Population trapping in the one-photon mazer

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Abstract

We study the population trapping phenomenon for the one-photon mazer. With this intent, the mazer theory is written using the dressed-state coordinate formalism, simplifying the expressions for the atomic populations, the cavity photon statistics, and the reflection and transmission probabilities. Under the population trapping condition, evidence for new properties of the atomic scattering is given. Experimental issues and possible applications are discussed.

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1 Introduction

Laser cooling of atoms is a rapidly developing field in quantum optics. Cold and ultracold atoms introduce new regimes in atomic physics often not considered in the past. In the last decade, Englert et al. [1] have demonstrated new interesting properties in the interaction of cold atoms with a micromaser field. More recently, Scully et al. [2] have shown that a new kind of induced emission occurs when a micromaser is pumped by ultracold atoms, requiring a quantum-mechanical treatment of the center-of-mass motion. They called this particular process mazer action to insist on the quantized z-motion feature of the induced emission.

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The detailed quantum theory of the mazer has been presented in a series of three papers by Scully and co-workers [3,4,5]. They showed that the induced emission probability is strongly dependent on the cavity mode profile. Analytical calculations were presented for the mesa and the sech \(^2\) mode profiles. For sinusoidal modes, WKB solutions were detailed.

Retamal et al. [6] showed that we must go beyond the WKB solutions for the sinusoidal mode case when we consider strictly the ultracold regime. Remarkably, they showed that the resonances in the emission probability are not completely smeared out for actual interaction and cavity parameters. In a recent work [7], we proposed a numerical method for calculating efficiently the induced emission probability for arbitrary cavity field modes. In particular, the gaussian potential was considered, thinking in open cavities in the microwave or optical field regime. Differences with respect to the sech \(^2\) mode case were found. Calculations for sinusoidal potentials were also performed and divergences with WKB results were reported, confirming results given in [6].

Zhang et al. [8] extended the concept of the mazer to the two-photon process by proposing the idea of the two-photon mazer. Their work was focused on the study of its induced emission probability in the special case of the mesa mode function. Under the condition of an initial coherent field state, they showed that this probability exhibits with respect to the interaction length the collapse and revival phenomenon, which have different features in different regimes. They are similar to those in the two-photon Jaynes-Cummings model only in the thermal-atom regime. Recently, Arun et al. [9] studied the mazer action in a bimodal cavity with particular incidence in the mode-mode correlations.

The collapses and revivals of the atomic excitation in the framework of the Jaynes-Cummings model was predicted in the early 1980s by Eberly and co-workers [10,11,12]. It is now well established (see e.g. Ref. [13] and references therein) that this phenomenon is a direct consequence of the interference of quantum Rabi floppings at various frequencies and of the granularity of the field. Experimental evidence for collapses and revivals has been reported by Brune et al. [14] and Rempe et al. [15] by use of a micromaser device. Fleischhauer and Schleich [16] showed later that the shape of each revival is a direct reflection of the shape of the initial photon-number distribution \(P_n\), assuming that the atom is prepared completely in the upper state or in the lower state and that the distribution \(P_n\) is sufficiently smooth. It was also noticed that, under some special conditions of the initial atom-field state, the revivals can be largely and even completely suppressed [17,18,19]. This phenomenon was denominated “population trapping” to refer, as noted by Yoo and Eberly [20], to a persistent probability of finding the atom in a given level in spite of the existence of both the radiation field and allowed transitions to other levels. The initial atom-field states giving rise to this phenomenon were called
“trapping states” in [21]. Let us mention that this denomination is actually used in various physical contexts whenever a degree of freedom is found unaltered in spite of the existence of an interaction able to change its value. For instance trapping states in the context of the micromaser theory have been predicted and very recently measured by Filipowicz et al. [22] and Weidinger et al. [23] respectively. Nevertheless, these trapping states do not relate with those responsible for the suppression of the revivals mentioned in [17,18,19].

An elegant explanation of the population trapping phenomenon has been proposed by Jonathan et al. [24], who noticed that the key to understand the collapse and revival patterns under very general conditions is to consider the joint initial properties of the atom-field system, even if this one is completely disentangled before the interaction. By defining an appropriate coordinate system, the dressed-state coordinates, they were able to yield simple analytical expressions for the atomic populations which exhibit the conditions needed for population trapping.

As the \( z \)-motion quantization introduces a new kind of induced emission dependent of the cavity mode profile, and thus strongly modifies the usual Jaynes-Cummings atom-field evolution, we study in this paper how the population trapping phenomenon is affected by this motion quantization and we show that the trapping states provide interesting new features to the mazer.

In Sec. 2, we write the quantum theory of the one-photon mazer by use of the dressed-state coordinate formalism as it was very efficient in the description of the population trapping phenomenon in the Jaynes-Cummings model [24]. General expressions are derived for the atomic populations and the cavity photon distribution after the interaction of the atom with the cavity. The theory is written for any initial pure state of the atom-field system (entangled or not). We consider zero temperature and no dissipation in the high-\( Q \) cavity. Sec. 3 is devoted to the population trapping phenomenon for the one-photon mazer and new properties of the trapping states in the scattering process are presented. Experimental issues are briefly discussed in Sec. 4. A brief summary of our results is given in Sec. 5.

2 The model

2.1 The Hamiltonian

We consider a two-level atom moving along the \( z \)-direction in the way to a cavity of length \( L \). The atom is coupled resonantly with an one-photon transition to a single mode of the quantized field present in the cavity. The atom-field
interaction is modulated by the cavity field mode function. The atomic center-of-mass motion is described quantum mechanically and the rotating-wave approximation is made. In the interaction picture, the Hamiltonian describing the system is

\[ H = \frac{p^2}{2M} + \hbar g u(z)(a^\dagger \sigma + a \sigma^\dagger), \tag{1} \]

where \( p \) is the atomic center-of-mass momentum along the \( z \)-axis, \( M \) is the atomic mass, \( \sigma = |b\rangle\langle a| \) (\(|a\rangle \) and \(|b\rangle\) are respectively the upper and lower levels of the atomic transition), \( a \) and \( a^\dagger \) are respectively the annihilation and creation operators of the cavity radiation field, \( g \) is the atom-field coupling strength and \( u(z) \) is the cavity field mode.

### 2.2 The wavefunctions

In the \( z \)-representation and in the dressed-state basis

\[
\begin{cases} 
|b, 0\rangle, \\
|\pm, n\rangle = \frac{1}{\sqrt{2}}(|a, n\rangle \pm |b, n+1\rangle),
\end{cases}
\tag{2}
\]

\(|n\rangle\) being the photon-number states, the problem reduces to the scattering of the atom upon the potentials \( V_{\pm}^n(z) = \pm \hbar g \sqrt{n+1} u(z) \) (see Ref. [3]). Indeed, the set of wavefunction components

\[ \psi_{n}^\pm(z, t) = \langle z, \pm, n|\psi(t)\rangle, \tag{3} \]

where \(|\psi(t)\rangle\) is the atom-field state satisfy the Schrödinger equation

\[ i\hbar \frac{\partial}{\partial t} \psi_{n}^\pm(z, t) = \left( -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial z^2} + V_{n}^\pm(z) \right) \psi_{n}^\pm(z, t). \tag{4} \]

The general solution of (4) is

\[ \psi_{n}^\pm(z, t) = \int dk \, \phi_{n}^\pm(k)e^{-i\frac{\hbar k^2 t}{2M}} \varphi_{n}^\pm(k, z), \tag{5} \]

where \( \varphi_{n}^\pm(k, z) \) is solution of the time-independent Schrödinger equation

\[ \left( \frac{\partial^2}{\partial z^2} + k^2 \mp \kappa_n^2 u(z) \right) \varphi_{n}^\pm(k, z) = 0, \tag{6} \]
with
\[ \kappa_n = \kappa \sqrt{n + 1} \] (7)
and
\[ \kappa = \sqrt{2Mg/\hbar}. \] (8)

The wavefunction component
\[ \psi_{-1}(z, t) = \langle z, b, 0 | \psi(t) \rangle \] (9)
satisfy a Schrödinger equation characterized with a null potential and is therefore not affected by the interaction of the atom with the cavity. The atom in the lower state cannot obviously interact with the cavity field that does not contain any photon. The component (9) describes a free particle problem.

We assume that, initially, the atomic center-of-mass motion is not correlated to the other degrees of freedom. We describe it by the wave packet
\[ \chi(z) \equiv \langle z | \chi \rangle = \int dk A(k) e^{ikz} \theta(-z), \] (10)
where \( \theta(z) \) is the Heaviside step function (indicating that the atoms are incident from the left of the cavity). No restrictions are made for the initial conditions of the atomic internal state and the cavity field state, except that pure states are only considered. By use of an expansion over the dressed-state basis (2), we may write
\[ |\psi(0)\rangle = |\chi\rangle \otimes \left( w_{-1} e^{i\chi_{-1}} |b, 0\rangle + \sum_{n=0}^{\infty} w_n e^{i\chi_n} |\beta_n\rangle \right), \] (11)
with
\[ |\beta_n\rangle = \cos \left( \frac{\theta_n}{2} \right) |+, n\rangle + e^{-i\phi_n} \sin \left( \frac{\theta_n}{2} \right) |-, n\rangle. \] (12)

The parameters \( w_n \in [0, 1], \theta_n \in [0, \pi] \) and \( \chi_n, \phi_n \in [0, 2\pi] \) are called dressed-state coordinates [24]. The normalisation condition is
\[ \sum_{n=-1}^{\infty} w_n^2 = 1 \] (13)
and the phase factor $\chi_{-1}$ may be set to 0 without loss of generality.

We consider therefore

$$
\begin{cases}
\psi_{-1}(z, 0) = c_{-1}\chi(z), \\
\psi^\pm_n(z, 0) = c^\pm_n\chi(z),
\end{cases}
$$

with

$$
\begin{cases}
c_{-1} = w_{-1}, \\
c^+_n = w_n e^{i\chi_n} \cos (\theta_n/2), \\
c^-_n = w_n e^{i(\chi_n-\phi_n)} \sin (\theta_n/2).
\end{cases}
$$

Inserting Eqs. (2) and (10) into Eq. (11), we get

$$
|\psi(0)\rangle = \int dz \int dk A(k) \times
\left( \sum_{n=0}^{\infty} \left[ S_{a,n} e^{ikz} \theta(-z)|z, a, n) \\
+ S_{b,n+1} e^{ikz} \theta(-z)|z, b, n + 1) \\
+ w_{-1} e^{ikz} \theta(-z)|z, b, 0) \right),
\right)
$$

with

$$
\begin{pmatrix}
S_{a,n} \\
S_{b,n+1}
\end{pmatrix} = \tilde{A}_n
\begin{pmatrix}
1 \\
1
\end{pmatrix}
$$

and

$$
\tilde{A}_n = \frac{w_n e^{i\chi_n}}{\sqrt{2}}
\begin{pmatrix}
\cos (\theta_n/2) & e^{-i\phi_n} \sin (\theta_n/2) \\
\cos (\theta_n/2) & -e^{-i\phi_n} \sin (\theta_n/2)
\end{pmatrix}
$$

After the atom has left the interaction region, the wavefunctions $\varphi^\pm_n(k, z)$ can be written as

$$
\varphi^\pm_n(k, z) = \begin{cases}
r^\pm_n(k) e^{-ikz} (z < 0) \\
t^\pm_n(k) e^{ik(z-L)} (z > L)
\end{cases},
$$
where $r_n^\pm(k)$ and $t_n^\pm(k)$ are respectively the reflection and transmission coefficient associated with the scattering of the particle of momentum $\hbar k$ upon the potential $V_n^\pm(z)$ (Eq. 6). The initial state components $\psi_n^\pm(z,0)$ have evolved into

$$
\psi_n^\pm(z, t) = c_n^\pm \int dk A(k) e^{-i k z^2 \hbar^2 t} [r_n^\pm(k)e^{-i k z} \theta(-z)
+ t_n^\pm(k)e^{i k (z-L)} \theta(z-L)]
$$

(20)

whereas the free particle wavefunction component $\psi_{-1}(z,0)$ becomes

$$
\psi_{-1}(z, t) = c_{-1} \int dk A(k) e^{-i k z^2 \hbar^2 t} e^{i k (z-L)} \theta(z-L).
$$

(21)

We thus obtain

$$
|\psi(t)\rangle = \int dz \int dk A(k)e^{-i k z^2 \hbar^2 t} \times \\
\left( \sum_{n=0}^{\infty} \left[ R_{a,n}(k)e^{-i k z} \theta(-z)|z, a, n\rangle
+ T_{a,n}(k)e^{i k (z-L)} \theta(z-L)|z, a, n\rangle
+ R_{b,n+1}(k)e^{-i k z} \theta(-z)|z, b, n+1\rangle
+ T_{b,n+1}(k)e^{i k (z-L)} \theta(z-L)|z, b, n+1\rangle
\right]
+w_{-1}e^{i k (z-L)} \theta(z-L)|z, b, 0\rangle ,
$$

(22)

in which

$$
\begin{pmatrix}
R_{a,n}(k) \\
R_{b,n+1}(k)
\end{pmatrix} = \tilde{A}_n
\begin{pmatrix}
r_n^+(k) \\
r_n^-(k)
\end{pmatrix},
$$

(23a)

$$
\begin{pmatrix}
T_{a,n}(k) \\
T_{b,n+1}(k)
\end{pmatrix} = \tilde{A}_n
\begin{pmatrix}
t_n^+(k) \\
t_n^-(k)
\end{pmatrix}.
$$

(23b)

If initially the electromagnetic field is in the state $|n\rangle$ and the atom is in the excited state $|a\rangle$, the only non-zero dressed-state coordinates are $w_{a} = 1$ and $\theta_{n} = \pi/2$. We get therefore

$$
\tilde{A}_n = \frac{1}{2}
\begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}
$$

(24)
and Eqs. (23) lead to the same results given by Meyer et al. [3] who considered in detail this case for the one-photon mazer.

2.3 Atomic populations

The reduced density matrix $\sigma(t)$ for the atomic internal degree of freedom is given by the trace over the radiation and the atomic external variables of the atom-field density matrix, that is its elements $i, j = a, b$ are

$$\sigma_{ij}(t) = \sum_{n} \int dz \langle z, i, n | \psi(t) \rangle \langle \psi(t) | z, j, n \rangle.$$  \hspace{1cm} (25)

The atomic populations $\sigma_{ii}$ follows immediately from Eq. (25):

$$\sigma_{ii}(t) = \sum_{n} \int dz |\langle z, i, n | \psi(t) \rangle|^2.$$  \hspace{1cm} (26)

Inserting Eqs. (16) and (22) into Eq. (26) and using Eqs. (17) and (23), we get for an incident atom of momentum $\hbar k$:

$$\sigma_{aa}(0) = \frac{1}{2} \left[ 1 - w_{-1}^2 + \sum_{n=0}^{\infty} w_n^2 \sin(\theta_n) \cos(\phi_n) \right],$$  \hspace{1cm} (27)

$$\sigma_{aa}(t) = \frac{1}{2} \left[ 1 - w_{-1}^2 + \sum_{n=0}^{\infty} w_n^2 \sin(\theta_n) \text{Re}(e^{i\phi_n K_n}) \right],$$  \hspace{1cm} (28)

where

$$K_n = r_n^+ r_n^- + t_n^+ t_n^-.$$  \hspace{1cm} (29)

The change of the atomic population $\sigma_{aa}$ induced by the interaction of the incident atom with the cavity radiation field is then given by

$$\delta\sigma_{aa} = \sigma_{aa}(t) - \sigma_{aa}(0),$$  \hspace{1cm} (30)

with the time $t$ chosen long after the interaction.

Thus we have

$$\delta\sigma_{aa} = \sum_{n=0}^{\infty} \Delta_n,$$  \hspace{1cm} (31)
with
\[ \Delta_n = \frac{w_n^2}{2} \sin(\theta_n) \left[ \text{Re} \left( e^{i\phi_n} K_n \right) - \cos(\phi_n) \right]. \] (32)

As expected, the component \( w_{-1} \) of the initial state \( |\psi(0)\rangle \) over the state \( |b,0\rangle \) does not play any role in the dynamics of the system.

We have to emphasize that in Eq. (31) \( \Delta_n \) cannot be interpreted strictly as the change in the \( \sigma_{aa} \) population induced by the interaction of the two-level atom with the cavity radiation field containing \( n \) photons. This is only true when the incident atom is prepared in the excited state. Indeed, if initially the internal atomic state is \( c_a |a\rangle + c_b |b\rangle \) and the field state is \( |n\rangle \) \( (n \geq 1) \), then the only non-zero dressed-state coordinates are \( w_n = |c_a|, \chi_n = \text{arg}(c_a), \theta_n = \pi/2, w_{n-1} = |c_b|, \chi_{n-1} = \text{arg}(c_b), \theta_{n-1} = \pi/2 \) and \( \phi_{n-1} = \pi \). We thus have in that case

\[ \delta\sigma_{aa} = \Delta_n + \Delta_{n-1}, \]
\[ = \Delta_n \text{ iff } c_b = 0. \] (33)

2.4 Photon statistics

The reduced density matrix \( \rho(t) \) for the cavity radiation field is given by the trace over the internal and external atomic degrees of freedom of the atom-field density matrix, that is its elements \( n, n' \) are

\[ \rho_{nn'}(t) = \sum_{i=a,b} \int dz \langle z, i, n|\psi(t)\rangle \langle \psi(t)|z, i, n'\rangle. \] (34)

The photon distribution \( P_n = \rho_{nn} \) follows immediately from Eq. (34):

\[ P_n(t) = \sum_{i=a,b} \int dz |\langle z, i, n|\psi(t)\rangle|^2. \] (35)

The change \( \delta P_n \) in the cavity photon distribution induced by the interaction of the cavity electromagnetic field with the incident atom is then given by

\[ \delta P_n = P_n(t) - P_n(0). \] (36)
Inserting Eqs. (16) and (22) into Eq. (35) and using Eqs. (17) and (23), we get for an incident atom of momentum $\hbar k$:

$$
\delta P_n = \begin{cases} 
\Delta_n - \Delta_{n-1} & (n \geq 1), \\
\Delta_n & (n = 0).
\end{cases} 
$$

(37)

We see that if the initial state is $|a, n\rangle$ we have

$$
\delta \sigma_{aa} + \delta P_{n+1} = 0,
$$

(38)

which gives an intuitive population conservation condition.

2.5 *Reflection and transmission probabilities*

The reflection and transmission probabilities of the incident atom upon the cavity are respectively given by

$$
R = \sum_{i=a,b} \sum_{n=0}^{\infty} \int_{-\infty}^{0} dz |\langle z, i, n|\psi(t)\rangle|^2, 
$$

(39a)

$$
T = \sum_{i=a,b} \sum_{n=0}^{\infty} \int_{L}^{\infty} dz |\langle z, i, n|\psi(t)\rangle|^2. 
$$

(39b)

Inserting Eq. (22) into Eqs. (39), we get for an incident atom of momentum $\hbar k$:

$$
R = \sum_{n=0}^{\infty} w_n^2 (\cos^2(\theta_n/2)|r_n^+|^2 + \sin^2(\theta_n/2)|r_n^-|^2),
$$

(40a)

$$
T = \sum_{n=0}^{\infty} w_n^2 (\cos^2(\theta_n/2)|t_n^+|^2 + \sin^2(\theta_n/2)|t_n^-|^2) \\
\quad + w_{-1}^2.
$$

(40b)

One verifies immediately that the results of Meyer et al. [3] about the reflection and transmission probabilities are well recovered by Eqs. (40) when their initial conditions are considered. Indeed, when the atom-field system is initially in the state $|a, n\rangle$, Eqs. (40) become

$$
R = \frac{1}{2}(|r_n^+|^2 + |r_n^-|^2), 
$$

(41a)

$$
T = \frac{1}{2}(|t_n^+|^2 + |t_n^-|^2). 
$$

(41b)
We get the same results if the atom-field system is initially in the state \( |b,n\rangle \) with \( n \geq 1 \), except that \( n \) must be replaced by \( n-1 \) in Eqs. (41). In the case \( n = 0 \), we have obviously \( T = 1 \).

2.6 Final remarks

All the results here above (about the atomic populations, the photon statistics, and the reflection and transmission probabilities) may be very easily generalized for any momentum wavefunction \( A(k) \) of the initial wave packet. The various expressions must simply be weighted by \( |A(k)|^2 \) and integrated over \( k \). For instance, Eq. (31) becomes

\[
\delta \sigma_{aa} = \int dk |A(k)|^2 \sum_{n=0}^{\infty} \Delta_n, \tag{42}
\]

where \( \Delta_n \) depends on \( k \) through the reflection and transmission coefficients, \( r_n^\pm(k) \) and \( t_n^\pm(k) \) respectively, in \( K_n \) (see Eq. (32)). The expressions obtained for all these various physical quantities are very simple in the framework of the dressed-state formalism, even though they are very general. They take a form much more complicated when the usual coordinates of the atom-field system are used (the complex coefficients \( c_a, c_b \) and \( c(n) \) of the atom-field states written as \( (c_a |a\rangle + c_b |b\rangle) \otimes \sum_n c(n)|n\rangle \)). Also entangled initial states may be considered by this formalism. The great advantage of the dressed-state coordinates was already pointed out by Jonathan et al. [24] who used them to express various physical quantities in the Jaynes-Cummings model.

3 Population Trapping

When the atom-field initial state is such that \( \sin(\theta_n) = 0 \), we get from Eq. (32) \( \Delta_n = 0 \), whatever the value of \( K_n \). In this case, we have

\[
\delta \sigma_{aa} = \delta \sigma_{bb} = \delta P_n = 0, \tag{43}
\]

indicating that the interaction of the atom with the cavity radiation field has no effect on the atomic populations \( \sigma_{ii} \) (\( i = a, b \)) and on the cavity photon distribution \( P_n \), whatever the cavity field mode function, whatever the cavity interaction length \( \kappa L \) and whatever the atomic initial velocity. We conclude that the mazer give rise to the perfect population trapping phenomenon, when considering zero temperature and no dissipation in the high-\( Q \) cavity. This property holds for the ultracold, intermediate and thermal-atom regimes, as
it is completely independent on the external atomic degree of freedom. For the same reason, it holds for any momentum wavefunction $A(k)$ of the initial wave packet.

The class of states verifying $\sin(\theta_n) = 0$, named perfect trapping states, are given by (see Ref. [24])

$$|\gamma^\pm\rangle = \frac{\gamma|a\rangle \pm |b\rangle}{\sqrt{1 + |\gamma|^2}} \otimes \sqrt{1 - |\gamma|^2} \sum_{n=0}^{\infty} \gamma^n |n\rangle,$$

where $\gamma$ is a complex number with $|\gamma| < 1$.

Indeed, rewriting these states in terms of the dressed-state basis, we find

$$|\gamma^\pm\rangle = \sqrt{\frac{1 - |\gamma|^2}{1 + |\gamma|^2}} \left( \sum_{n=0}^{\infty} \sqrt{2} \gamma^{n+1} |\pm, n\rangle \pm |b, 0\rangle \right).$$

(45)

For each $n$ there is only a single dressed-state present in the sum of expression (45). Depending on whether it is $|+, n\rangle$ or $|-, n\rangle$, we have respectively $\sin(\theta_n/2) = 0$ or $\cos(\theta_n/2) = 0$, and so $\sin(\theta_n) = 0$ in any case. This give rise to another very interesting feature of the perfect trapping states. The reflection and transmission probabilities (40) become

$$R = \sum_{n=0}^{\infty} w_n^2 |r_n^\pm|^2,$$

(46a)

$$T = \sum_{n=0}^{\infty} w_n^2 |t_n^\pm|^2 + w_{-1}^2,$$

(46b)

with

$$w_n = \sqrt{\frac{1 - |\gamma|^2}{1 + |\gamma|^2}} \sqrt{2} \gamma^{n+1},$$

(47a)

$$w_{-1} = \sqrt{\frac{1 - |\gamma|^2}{1 + |\gamma|^2}}.$$

(47b)

Remarkably, the particle moving along the $z$–axis is only sensitive to either a superposition of the potentials $V_n^+(z)$ (for the $|\gamma^+\rangle$ states) or a superposition of $V_n^-(z)$ (for the $|\gamma^-\rangle$ states), but never to both. So, it is possible to imagine an experimental set-up where the particles would encounter only an effective potential well or only an effective potential barrier, inhibiting the simultaneous action of them.
Another important characteristic is that the perfect trapping states do not make the cavity transparent to the incident atoms, as in the conventional micromaser, because the reflection coefficient $R$ is not nullified. In this way, we may say that while there is no change in the atomic and field populations, the quantization of the $z$–motion leads to an observable mechanical effect of the atom-field interaction under the perfect trapping condition.

Inserting (47) into (46), we get

$$R = 2 \frac{1 - |\gamma|^2}{1 + |\gamma|^2} |\gamma|^2 \sum_{n=0}^{\infty} |\gamma|^{2n} |r_n^\pm|^2, \quad (48a)$$

$$T = 2 \frac{1 - |\gamma|^2}{1 + |\gamma|^2} |\gamma|^2 \left( \sum_{n=0}^{\infty} |\gamma|^{2n} |t_n^\pm|^2 + \frac{1}{2} |\gamma|^{-2} \right). \quad (48b)$$

In the ultracold regime ($k \ll \kappa_n$), the transmission probability through the potential barrier $V_n^+(z)$ is negligible and we may consider $|t_n^+| = 0$ and $|r_n^+| = 1$, whatever the cavity field mode. We thus have for the $|\gamma^+\rangle$ state

$$R = \frac{2 |\gamma|^2}{1 + |\gamma|^2}, \quad (49a)$$

$$T = \frac{1 - |\gamma|^2}{1 + |\gamma|^2}. \quad (49b)$$

When $|\gamma| \to 0$, we have $R \to 0$ and the cavity acts as a transmitter system. Inversely, when $|\gamma| \to 1$, we get $R \to 1$ and the cavity acts as a reflector system. By varying $|\gamma|$ between 0 and 1, we tune the mazer from a perfect transmitter to a perfect reflector. In both case, the atom-cavity interaction does not perturb in any way the internal state of the system. This interesting property may be easily understood like this. In the state $|\gamma^+\rangle$, the ratio between the wavefunction components over the states $|b, 0\rangle$ and $|+, n\rangle$ is given by (see eq. (45))

$$\frac{\omega_{-1}}{\omega_n} = \frac{1}{\sqrt{2} |\gamma|^{n+1}}. \quad (50)$$

As $|\gamma|$ decreases, the wavefunction component over the state $|b, 0\rangle$ increases and this state does not give rise to any interaction between the atom motion and the cavity. No reflection may occur in this case. Inversely, as $|\gamma|$ increases, the wavefunction components over the states $|+, n\rangle$ dominate and these states give rise to diffusion processes against the potential barriers $V_n^+(z)$. In the ultracold regime, this explains why the atoms are reflected with high probability.

The tuning of the mazer from a perfect transmitter to a perfect reflector system is a general property of the mazer when the atom-field internal state is
initially prepared in the trapping state $|\gamma^+\rangle$. This property holds for any cavity field mode and is independent of the cavity length (as it is based on Eqs. (49) valid in these general conditions). More restrictively, this property may also be observed using the $|\gamma^-\rangle$ states as these states verify also the relation (50). In this situation, when $|\gamma|$ varies from 0 to 1, the wavefunction components over the states $|-n, n\rangle$ increase and the particle is diffused more and more over the potential wells $V_n^-(z)$. In the ultracold regime, this may contribute to reflect as well the atoms, depending on the cavity mode function. This is in particular the case for the mesa mode function ($u(z) = 1$ inside the cavity and zero elsewhere) as it is illustrated on Fig. 1. This figure represents the reflection probability (48a) with respect to $|\gamma|$ for the $|\gamma^+\rangle$ and $|\gamma^-\rangle$ states. The reflection coefficient $r^\pm_n(k)$ have been calculated using the results of Löffler et al. [4]. The curve corresponding to the $|\gamma^-\rangle$ state (denoted $R^-$ on the figure) is specific to the mesa mode function. This is not the case for the curve corresponding to the $|\gamma^+\rangle$ state (denoted $R^+$) which, as expected, fits perfectly the general result (49a) valid for any cavity field mode.

4 Experimental Issues

The feasibility of a mazer-type experiment in the microwave or optical domain has been discussed by Löffler et al. [4] and Retamal et al. [6]. High-$Q$ cavities are needed to fulfill the strong coupling condition and avoid the spontaneous emission during the interaction of the atom with the cavity. Also, kinetic energy significantly lower than the interaction energy $\hbar g (k/\kappa_n \ll 1)$ are required to be in the ultracold regime. Such experimental condition have been achieved for the first time recently by Hood et al. [25] in the optical domain (see also [26,27]). Nevertheless, these results will have still to be improved to test the effects presented in this work.
The population trapping condition, that preserves the internal atomic state while keeping special scattering properties, are attractive from an experimental point of view, since they do not have any restriction about the cavity field mode, the cavity interaction length and the initial atomic velocity. The fact that these perfect trapping states present only one kind of potential, a barrier or a well, to the incident ultracold atoms may be used for new cavity QED scattering experiments [28].

5 Summary

In this paper, we have studied the population trapping phenomenon for the one-photon mazer. With the aim of such study in view, we have written the quantum theory of the mazer by use of the dressed-state coordinate formalism. Simple expressions for the atomic populations, the cavity photon statistics, and the reflection and transmission probabilities have been given for any initial pure state of the atom-field system.

We have demonstrated that the population trapping phenomenon is not only preserved in the ultracold regime but also exhibits new properties. When the atom-field system is prepared in a perfect trapping state, the scattering process becomes very particular. Instead of “feeling” both a potential barrier and a potential well, atoms passing through the cavity are only sensitive to one of the potential components. Also, the atomic and field populations are not changed while having the possibility of a non-zero reflection coefficient.

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References


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