which dimensions are picked up for expansion is left unanswered. Presumably, quantum or thermal fluctuations determine this randomly. However, as discussed in [1], unwinding interactions are very short ranged since this can only happen when the world-volumes physically intersect. Moreover in BV mechanism three dimensions are picked up when the strings fail to intersect each other effectively i.e. when the size of the universe exceeds a critical value. Therefore, it is plausible that at the time of decompactification the universe contains many “causal patches” with respect to unwinding interactions and different dimensions become large in different patches.

The second difficulty is that, in principle, two $p$-branes with different values of $p$ may find each other to change the winding number. Similar interactions are known to exist in string/M theory. For instance, in the presence of background Ramond-Ramond fluxes, Myers effect [21] causes a point-like graviton to expand into a spherical D3-brane. It is also well known that a D-brane can absorb closed strings so that the quantum numbers (like the winding number) are encoded in the D-brane world-volume fields. Therefore, it seems reasonable to expect that in nine spatial dimensions strings may efficiently interact with seven or eight dimensional branes to change the winding number. This causes, without any difficulty, all winding states to remain in thermal equilibrium with the rest of the modes.

Finally, as discussed in [20], a uniform distribution of winding $p$-branes along a transverse direction cause it to expand. Therefore, for more than one brane wrapping different dimensions one has to take into account this effect. Consider, for instance, a toy model of strings with two compact dimensions, named $x$ and $y$. The strings winding $x$ look like point particles with respect to $y$. Homogeneity requires a uniform distribution of such strings in $y$ direction and this forces $y$ to expand (see Fig. 1). In this case one has to compare this expansion to the contraction forced by strings winding $y$ to decide whether decompactification occurs.

![FIG. 1: The closed strings winding along $x$ and $y$ directions (the torus is identified along $x$-direction). Strings winding $x$ force $y$ to expand. One has to compare this expansion to the contraction forced by the strings winding $y$.](image)

It is possible to avoid the first two complications mentioned above by simply assuming that the observed three dimensional space is topologically non-compact. This, one may hope, to be fixed by an unknown stringy consistency condition. In this case, there is no problem for these dimensions to expand. During this expansion, the density of branes winding compact dimensions start to decrease in the three dimensional subspace. At some point, these branes will be diluted enough so that they generically will not be able to find each other to intersect. This will cause winding modes to fall out of equilibrium and one hopes this may stop the expansion of the extra dimensions.

In this paper, our aim is to analyze the third point raised above more carefully. Assuming that the observed dimensions are topologically non-compact and the remaining ones form a flat torus, we study toy cosmological models involving various winding brane configurations.

In [20], two non-intersecting branes wrapping different internal dimensions has been considered to
compare the relative strengths of the expansion and contraction forced by \( p \)-branes in pure Einstein gravity. Interestingly, it was found that whether the compact dimensions are getting large or small depends on the dimension of the observed space. Namely, for the two brane case, the internal dimensions are stabilized when the observed space is three dimensional. Above three dimensions the compact space contracts and below three it expands. As we will see, there is a similar behavior for intersecting branes.

The main framework of [20] was the pure Einstein gravity. In this paper, we extend the formalism to dilaton gravity. The dilaton played a crucial role in the development of string cosmology. Especially, the large-small scale symmetry of string theory called T-duality, which can only be established in the presence of dilaton, has been argued to resolve the initial big bang singularity in a natural way. The vacuum expectation value of the dilaton also determines the string coupling constant and string theory enjoys a strong-weak coupling symmetry called S-duality. We also discuss the action of S and T-duality transformations in our cosmological context.

For a single \( p \)-brane the energy momentum tensor can be determined by coupling the brane action to the Einstein-Hilbert action. This is a standard technique which has been used to determine the geometry outside a macroscopic \( p \)-brane, see for instance [22]. The energy momentum tensor obtained in this way supports a delta function singularity at the position of the brane along the transverse directions. The delta function is smoothed out if one assumes a gas of such \( p \)-branes in a continuum approximation [20].

Let us also emphasize that, in this paper we ignore the fluctuations of the brane coordinates and other world-volume fields. This is a classical approximation to the brane dynamics. Therefore, our considerations can only be trusted at some late time after the big bang when the temperatures are low enough so that the brane fields are not exited.

The organization of the manuscript is as follows. In section II we review the main formalism. In section III we study general aspects of a cosmology with non-intersecting winding branes in pure Einstein gravity and argue that one should have at most two branes to prevent cosmological expansion of the internal space. We also construct solutions for intersecting \( p \)-branes. In section IV we consider the dilaton gravity and present cosmological solutions for single and intersecting D-branes. We also discuss T-duality invariance of the solutions. In the last section, we summarize our findings and speculate on a scenario based on our results.

II. THE MAIN FORMALISM

Let us consider the following \( d \)-dimensional metric which is of interest in cosmological applications

\[
ds^2 = -e^{2A}dt^2 + \sum_i e^{2B_i}dx^i dx^i.
\]  

(1)

Here the metric functions \( A \) and \( B_i \) depend only on time \( t \). Using \( t \)-reparametrization invariance one can set \( A = 1 \). This is the most preferred gauge choice in the literature. However, there is another very useful gauge condition in which the Ricci tensor simplifies considerably. It is easy to verify that, imposing \( A = \sum B_i \), the Ricci tensor takes the form

\[
R_{00} = e^{-2A}(-A'' + A'^2 - \sum_i B_i'^2),
\]

\[
R_{ij} = e^{-2A}B_i'' \delta_{ij},
\]  

(2)

In our conventions, the connection one-forms and curvature two-forms are given by \( d\omega^\mu + \omega^\mu_\rho \wedge \omega^\rho = 0 \) and \( R^\theta_\rho = d\omega^\theta_\rho + \omega^\theta_\rho \wedge \omega^\theta_\rho \). The Ricci tensor is defined by \( R_{\mu\nu} = R^\rho_{\mu\rho\nu} \).
where the hatted indices refer to the orthonormal frame \((e^A dt, e^B dx^i)\) and \(\dot{}\) denotes differentiation with respect to \(t\). We impose this gauge in solving the field equations and then switch to the proper time coordinate after we obtain the metric functions.

Generically, one can consider the following energy momentum tensor for matter

\[ T_{\dot{\mu} \dot{\nu}} = \text{diag}(\rho, p_i), \]

where \(p_i = \omega_i \rho\) with constant \(\omega_i\). Energy momentum conservation \(\nabla_\mu T^{\mu\nu} = 0\) gives

\[ \rho = \rho_0 \exp \left[ -\sum_i (1 + \omega_i) B_i \right], \]

where \(\rho_0\) is a constant.

In string frame the dynamics of the metric and the dilaton are governed via the action

\[ S_{\text{bulk}} = \frac{1}{\kappa^2} \int d^{10}X \sqrt{-g_s} e^{-2\phi} \left[ R_s + 4(\nabla \phi)^2 \right], \]

where \(\kappa^2 = (2\pi)^7 l_s\) and \(l_s\) is the string length. We assume \(4 + 6\) splitting of the space-time coordinates \(X^\mu\) where the first three spatial coordinates are non-compact and the other six are topologically \(S^1\).

Ignoring the world-volume fields, the action for a single \(p\)-dimensional D-brane is given

\[ S_p = -2T_p \int d^{p+1} \xi e^{-\phi} \sqrt{-g_s}, \]

where \(\xi^\alpha = (\tau, \sigma^a)\) are world-volume coordinates, \(g_{\alpha\beta}^s\) is the pull back of the string metric and \(T_p\) is the D-brane tension which can be fixed by the string length. The complete effective action is

\[ S = S_{\text{bulk}} + S_p. \]

We note that the string coupling constant \(g_s\) is given by \(e^\phi\).

The action \[\text{S}\] is invariant under the S-duality transformations

\[ \phi \rightarrow -\phi, \]
\[ g_{\mu\nu} \rightarrow e^{-\phi} g_{\mu\nu}^s. \]

The total action \[\text{S}\] is not invariant under \[\text{S}\]. However, the transformed action can simply be interpreted as a fundamental (S-dual) brane coupled to the bulk fields. While the coupling of original D-brane to the bulk is inversely proportional to the string constant \(g_s\), the coupling of the S-dual brane is proportional to \(g_s^{(1-p)/2}\).

To see the action of T-duality we take a special configuration where the world-volume coordinates are identified with some of the space-time directions. We also assume that the space-time metric is diagonal and there is a Killing direction named \(x\). Then the total action \[\text{S}\] is invariant under the T-duality transformations

\[ \phi \rightarrow \phi - \frac{1}{2} \ln(g_{xx}^s), \]
\[ g_{xx}^s \rightarrow 1/g_{xx}^s. \]

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2 To be more precise, type IIA string theory is mapped to M-theory under S-duality. For instance type IIA D2-branes are related to membranes in eleven dimensions. On the other hand, type IIB theory is self-dual; D1-branes are mapped to fundamental strings, D3-branes are self-dual and D5-branes are mapped to magnetically charged 5-branes of IIB theory. In this paper, we ignore all these stringy phenomena (which are of course very important in developing string cosmology) and see \[\text{S}\] as a symmetry of the effective action \[\text{S}\].

3 Actually, to establish the exact T-duality invariance of string theory along a Killing coordinate, one should assume that the isometry is compact. Here, we simply consider the invariance of the effective action and non-compactness of an isometry is not a problem.
where $x$ is identified as an additional brane coordinate if it is not along the brane directions, or otherwise it is ignored as a brane coordinate. Thus under a T-duality transformation a $p$-dimensional D-brane can be mapped to a $(p - 1)$ or $(p + 1)$-dimensional D-brane. Note that T-duality of the effective action can be established in the presence of D-branes and not the S-dual fundamental branes.

In solving field equations, we find it convenient to work in the canonical Einstein frame which is defined by

$$g^{\phi}_{\mu \nu} = e^{\phi/2} g_{\mu \nu}.$$  

(10)

In terms of the Einstein metric $g_{\mu \nu}$, the total action (7) can be written as

$$S = \frac{1}{\kappa^2} \int d^{10} X \sqrt{-g} \left[ R - \frac{1}{2} (\nabla \phi)^2 \right] - 2 T_p \int d^{p+1} \xi e^{\alpha \phi} \sqrt{-\gamma},$$

(11)

where $a_p = (p - 3)/4$ and $\gamma_{\alpha \beta}$ is the pull back of the Einstein metric.

From (11), one can determine the energy momentum tensor for a single D-brane and the dilaton

$$\sqrt{-g} T^\mu_\nu = - T_p \int d^{p+1} \xi e^{\alpha \phi} \sqrt{-\gamma} \gamma^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\nu \delta[X - X(\xi)],$$

(12)

$$\sqrt{-g} T^\phi_\mu_\nu = \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{4} g_{\mu \nu} (\nabla \phi)^2$$

(13)

so that the field equations can be written as

$$R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \kappa^2 T^\mu_\nu + T^\phi_\mu_\nu,$$

(14)

$$\nabla^2 \phi = 2 a_p T_p \kappa^2 \int d^{p+1} \xi e^{\alpha \phi} \sqrt{-g} \delta[X - X(\xi)].$$

(15)

To work in pure Einstein gravity one can set $a_p = 0$ and ignore the dilaton.

As it is clear from the above formulas, for a single D-brane there is a generic delta function singularity at the position of the brane along the transverse directions. In a cosmological setting, it is natural to take a gas of such D-branes in a continuum approximation (this is similar to assuming continuous charge distributions in electromagnetism, although the unit of charge is quantized) and this smooths out the singularity by an integration over transverse dimensions [20]. We assume that the branes are uniformly distributed and in this case one should replace the delta function by a constant which is the average number of D-branes per unit comoving transverse volume.

In the following sections, we take time dependent metric functions and dilaton corresponding to certain brane configurations and try to determine the cosmological evolution. Our aim is to understand the main dynamical aspects and to find the brane configurations which give contracting or stabilized internal dimensions.

## III. COSMOLOGY OF BRANES IN EINSTEIN GRAVITY

In this section we concentrate on pure Einstein gravity and set $a_p = \phi = 0$. We first consider non-intersecting branes winding different dimensions and argue (under some simplified assumptions) that such a configuration cannot prevent cosmological expansion of all internal dimensions unless there are only...
two branes. We then present exact solutions for intersecting branes. In this section we also set $\kappa^2 = 1$ and do not fix the dimension of the space-time.

We take a $d$-dimensional metric of the form \(1\) and consider a uniform, continuum gas of toroidal $p$-branes, which can be described by the map $t = \tau$, $x^a = \sigma^a$. From \(2\), the energy momentum tensor can be written as \(3\), where the constants $\omega_i$ and $\rho_0$ are given by

\[
\omega_i = \begin{cases} 
-1 & \text{winding (brane) dimension,} \\
0 & \text{transverse dimension,} 
\end{cases}
\]

\[
\rho_0 = n T_p
\]

and $n$ is the number of branes per unit comoving $(d - p - 1)$-dimensional transverse volume. We assume that the first $m$ of $d - 1$ spatial coordinates are non-compact playing the role of observed space. Therefore $\omega_i = 0$ for $i = 1, \ldots, m$. We also choose $B_1 = \ldots = B_m \equiv B$ for isotropy.

It is clear from \(16\) that wrapped branes behave like pressureless dust with respect to the directions they are distributed. On the other hand, they apply negative pressure along the winding dimensions. One would naively claim that negative pressure would cause these dimensions to expand as in the case of vacuum energy domination during inflation. However, in Einstein gravity, negative pressure does not necessarily imply expansion. Indeed, even in de Sitter phase of the early universe, one would have an exponential contraction with a negative Hubble constant. Therefore, the situation highly depends on the initial conditions.

Adding $k$-different branes with dimensions $p_l$ $(l = 1, \ldots, k)$, one has $T_{\mu\nu} = \sum_l T^{(l)}_{\mu\nu}$. From the structure of the energy momentum tensor, it is easy to see that

\[
g^{\mu\nu}T^{(l)}_{\mu\nu} = -(p_l + 1)T^{(l)}_{00} < 0.
\]

Therefore the trace of the energy momentum tensor is negative. Using \(2\), the $(ij)$ components of the Einstein equations \(14\) for $i, j \leq m$ imply

\[
(d - 2)B'' = -g^{\mu\nu}T_{\mu\nu}e^{-2A}.
\]

Thus $B'' > 0$. On the other hand $(00)$ component of \(14\) gives

\[
A'' - B'' = T_{00} e^{-2A}.
\]

We have $A'' > B''$ since $T_{00} > 0$.

Now assume that the branes are not intersecting so that each internal dimension belongs to a single brane and we have $d = 1 + m + \sum_l p_l$. We take the metric functions of the same brane to be equal. Thus there are $k$ different functions multiplying extra dimensions which we label as $C^{(l)}$. Using \(18\), Einstein equations \(14\) for each brane direction give

\[
C''^{(l)} = -T_{00}^{(l)}e^{-2A} + B''
\]

Multiplying the above formula with $(p_l + 1)$, summing over $l$, using \(17\) and \(18\) we obtain

\[
\sum_{l=1}^k (p_l + 1)C''^{(l)} + (m - k - 1)B'' = 0.
\]

By integrating \(21\), one can fix one of the metric functions in terms of the others up to the linear time dependent terms. Generically, $(00)$ component of \(14\) constrains the related undetermined integration constants. (The $(00)$ component is special because of the gauge we choose where $g_{00}$ is determined by the other metric functions.) As we will see, for many brane configurations a power law ansatz for the metric
is sufficient to solve the Einstein equations. If this is the case then the linear time dependent functions which arise in equations like (21) modify this power law behavior. However, it is easy to see that in the (00) component of (14) the terms coming from these functions have a different $t$ dependence. As a result they do not mix with the other contributions, should sum up to zero separately and thus can be ignored consistently. The only counterexample we find is in the context of dilaton gravity where the power law ansatz fails and we will be forced to keep these linear $t$ terms giving a different evolution (see the metric (17)).

To proceed, let us now concentrate on power law solutions so that the functions $A$, $B$ and $C^{(i)}$ are proportional to $\ln(t)$ and ignore linear $t$ terms in (21). Then, for $m < k + 1$, (21) implies that at least one of the functions $C^{(i)}$ should have the same sign with $B$, i.e. one of the internal dimensions should behave like the observed ones and expand $^5$. For $m = k + 1$ either all the internal dimensions are stabilized or while some of them grow the others contract. For $m > k + 1$, all extra dimensions can get small.

This has already been anticipated in (21). Indeed, we expect all internal dimensions to have the same behavior i.e. the sign of $(m - k - 1)$ determines whether the extra dimensions expand, contract or stabilized.

Since physically we have $m = 3$, the above argument strongly suggests that one should have at most two non-intersecting branes to prevent cosmological expansion of the internal space. For instance, six internal dimensions cannot be held small by strings. Even for the two brane case, which has been studied in detail in (20), the internal dimensions are stabilized. Including the states corresponding to other fields and momentum modes of the branes, one would expect to obtain enlarging extra dimensions.

Having established the general picture for non-intersecting winding branes, let us now consider some intersecting configurations to have a more complete understanding. We first take two branes with dimensions $p + s$ and $q + s$ intersecting over an $s$-brane. The $d$-dimensional metric can be written as

$$ds^2 = -e^{2A}dt^2 + e^{2B}dx^i dx^i + \sum_l e^{2C_l}dy^{a_l} dy^{a_l},$$

where $i = 1, \ldots, m, a_l = p, q, s$ for $l = 1, 2, 3$ respectively and we again impose the gauge $A = MB + pC_1 + qC_2 + sC_3$. The first brane with tension $T_1$ has the world-volume coordinates $(y^{a_1}, y^{a_3})$ and the second brane with tension $T_2$ wraps over the torus parametrized by $(y^{a_2}, y^{a_3})$. Using (16), the total energy momentum tensor corresponding to this configuration can be written as

$$T_{\hat{0}\hat{0}} = n_1 T_1 \exp[-MB - qC_2] + n_2 T_2 \exp[-MB - pC_1],$$

$$T_{ij} = 0,$$

$$T_{a_1b_1} = -n_1 T_1 \exp[-MB - qC_2] \delta_{a_1b_1},$$

$$T_{a_2b_2} = -n_2 T_2 \exp[-MB - pC_1] \delta_{a_2b_2},$$

$$T_{a_3b_3} = -n_1 T_1 \exp[-MB - qC_2] \delta_{a_3b_3} - n_2 T_2 \exp[-MB - pC_1] \delta_{a_3b_3},$$

where $n_1$ and $n_2$ are the number of branes per unit respective comoving transverse volumes. From (21), the Einstein equations can be written as

$$A'' - A'^2 + MB'^2 + pC_1'^2 + qC_2'^2 + sC_3'^2 = C''_3, $$

$$(d - 2)B'' = (p + s + 1) F_1 + (q + s + 1) F_2,$$

$^5$ To see that the observed space expands, we start with $A' > 0$. This implies that original time coordinate $t$ and the proper time are inversely proportional. Since $B'' > 0$, we have $B = -m \ln(t)$ for some positive $m$. Therefore $e^B$ will be a decreasing function of $t$ and an increasing function of the proper time.
\[(d - 2)C_1' = -(m + q - 2)F_1 + (q + s + 1)F_2,\]
\[(d - 2)C_2' = (p + s + 1)F_1 - (m + p - 2)F_2,\]
\[(d - 2)C_3' = -(m + q - 2)F_1 - (m + p - 2)F_2,\]

where
\[F_1 = n_1 T_1 \exp[mB + 2pC_1 + qC_2 + 2sC_3],\]
\[F_2 = n_2 T_2 \exp[mB + pC_1 + 2qC_2 + 2sC_3].\]

Although it looks quite complicated, it is remarkable that \((24)\) admits an exact power law solution. To determine the metric, let us first note that the last four equations in \((24)\) give
\[(m - s - 3)B + (p + s + 1)C_1 + (q + s + 1)C_2 = 0,\]
\[B - C_1 - C_2 + C_3 = 0,\]
where we ignore linear integration terms (see the discussion below \((21)\)). Using \((27)\), one can express the second and the third equations of \((24)\) in terms of \(B\) and \(C_1\) alone. To solve them, we assume
\[B = b_1 \ln(t) + \ln b_2,\]
\[C_1 = c_1 \ln(t) + \ln c_2,\]
where \(b_1, b_2, c_1\) and \(c_2\) are constants. A straightforward calculation shows that these constants can be determined uniquely. Using \((27)\) and the gauge choice for \(A\), one can then determine all the metric functions. The first equation in \((24)\) is then to be checked for consistency and it is satisfied identically.

On the other hand, the constants \(b_2\) and \(c_2\) should be determined to be non-zero which imposes
\[pq(9 - s) + 4sp + 4qs - mq(s - 1) - mp(s + q - 1) \neq 0.\]

This condition is obeyed for the physically important case of \(m = 3\). All these calculations involve trivial algebraic manipulations which can be performed by using a computer program.

After solving all the unknown metric functions, one can switch to the proper time coordinate (which we again denote by \(t\)) to get
\[ds^2 = -dt^2 + R_m^2 dx^i dx^i + \sum_l R_l^2 dy^{al} dy^{al},\]

where
\[\ln R_m = 2\Delta^{-1}[p + q + 2pq + ps + qs] \ln(\alpha_m t),\]
\[\ln R_1 = -2q\Delta^{-1}[m - s - 3] \ln(\alpha_1 t),\]
\[\ln R_2 = -2p\Delta^{-1}[m - s - 3] \ln(\alpha_2 t),\]
\[\ln R_3 = -\Delta^{-1}[2(m - 2)(p + q) + 4pq] \ln(\alpha_3 t).\]

Here \(\alpha_m, \alpha_l\) are dimensions-full constants determined by \(T_1, T_2, n_1, n_2\) and the positive number \(\Delta\) is given by
\[\Delta = pq(3 + s) + m(1 + s)(p + q) + mpq.\]

The values of \(\alpha_m\) and \(\alpha_l\) set the units of length for each direction and they can be eliminated by scalings of \(x\) and \(y\) coordinates. Note that this also modifies the energy momentum tensor.
Setting \( s = 0 \), ignoring \( R_3 \) and the corresponding coordinates, \( \text{(30)} \) becomes the metric constructed in \( \text{(20)} \) for two non-intersecting branes. This is a consistency check on both solutions. However, let us emphasize that setting spatial brane dimensions to zero is not always possible. For instance one cannot let \( p = 0 \) further to obtain the solution for a single winding brane. To see this, we note that the trace of the energy momentum tensor is proportional to \( p + 1 \) and setting \( p = 0 \) does not remove the trace part.

From \( \text{(31)} \), one finds that the observed space expands and the intersecting directions contract. On the other hand, the sign of \( m - s - 3 \) determines the fate of the relative transverse directions \(^6\). When \( m < s + 3 \), these dimensions expand. For \( m = s + 3 \) they are stabilized and for \( m > s + 3 \) they contract. Therefore, for the physically important case of \( m = 3 \) one finds that the intersecting dimensions diminish and the relative transverse coordinates enlarge. Comparing with the non-intersecting solution which gives stable extra dimensions, this behavior suggests that the internal space can be seen as an elastic balloon; when some directions are forced to contract the others tend to expand.

Using \( \text{(30)} \), it is also possible to obtain the special intersection where an \( s \)-brane is located inside a \( (p + s) \)-brane by setting \( q = 0 \) in \( \text{(31)} \) and ignoring the corresponding coordinates (as we pointed out this is not a trivial operation and one should check the steps carefully). In this way, one can obtain the following metric

\[
ds^2 = -dt^2 + (\alpha_m t)^{\frac{4}{m-2}} dx^i dx^i + dy^{a_1} dy^{a_1} + \left( \alpha_3 t \right)^{\frac{4(m-2)}{m(m+3)}} dy^{a_2} dy^{a_3},
\]

where \( (y^{a_1}, y^{a_1}) \) and \( (y^{a_3}) \) are the world-volume coordinates of \( (p + s) \) and \( s \)-dimensional branes, respectively, and we scale \( y^{a_1} \) to set \( \alpha_1 = 1 \). In the \( p \)-dimensional subspace spanned by \( y^{a_1} \), the contraction forced by the winding \( (p + s) \)-branes is compensated by the expansion forced by the gas of \( s \)-branes. Ignoring this part, \( \text{(30)} \) is precisely the metric corresponding to \( s \)-dimensional winding branes which has been found in \( \text{(20)} \).

Finally, we consider adding a third \((p + q + s)\)-dimensional brane to \( \text{(30)} \) winding over all extra dimensions. This is a triple intersection. One should modify \( \text{(28)} \) and \( \text{(24)} \) by adding terms coming from the third brane source. The rest of the calculation can be carried out as follows. The last four equations in \( \text{(23)} \) give a relation between \( B \) and \( C_1 \). Using this relation, one can eliminate one of the functions and obtain three differential equations for three functions which can be solved using an ansatz like \( \text{(28)} \). It is then possible to determine all metric functions and the first equation in \( \text{(24)} \) is satisfied identically. In the proper time coordinate the final result can be written as

\[
ds^2 = -dt^2 + (\alpha_m t)^{\frac{4}{m-2}} dx^i dx^i + dy^{a_1} dy^{a_1} + dy^{a_2} dy^{a_2} + \left( \alpha_3 t \right)^{\frac{4(m-2)}{m(m+3)}} dy^{a_3} dy^{a_3},
\]

where we scale \( y^{a_1} \) and \( y^{a_2} \) to set \( \alpha_1 = \alpha_2 = 1 \). Ignoring these coordinates, this is again precisely the metric for a gas of winding \( s \)-branes. Let us remind that in \( \text{(31)} \), there are three branes which has the world-volume coordinates \( (y^{a_1}, y^{a_1}), (y^{a_2}, y^{a_3}) \) and \( (y^{a_1}, y^{a_2}, y^{a_3}) \), respectively. It is remarkable that, as in the case of two non-intersecting branes, in certain directions the expansion and contraction forced by the branes cancel each other. This suggests that it may be possible to establish a superposition rule for wrapped branes and this might have important cosmological applications.

Before closing this section, we would like to address an important issue: since the energy of the winding branes increases as the internal dimensions expand, one may think that in the above solutions the total energy is not conserved when this happens. This claim is not correct since all energy momentum tensors considered so far obey \( \nabla_{\mu} T^{\mu\nu} = 0 \) which guarantees conservation of energy. One may then wonder where

\(^6\) A coordinate is defined to be a relative transverse one if it belongs to a brane but transverse to another one.
this extra energy comes from? To answer this question, let us remind that we employ an approximation where the branes are assumed to be continuously distributed. Therefore, in addition to the winding energy one can talk about energy corresponding to this continuous distribution. When the winding energy increases the energy of the distribution decreases and the total energy remains constant.

How we should understand the flow between winding and distribution energies in the real situation where the continuum approximation is not valid. Since we assumed that the branes fall out of thermal equilibrium and do not interact with each other, it seems odd to say that the increase in the energy of a single winding brane is compensated by a decrease in the energy of the others. To understand this let us consider a two dimensional cylinder with winding strings on it (see Fig. 2). When the strings are dense enough they are able to prevent the radial expansion. However, as the cylinder undergoes a linear cosmological expansion, the number of strings per unit length decreases. In this case, winding strings may not be able to prevent the radial expansion of the space in between them. Of course, whether this happens is determined by the dynamical field equations. If it turns out to be the case, then the space whose radial expansion is prevented by the strings on the cylinder becomes of measure zero in time and one sees a radially expanding cylinder.

![Fig. 2: Strings winding a two dimensional cylinder. As the cylinder expands horizontally the number of strings per unit length decreases. As a result, the strings may not be able to prevent the radial expansion of the space in between them.](image)

**IV. ADDING DILATON**

In this section, we construct solutions for winding branes in dilaton gravity. Adding dilaton, one discovers new physical phenomena to study. For instance, it is possible to define string and Einstein frame metrics and generically one would expect to observe different cosmological behavior in each frame. As we will see, T-duality invariance plays a key role in a cosmology D-brane gases in dilaton gravity.

Let us first consider a single winding D-brane, which can be described by following ten dimensional metric

$$ds^2 = -e^{2A}dt^2 + e^{2B}dx^i dx^i + e^{2C}dy^a dy^a,$$

(35)

where $i, j = 1, \ldots, m$ and $a, b = 1, \ldots, p$, $m + p = 9$ and $A = mB + pC$. As before, we take $p$-dimensional D-branes wrapping the internal circular dimensions $y^a$ and uniformly distributed in the observed space spanned by $x^i$. We carry out the computation in Einstein frame and then switch to string frame. Using (12) and (13), it is easy to write down the total energy momentum tensor (we set $\kappa^2 = 1$)

$$T_{\hat{0}\hat{0}} = n T_1 e^{-mB - a \phi} + \frac{1}{4} \phi'^2 e^{-2A},$$

$$T_{\hat{i}\hat{j}} = \frac{1}{4} \phi'^2 e^{-2A},$$

$$T_{\hat{a}\hat{b}} = -n T_1 e^{-mB - a \phi} \delta_{a b} + \frac{1}{4} \phi'^2 e^{-2A} \delta_{a b},$$

(36)
where \( a_p = (p - 3)/4 \). Due to the non-trivial coupling in \( \pi_1 \) only the total \( T_{\mu\nu} \) is conserved provided that the equations of motion are satisfied. The Einstein and the dilaton field equations can be written as

\[
\begin{align*}
A'' - A'^2 + mB'^2 + pC'^2 + \frac{1}{2}\phi'^2 &= C'', \\
B'' &= \frac{p + 1}{m + p - 1} \pi_p \exp[\pi B + 2pC + a_p\phi], \\
C'' &= -\frac{m - 2}{m + p - 1} \pi_p \exp[\pi B + 2pC + a_p\phi], \\
\phi'' &= -2\pi_p a_p \exp[\pi B + 2pC + a_p\phi].
\end{align*}
\] (37)

The last equation indicates that the dilaton can effectively be seen as an additional internal dimension. This is not surprising since the circular coordinate, on which one can relate M-theory to type IIA string theory by a compactification, is parametrized by the vacuum expectation value of the dilaton.

We first attack these equations without fixing the values of \( m, p \) and \( a_p \) to see the effect of adding dilaton in Einstein gravity. Later we focus on the case implied by string theory. In accordance with our main strategy, using the last three equations in (37) one can determine \( C \) and \( \phi \) in terms of \( B \). The second equation in (37) can then be used to solve \( B \) by an ansatz like (28) and this determines all metric functions. Switching to the proper time coordinate one can finally obtain

\[
\begin{align*}
\text{d}s_E^2 &= -\text{d}t^2 + R^2_m \text{d}x^i \text{d}x^i + R^2_p \text{d}y^a \text{d}y^a, \\
\text{e}^\phi &= R\phi
\end{align*}
\] (38)

where

\[
\begin{align*}
\ln R_m &= \frac{2(p + 1)}{m(p + 1) + 2(m + p - 1)a_p^2} \ln(\alpha t), \\
\ln R_p &= -\frac{2(m - 2)}{m(p + 1) + 2(m + p - 1)a_p^2} \ln(\alpha t), \\
\ln R_\phi &= -\frac{4a_p(m + p - 1)}{m(p + 1) + 2(m + p - 1)a_p^2} \ln(\alpha t).
\end{align*}
\] (39)

For \( a_p = 0 \), the dilaton decouples and we find the single winding brane solution of (20). Adding dilaton modifies the power law but does not change the physically expected behavior where the observed space expands and the compact space contracts due to winding of branes.

In (39), the constant \( \alpha \) should be determined to be non-zero. One can check that this implies

\[
a_p^2 \neq \frac{m - mp + 4p}{2(m + p - 1)}.
\] (40)

When (40) is not satisfied the power law ansatz which has been used to solve the above differential equations cannot be used.

Returning now to string theory, we have \( m + p = 9 \) and \( a_p = (p - 3)/4 \). It is remarkable that for these values (40) is not obeyed. Therefore, (37) cannot be used to describe the cosmology of winding branes in string theory. To obtain the proper solution, we first followed our main strategy. Namely we first determined \( C \) and \( \phi \) in terms of \( B \) neglecting linear \( t \) terms. After solving \( B \) from the second equation in (37) we obtained all metric functions. However, a quick analysis showed that the first equation in (37) cannot be satisfied in this way.

Therefore, this time one has to keep linear \( t \) terms in obtaining relations between \( B, C \) and \( \phi \). Keeping them, (37) gives

\[
C = \frac{p - 7}{p + 1} B + ct,
\] (41)
\[ \phi = \frac{4(3-p)}{p+1} B + d t, \]

where \( c \) and \( d \) are constants. Using (11), the second equation in (37) becomes

\[ 8 B'' = n T_p (p+1) \exp[(2pc + ap)dt], \] (42)

which can be solved as

\[ B = \frac{(p+1)nT_p}{8(2pc + ap)^2} \exp[(2pc + ap)dt] + bt, \] (43)

where \( b \) is a constant. Using (41), (43) and the gauge condition for \( A \), one finds that (37) are satisfied provided that

\[ -\frac{16b(2pc + ap)}{p+1} - p^2 c^2 + p c^2 + \frac{d^2}{2} = 0. \] (44)

It is possible to eliminate \( b \), \( c \) or \( d \) by scaling \( t \) appropriately. Since two remaining undetermined constants are constrained by (44), there is a single parameter solution space. However, we would like to construct a solution which is invariant under T-duality transformations. To determine the constants for this solution, we first switch to string frame. From (10), the string frame metric functions are defined by

\[ A_s = A + \frac{\phi}{4} = -\frac{12}{p+1} B + (pc + \frac{d}{4})t, \]
\[ B_s = B + \frac{\phi}{4} = \frac{4}{p+1} B + \frac{d}{4} t, \]
\[ C_s = C + \frac{\phi}{4} = \frac{4}{p+1} B + (c + \frac{d}{3})t. \] (45)

For invariance under a T-duality transformation \( \theta \), one should have \( B_s = -C_s \), and this imposes \( c = -d/2 \). On the other hand, by scaling \( t \), one can set \( c = 2/(p+1) \) and (41) can be used to find \( b \). This gives

\[ A_s = 3f + t, \]
\[ B_s = f - t/3, \]
\[ C_s = -f + t/3, \]
\[ \phi = -(p-3)f + \frac{p-6}{3}t, \] (46)

where \( f = \lambda e^{3t} \) and \( \lambda = nT_p/18 \). Defining a new time coordinate by \( t \to e^t \), we obtain the string frame metric and the dilaton

\[ ds_S^2 = -e^{6\lambda t^3} dt^2 + t^{-2}\hat{e}^{2\lambda^3} dx^i dx^i + t^{-1}e^{-2\lambda^3} dy^a dy^a, \]
\[ e^\phi = \frac{e^{(p-6)}}{t^{(p-3)p}e^{(p-3)|\lambda|^3}}. \] (47)

One can easily see that applying a T-duality transformation along \( x \) direction gives a solution for a gas of \((p+1)\)-dimensional D-branes. Similarly, a T-duality along \( y \) direction reduces the brane dimension by one. It is also straightforward to calculate the S-dual solution from (3).

One would claim that (47) gives exponential scaling functions for the observed and the internal spaces. However, note that \( t \) is not the proper time coordinate. It is indeed possible to show that when expressed in terms of the proper time \( \tau \), which can be defined as

\[ d\tau = e^{3\lambda t^3} dt, \] (48)
(47) is close to a power low solution. To see this, one can integrate (48) so that as $t \to -\infty$, $\tau \to 0$. When $|t|$ is large (48) gives $\tau \sim e^{3\lambda t^3}/(9\lambda t^2)$. On the other hand when $t$ is small, one finds $\tau \sim t$. The Hubble parameters with respect to $\tau$ can be calculated as

$$H_m = -\frac{d}{d\tau} \ln R_m = \frac{(9\lambda t^3 - 1)}{3t} e^{-3\lambda t^3}.$$

(49)

When $t$ is very large and very small one finds $H_m \sim \pm 1/(3\tau)$, respectively, which indicates a cosmological evolution obeying a power law $R_m \sim \tau^{\pm 1/3}$.

At $\tau = 0$ ($t = -\infty$), the observed space has zero size and the scale function of the compact space diverges. As $\tau$ increases, the observed space expands and the internal space contracts where the metric functions have power law dependences on the proper time. However, at some $\tau$ (given by $t = 0$) the scale factor of the observed space diverges and the compact space has zero size. After that time the observed space starts to diminish and the compact space enlarges. This period is followed by an expansion of the observed space and the contraction of the internal space, which is again close to a power law.

In string theory, (47) can be trusted when the curvatures and the string coupling $g_s$ are small. For example, one cannot trust the geometry near $t = 0$ since the curvatures are large. Similarly, for $p > 3$, one cannot use (47) near $t \sim -\infty$ since string coupling diverges. This shows that (47) can be trusted when $|t|$ is large. Moreover, for $p > 3$, $t$ should take positive values and for $p < 3$ it should be negative. In this range of validity, we discover the physically expected behavior; that is the observed space expands and the extra dimensions contract.

Having studied the single D-brane case, let us now consider an intersecting brane configuration. Assume that in the six dimensional compact space there are $(p+s)$ and $(q+s)$-dimensional D-branes with tensions $T_1$ and $T_2$ intersecting over an $s$-dimensional submanifold, where $p + q + s = 6$. As before, we carry out calculations in Einstein frame and then switch to the string frame. The metric can be written as

$$ds^2 = -e^{2A} dt^2 + e^{2B} dx^i dx^i + \sum_l e^{2C_l} dy^{a_l} dy^{a_l},$$

(50)

where $i = 1, 2, 3$, $a_l = p, q, s$ for $l = 1, 2, 3$ respectively and we again impose the gauge $A = mB + pC_1 + qC_2 + sC_3$. The first brane wraps on the torus parametrized by $(y^{a_1}, y^{a_2})$ and the second brane has the toroidal world-volume coordinates $(y^{a_2}, y^{a_3})$. Using (12) and (13) the total energy momentum tensor can be calculated as

$$T_{00} = n_1 T_1 \exp[-mB - qC_2 + a_p \phi] + n_2 T_2 \exp[-mB - pC_1 + a_q \phi],$$

$$T_{ij} = 0,$$

$$T_{a_1b_1} = -n_1 T_1 \exp[-mB - qC_2 + a_p \phi] \delta_{a_1b_1},$$

$$T_{a_2b_2} = -n_2 T_2 \exp[-mB - pC_1 + a_q \phi] \delta_{a_2b_2},$$

$$T_{a_3b_3} = -n_1 T_1 \exp[-mB - qC_2 + a_p \phi] \delta_{a_1b_3} - n_2 T_2 \exp[-mB - pC_1 + a_q \phi] \delta_{a_3b_2},$$

(51)

where $a_p = (p + s - 3)/4$ and $a_q = (q + s - 3)/4$. From (51), the field equations (14) and (15) can be written as

$$A'' = A'^2 + mB'^2 + pC_1'^2 + qC_2'^2 + sC_3'^2 + \frac{1}{2} \phi'^2 = C_3'',$$

$$8 B'' = (p + s + 1) F_1 + (q + s + 1) F_2,$$

$$8 C_1'' = -(q + 1) F_1 + (q + s + 1) F_2,$$

$$8 C_2'' = (p + s + 1) F_1 - (p + 1) F_2,$$

$$8 C_3'' = -(m + q - 2) F_1 - (m + p - 2) F_2,$$

$$\phi'' = -2a_q F_1 - 2a_q F_2$$

(52)
where

\[ F_1 = n_1 T_1 \exp[mB + 2pC_1 + qC_2 + 2sC_3 + a_p \phi], \]  
\[ F_2 = n_2 T_2 \exp[mB + pC_1 + 2qC_2 + 2sC_3 + a_q \phi]. \]  

To solve these equations we first note that, up to the linear \( t \) terms which we ignore, (52) implies

\[
(1 + q + s)C_2 = sB - (p + s + 1)C_1, \\
(1 + q + s)C_3 = -qB + (q - p)C_1, \\
(1 + q + s) \phi = -2sB + 2(p - q)C_1.
\]

These relations can be used to express the second and the third equations of (52) in terms of \( B \) and \( C_1 \) alone. To solve them, one can use the ansatz (28) and see that all constants are uniquely fixed. From (55) and gauge condition \( A = mB + pC_1 + qC_2 + sC_3 \), all metric functions and the dilaton can be determined.

It is also possible to show that the first equation in (52) is satisfied identically. Switching to the proper time coordinate we finally obtain the following solution in *Einstein frame*

\[
ds_E^2 = -dt^2 + (\alpha) \frac{16 + 2q}{8 - p - q} dx^i dx^j + (\alpha) \frac{16 + 2q}{8 - p - q} (dy^{a_1} dy^{a_1} + dy^{a_2} dy^{a_2}) + (\alpha) \frac{16 + 2q}{8 - p - q} dy^{a_3} dy^{a_3},
\]

\[
e^\phi = (\alpha) \frac{16 + 2q}{8 - p - q},
\]

where \( \alpha \) is a *positive* dimension-full number.

The metric in (56) shows that the observed space expands (close to a power law corresponding to pressureless dust) and the intersecting dimensions contract. On the other hand, the relative transverse directions also enlarge. Comparing with the intersecting brane solution in pure Einstein gravity for \( m = 3 \), we see that adding dilaton modifies the power law but does not change the main physical behavior.

Let us now switch to the *string frame* metric defined by (11). In terms of the proper time coordinate we find

\[
ds_S^2 = -dt^2 + (\alpha) \frac{8}{2 - p - q} dx^i dx^j + (dy^{a_1} dy^{a_1} + dy^{a_2} dy^{a_2}) + (\alpha) \frac{8}{2 - p - q} dy^{a_3} dy^{a_3},
\]

As in the previous cases, we see that the observed space expands and the intersecting directions contract. On the other hand, the dilaton stabilizes expansion of the relative transverse space parametrized by \((y^{a_1}, y^{a_3})\). This is remarkable compared to the behavior we observed in (50).

One can easily see from (57) that a T-duality transformation along \( x \) shifts \( s \rightarrow s + 1 \) and decreases the dimension of the observed space by one. This gives a solution describing \((p + s + 1)\) and \((q + s + 1)\)-dimensional branes intersecting over an \((s+1)\)-dimensional internal space. On the other hand, a T-duality along \( y^{a_1} \) gives a metric for the intersection of \((p + s - 1)\) and \((q + s + 1)\)-dimensional branes over an \(s\)-dimensional space. Transformations along \( y^{a_2} \) and \( y^{a_3} \) can be interpreted similarly. Note that the value of \( p + q \) is not altered under the T-duality.

It is possible to argue that T-duality invariance of string theory can be used to analyze more general brane configurations. For the moment, let us assume that there are many number of different D-branes winding the compact internal space. One can group the space-time coordinates into three disjoint sets: the observed, the relative transverse and the intersecting ones. If a coordinate is common to all branes it is an intersecting dimension. If it belongs to a brane but it is transverse to another one, it is a relative transverse coordinate. If there are no branes wrapping on it, it is one of the observed coordinates. One can easily see that the set of relative transverse coordinates is invariant under T-duality transformations. On the other hand, the observed coordinates are interchanged with the intersecting ones. Since physically one
expects that the observed space expands, T-duality invariance dictates that the intersecting dimensions should contract. On the other hand, the relative transverse coordinates should be stabilized since one can interchange two of them by a T-duality. Therefore, T-duality invariance of the effective action implies that the internal dimensions cannot expand. This is contrary to the behavior we observed in Einstein gravity and shows the importance of T-duality in string cosmology.

V. CONCLUSIONS

In this paper, we study cosmology of winding branes in both pure Einstein and dilaton gravities. The energy momentum tensor is obtained by coupling the brane action to the gravity action. We take a uniform gas of such branes and utilize a continuum approximation. The brane fluctuations and the world-volume fields are ignored in our analysis. Generically, we see that the internal dimensions wrapped by $p$-branes tend to contract. On the other hand, the gas of such winding $p$-branes force the transverse dimensions to expand.

In pure Einstein gravity, cosmological evolution in the presence of single and two non-intersecting branes has been determined in [20]. The single brane wrapping over all extra dimensions cause them to diminish. On the other hand for two branes winding different internal dimensions, the expansion and the contraction is stabilized when the observed space is three dimensional. We argue that if there are three or more non-intersecting winding branes the cosmological expansion of the internal space cannot be prevented. We also consider intersecting brane configurations. In this case, we find that the common brane directions contract while the relative transverse directions tend to enlarge or stay stabilized.

Adding dilaton to pure Einstein gravity, we see that the above conclusions do not change appreciably. For instance, two intersecting branes give expanding relative transverse directions.

On the other hand, the dilaton has a remarkable effect in string frame; it stabilizes the expansion of the relative transverse coordinates. We argued that, T-duality invariance of the effective action can be used to determine the main cosmological behavior for more general systems. Namely, one expects to find an expanding observed space, stable relative transverse directions and contracting intersecting dimensions.

Is it possible to determine which brane configurations can play a role in keeping the extra dimensions small and which ones should be eliminated? To answer this question let us point out that till now we have ignored two important issues. The first one is that we have only considered winding modes of the branes. To have a more complete scenario one should include effects of other sources, like the momentum modes, which are expected to force all dimensions to expand. Secondly, there is now considerable evidence indicating that the universe underwent an exponentially growing period, called inflation, forced by a positive cosmological constant. Inflationary paradigm is successful in explaining away the basic shortcomings of standard cosmology and its predictions are confirmed with the astrophysical observations so far. Phenomenologically one expects 60 or 70 e-foldings during inflation. As discussed in [20], with a positive cosmological constant the brane terms have negligible effects and can be ignored. Thus, internal space was also subject to an exponential growth. Assuming that the compact dimensions started out at about Planck length, one finds that their size grew to about $10^{-3}m$ after inflation. This indicates that the configuration dominating the subsequent cosmological evolution should force all internal dimensions to contract and the results of this paper show that this can only be realized when there is a single brane wrapping over all extra dimensions.

It is then possible to speculate on the following scenario. The ten dimensional universe was born in a hot, dense state at about the Planck length. For some reason (which hopefully will be found by string/M theory) three spatial coordinates were started to be non-compact and the remaining ones parametrize a six dimensional compact internal space. The universe then underwent an inflationary period where all
dimensions were subject to exponential growth. The inflation was followed by an expansion close to a power law. During this period winding branes could not prevent the cosmological expansion but they slowed it down. However, in a short time all winding Dp-branes with \( p < 6 \) were annihilated. One can claim that they could all be absorbed by D6-branes since they were wrapping over all extra dimensions leaving no room for other branes. Then the internal space started to contract and this might explain the hierarchy between the sizes of the extra and the observed dimensions.

To see that such a scenario can produce phenomenologically viable numbers for the size of the internal space, let us ignore all other sources and consider the cosmological evolution after the inflation when there is a single brane wrapping over all extra dimensions. The scale functions of the observed space \( R_o \) and the internal space \( R_i \) are related by \( R_i = R_o^{-1/(p+1)} \) in pure Einstein gravity \(^2\) (where \( p \) is the number of extra dimensions) and \( R_i = 1/R_o \) in dilaton gravity. The estimates obtained in \(^2\) indicate that in pure Einstein gravity \( p > 1 \) is not viable phenomenologically \(^7\). To be more precise, let us assume that the diameter of presently observable universe (our current Hubble volume), which is of the order of \( 10^{27} m \), was about \( 10^{-5} m \) at the end of inflation. Therefore, the scale function of the observed space for this time interval can be estimated to be \( R_o \sim 10^{32} \). Using this number, the scale function of the internal space becomes \( R_i \sim 10^{-16} \) in pure Einstein gravity for \( p = 1 \) and \( R_i \sim 10^{-32} \) in string cosmology. This gives the current size of the internal space to be \( 10^{-21} m \) in Einstein gravity and \( 10^{-37} m \) in string theory. Doing the same exercise for \( p = 6 \) in pure Einstein gravity one finds unacceptable estimates. It is remarkable that the dilaton modifies the relation between the scale functions in such a way that even for six dimensional branes wrapping over all extra dimensions one can obtain phenomenologically viable estimates.

We conclude by noting that in string theory it is possible to obtain cosmological backgrounds with contracting extra dimensions without introducing winding brane sources explicitly (see for instance S-brane solutions \(^{24,28}\)). In these solutions there are non-zero form-fluxes and this is a different way to describe branes in a supergravity context. It would be interesting to find a connection between the solutions presented in this paper and the other cosmological backgrounds related to S-branes.

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\(^7\) The analysis of \(^2\) ignored one possible caveat that in reality the extra dimensions do not grow in the same amount as the observed space during inflation due to winding branes. However, a numerical study shows that, although the situation may depend on the values of the brane tension and the energy density driving inflation, the difference cannot be accommodated in this way \(^2\).


