Addendum to: Update on neutrino mixing in the early Universe

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Abstract

In the light of the recent WMAP results we update the constraints on a class of non standard BBN models with a simultaneous combination of non standard neutrino distributions and extra effective number of neutrinos in the expansion rate. These models can be described in terms of the two parameters $\Delta N^\text{tot}_\nu$, constrained by the primordial Helium abundance $Y_p$ measurement, and $\Delta N^\rho_\nu$, constrained by a combination of CMB and primordial Deuterium data. Small deviations from standard Big Bang Nucleosynthesis are suggested. Different non standard scenarios can be distinguished by a measurement of the difference $\Delta N^f_\nu = \Delta N^\text{tot}_\nu - \Delta N^\rho_\nu$. From the current data we estimate $\Delta N^f_\nu \simeq -1.4^{+0.9}_{-1.4}$, slightly disfavouring solutions with a low expansion rate, characterized by $\Delta N^f_\nu = 0$ and negative $\Delta N^\rho_\nu$. From the new WMAP upper bound on the absolute neutrino mass scale we show how active-sterile neutrino mixing could be still a viable explanation only for high values of $Y_p \gtrsim 0.24$, while it would be ruled out by low values $Y_p \lesssim 0.24$. In this second case the existence of large positive neutrino chemical potentials $\xi_i \sim 0.05$, implying $\Delta N^\rho_\nu \simeq 0$, would be a possible explanation of the data within the analyzed class of non standard BBN models.
I. INTRODUCTION

In a previous paper [1] we showed how the new CMB measurements of the baryon to photon ratio, \( \eta \), are able to put stringent constraints on a large class of non standard BBN models where, together with a usual variation of the expansion rate due to the presence of extra degrees of freedom, distortions of the electron neutrino distribution are also present. This class of models can be described in terms of two parameters [2]. The usual extra effective number of neutrinos, modifying the standard expansion rate, \( \Delta N^\rho_\nu \), is defined as

\[
\Delta N^\rho_\nu = \sum_X \frac{\rho_X}{\rho_0} - 3,
\]

where \( \rho_X \) is the energy density of the \( X \)-particle species, including the three ordinary neutrinos plus possible new ones, and \( \rho_0 = (7\pi^2/120) T^4 \) is the energy density of one standard neutrino species. The second parameter is the total extra effective number of neutrinos \( \Delta N^{\text{tot}}_\nu \) defined as:

\[
\Delta N^{\text{tot}}_\nu = \frac{Y_{BBN}^p(\eta, \Delta N^\rho_\nu, \delta f_{\nu_e}) - Y_{SBBN}^p}{0.0137}.
\]

The difference \( \Delta N^{\text{tot}}_\nu - \Delta N^\rho_\nu \) is a quantity that, in the class of models that we are considering, has to be entirely ascribed to the effect of deviations of the electron electron neutrino distribution from the standard Fermi-Dirac with zero chemical potential, \( \delta f_{\nu_e} = f_{\nu_e} - f_{\nu_e}^0 \). If \( \delta f_{\nu_e} = 0 \) then \( \Delta N^{\text{tot}}_\nu = \Delta N^\rho_\nu \) and simply [3]:

\[
Y_{BBN}^p(\eta, \Delta N^\rho_\nu, \delta f_{\nu_e} = 0) \simeq Y_{SBBN}^p(\eta) + 0.0137 \Delta N^\rho_\nu \tag{3}
\]

The standard BBN prediction for \( Y_p \) is well described by the following expansion around \( \eta = 5 \) [3]:

\[
Y_{SBBN}^p \simeq 0.2466 + 0.01 \ln \left( \frac{\eta}{5} \right) \tag{4}
\]

with \( \eta \) the baryon to photon ratio in units of \( 10^{-10} \). The presence of non zero \( \delta f_{\nu_e} \) affects mainly the primordial Helium abundance, \( Y_p \), while its effect can be safely neglected in the Deuterium abundance, \( (D/H) \), also considering that we will be interested in small deviations. With this approximation the \( D/H \) abundance is well described by the following expression [1]:

\[
(D/H)^{BBN}(\eta, \Delta N^\rho_\nu) \simeq \left[ 3.6 \cdot 10^{-5} \left( \frac{\eta}{5} \right)^{1.6} \right] \left( 1 + 0.135 \Delta N^\rho_\nu \right)^{0.8} \tag{5}
\]

With these analytical expressions a simultaneous measurement of the primordial abundances \( (D/H) \) and \( Y_p \), and of the baryon to photon ratio \( \eta \) can be easily translated into a ‘measurement’ of \( \Delta N^{\text{tot}}_\nu \) and \( \Delta N^\rho_\nu \). For the Helium abundance \( Y_p \) we used in [1] \(^1\) both high values

\[^1\]We indicate 68\% c.l. errors for all quantities unless differently indicated.
\( Y_p^{\text{exp}} = 0.244 \pm 0.002, \quad (6) \)

and low values [5]
\[ Y_p^{\text{exp}} = 0.234 \pm 0.003, \quad (7) \]

while for the Deuterium abundance we used [6]
\[ (D/H)^{\text{exp}} = (3.0 \pm 0.4) \times 10^{-5}. \quad (8) \]

For \( \eta \) we used the DASI and BOOMerANG result [7]:
\[ \eta^{CMB} = 6.0^{+1.1}_{-0.8} \quad (9) \]

From low values of Helium and assuming gaussian errors for all quantities, we obtained at 1 \( \sigma \):
\[ \Delta N_{\nu}^{\text{tot}} = -1.05 \pm 0.25, \quad (10) \]

while from high values of Helium we obtained
\[ \Delta N_{\nu}^{\text{tot}} = -0.3 \pm 0.2. \quad (11) \]

Using the primordial Deuterium abundance measurement (cf. (8)), from the expression (5) we could estimate \( \Delta N_{\nu}^p \) obtaining
\[ \Delta N_{\nu}^p = 1 \pm 4. \quad (12) \]

These results were implying, at 3 \( \sigma \) the following bounds [1]
\[ \Delta N_{\nu}^{\text{tot}} < 0.3, \quad \Delta N_{\nu}^p \lesssim 13 \quad (13) \]

In particular the bound on \( \Delta N_{\nu}^{\text{tot}} \) was used to conclude that all four neutrino mixing models are in disagreement with cosmology and thus ruled out. This result has been then also confirmed by the improved solar and atmospheric neutrino data from the SNO [8] and SuperKamiokande [9] experiments without use of cosmological bounds [10]. In the next section we will update these results in light, mainly, of the recent results from the WMAP experiment [11] and we will see how the data suggest possible deviations from a standard picture.

**II. UPDATED REFERENCE VALUES AND RESULTS**

The WMAP collaboration finds \( \Omega_b h^2 = 0.0224 \pm 0.0009 \) [11] corresponding to:
\[ \eta^{CMB} = 6.15 \pm 0.25 \quad (14) \]

This measurement is so precise that now, when estimating \( \Delta N_{\nu}^{\text{tot}} \), the experimental error on \( Y_p \) is dominant compared to that one on \( \eta \). Using high values of \( Y_p^{\text{exp}} \) (cf. (6)) we find at 1 \( \sigma \):
\[ \Delta N_{\nu}^{\text{tot}} = -0.35 \pm 0.15 \]  

This means that now a 3\(\sigma\) upper bound is given by
\[ \Delta N_{\nu}^{\text{tot}} < 0.1 , \]  

that is quite more stringent compared to the previous estimation. Even using a range of values for \(Y_p^{\text{exp}}\) that is a compromise between low and high values and takes into account the discrepancy as a systematic uncertainty [12],
\[ Y_p =! 0.238 \pm 0.002 \pm 0.005 , \]  

we find
\[ \Delta N_{\nu}^{\text{tot}} = -0.8 \pm 0.4 , \]  

implying a 3\(\sigma\) bound
\[ \Delta N_{\nu}^{\text{tot}} < 0.4 . \]  

Both results confirm our previous conclusion for which \(\Delta N_{\nu}^{\text{tot}}\) as high as 1 is highly disfavoured, thus ruling out all 4 neutrino mixing models. However now both results seem to point out, at 2\(\sigma\), to a negative value of \(\Delta N_{\nu}^{\text{tot}}\), suggesting the presence of non standard BBN effects. We can also update the estimation of \(\Delta N_{\rho}^{\nu}\) using the new \(\eta\) measurement from CMB and a new primordial Deuterium abundance measurement, slightly lower than the previous one [13]
\[ (D/H)^{\text{exp}} = (2.78^{+0.44}_{-0.38}) \times 10^{-5} , \]  

finding
\[ (\Delta N_{\rho}^{\nu})^{\text{BBN}} = 0.7 \pm 2.1 \]  

As already anticipated in [1], the error has been highly reduced by the great improvement in the \(\eta\) determination from CMB and it is now dominated by the error on \(D/H\). However, differently from \(Y_p\), a better determination of \(\eta\) can further reduce the error on \(\Delta N_{\rho}^{\nu}\) at the level of \(\sim 1.5\).

It is interesting to note that the value from BBN is comparable to the direct determination from CMB. In [14], combining the WMAP data with the 2dF redshift survey and using the value on the Hubble constant from the HST Key Project, \(h = 0.72 \pm 0.08\) [15], the authors find:
\[ (\Delta N_{\rho}^{\nu})^{\text{CMB}} = 0.5^{+1.8}_{-0.9} \]  

Assuming that, between the nucleosynthesis and the recombination time, the quantity \(\Delta N_{\rho}^{\nu}\) does not change and thus that \((\Delta N_{\rho}^{\nu})^{\text{BBN}} = (\Delta N_{\rho}^{\nu})^{\text{CMB}}\), one can then combine the two values. We will still use a gaussian distribution approximation for a qualitative estimation.
From the likelihood distribution given in [14], this does not seem to be a very good approximation at values larger than the central one, while it is reasonably good for smaller values. With this clarification, we can then estimate a CMB-Deuterium combined value

$$\Delta N_\nu^\rho \simeq 0.6^{+1.4}_{-0.8}.$$  \hspace{1cm} (23)

In this way we get a much more stringent $2\sigma$ ($3\sigma$) upper bound:

$$\Delta N_\nu^\rho \lesssim 3.4(4.8)$$  \hspace{1cm} (24)

Certainly a more precise calculation, taking into account the exact distribution, would be desirable but the final result should not be much different from this qualitative estimation.

### III. POSSIBLE SCENARIOS

These new results show that deviations from Standard BBN, if they exist, are small. This means that Standard BBN is in any case, in first approximation, a very good description of all data. This result is mainly due to the fact that the Deuterium abundance is in very good agreement with the CMB prediction. At the same time the measured primordial Helium abundance, $Y_p$, suggests the possible presence of small deviations whose detection is now possible mainly to the great precision of CMB in measuring the baryon asymmetry. However, for an assessment of such a hint, it will be necessary to reduce the large systematic uncertainties on $Y_p$ and it will be also necessary to investigate even more accurately on the robustness of the $\eta$ determination from CMB. In the following we will assume that such a hint is suggestive of non standard BBN effects and we will discuss some possible scenarios that could explain these deviations. An important role in our discussion is given by the quantity $\Delta N_{f\nu} = \Delta N_{\nu}^{\text{tot}} - \Delta N_\nu^\rho$. From (23) and (18) we can estimate:

$$\Delta N_{f\nu} \simeq -1.4^{+0.9}_{-1.4}$$  \hspace{1cm} (25)

### A. Low expansion rate

A minimal possibility is to interpret the data saying that $\Delta N_\nu^{\text{tot}} - \Delta N_\nu^\rho = 0$. In this case the combined measurement of $\Delta N_\nu^{\text{tot}}$ would point out to a negative value, mainly because of the low value of $Y_p$ [16]. This possibility would suggest a highly non standard modification of the expansion rate during the BBN time, more precisely a lower expansion rate. Usually the presence of new particle species would lead to a higher expansion rate and so such a possibility should rely on some drastic change of the radiation dominated picture during the BBN period. Moreover note that, from the Eq. (25), the measurements slightly favour a value $\Delta N_{f\nu} \neq 0$. 

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B. Degenerate BBN

A well known modification of the Standard BBN is to introduce neutrino chemical potentials in the thermal distributions [18]. An electron neutrino chemical potential \((\xi_e = \mu_e/T)\) would yield

\[ \Delta N^\text{tot}_\nu \simeq -16 \xi_e. \quad (26) \]

The observed \(Y_p\) (cf. (17) ) would be then explained by having

\[ \xi_e = 0.05 \pm 0.025 \quad (27) \]

It has been shown [17] that the existing information on neutrino mixing makes possible to conclude that before the onset of BBN arbitrary initial neutrino chemical potentials would be almost equilibrated in a way that \(\xi_\nu \simeq \xi_\tau \simeq \xi_e\). The presence of chemical potentials would thus correspond to

\[ \Delta N^\rho_\nu \simeq 3 \left[ \frac{30}{\pi} \left( \frac{\xi_e}{\pi} \right)^2 + \frac{15}{\pi} \left( \frac{\xi_e}{\pi} \right)^4 \right] \simeq 3 \times 10^{-3} \ll \Delta N^\text{tot}_\nu \quad (28) \]

Therefore in this scenario the expansion rate would be practically standard and the deviations would entirely arise from non standard electron neutrino distribution.

C. Active sterile neutrino oscillations

It has been shown in many papers [19] that a small mixing between active neutrinos and new light sterile neutrinos can produce in general a negative value of \(\Delta N^f_\nu\) together with \(\Delta N^\rho_\nu \geq 0\). In a simplified two neutrino mixing the value of \(\Delta N^f_\nu\) is highly dependent on the value of the parameter \(\Delta m^2_{is} = m^2_s - m^2_i\). Usually the possibility to introduce active-sterile neutrino oscillations was motivated by the LSND anomaly [21]. However an explanation of the LSND anomaly in terms of active -sterile neutrino oscillations, compatible with the solar and atmospheric neutrino data would yield, as already mentioned, \(\Delta N^\text{tot}_\nu = \Delta N^\rho_\nu \sim 1\) [1,22]. At the same time the new WMAP bound on the neutrino masses, \(m_i \leq 0.23\,\text{eV}\) [11], is now also incompatible with such an explanation of the LSND anomaly [23].

The possibility to generate a negative \(\Delta N^\text{tot}_\nu\) requires a negative value of \(\Delta m^2 = m^2_s - m^2_i\) and very small mixing angles \((\sin^2 2\theta \ll 10^{-4} [19,1])\). Values of \(m_i \leq 0.23\,\text{eV}\) imply thus \(|\Delta m^2_{is}| \leq 5 \times 10^2\,\text{eV}^2\). In [20] it was shown how such maximum value, together with very small mixing angles, would produce \(\Delta N^f_\nu \geq -0.3\). For an inverted full hierarchical case the corresponding \(\Delta m^2_{is} \sim 10^2\,\text{eV}^2\) and in this case \(\Delta N^f_\nu \sim -0.13\). These values have to be considered as maximal because in the reality one should consider a 3 + 1 mixing and, though full calculations are still missing, one can expect that part of electron neutrino asymmetry is actually shared with the other two flavours. This means that the small effect could reconcile the observed \(\eta_B\) from CMB only with high values of \(Y_p\) (cf. (6)). In a two neutrino mixing small positive values of \(\Delta N^f_\nu\) are also possible, for larger mixing angles, but this would go at expenses of \(|\Delta N^f_\nu|\), making it even smaller [1]. Having more than one
sterile neutrino flavour would make possible to have $\Delta N^b_\nu \simeq -0.3$ and positive $\Delta N^o_\nu$ but in this case the total $\Delta N^\text{tot}_\nu$ would be larger than $-0.3$. This possibility is however interesting, since it would be a way to distinguish active-sterile neutrino oscillations from a degenerate BBN scenario. Another way would be the detection of the effects of a possible formation of neutrino domains [24], like inhomeogeneities in the primordial Deuterium abundance [24] or an associated possible production of gravitational waves [25].

IV. CONCLUSIONS

In future years a better understanding of sistematic uncertainties in the measured $Y_p$ could strengthen or disprove the hint of non standard BBN effects. At the same time improved data from CMB experiments should both be able to measure $\Delta N^o_\nu$ with a precision of $\sim 0.1$ and make even more robust and precise the determination of $\eta_B$. If the primordial Helium anomaly will be confirmed, implying negative $\Delta N^\text{tot}_\nu < 0$, then a key quantity in discriminating among different explanations is the quantity $\Delta N^\text{tot}_\nu - \Delta N^o_\nu$. If this will prove to be not zero and negative, then low expansion rate scenarios will be ruled out, as already slightly suggested from current data, and a scenario with large chemical potentials would be possible explanation if at the same time $\Delta N^o_\nu \sim 0$. In the case that $|\Delta N^\text{tot}_\nu| < 0.3$, then active-sterile neutrino mixing can be a viable explanation too and if this is also accompanied by a positive value of $\Delta N^o_\nu$, then it will be actually favoured, since degenerate BBN would be ruled out.

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