Kinetic approach to electroweak baryogenesis

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Abstract

After a short review of baryogenesis mechanisms, we focus on the charge transport mechanism at the electroweak scale, effective at strong electroweak phase transitions. Starting from the one-loop Schwinger-Dyson equations for fermions coupled to bosons, we present a derivation of the relevant kinetic equations in the on-shell and gradient approximations, relevant for the thick wall baryogenesis regime. We then discuss the CP-violating source from the semiclassical force in the flow term, and compare it with the source arising in the collision term of the kinetic equation. Finally, we summarize the results concerning the chargino mediated baryogenesis in the Minimal Supersymmetric Standard Model.

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1 Introduction

The necessary requirements on dynamical baryogenesis at an epoch of the early Universe are provided by the Sakharov conditions: (I) baryon number (B) violation, (II) charge (C) and charge-parity (CP) violation and (III) departure from thermal and kinetic equilibrium. The Sakharov conditions may be realised at the electroweak transition \cite{1}, provided the transition is strongly first order. C and CP violation are realised in the standard model (SM) for example through the Cabibbo-Kobayashi-Maskawa (CKM) matrix of quarks. At high temperatures B is violated through sphaleron transitions, and may be responsible for the observed baryon asymmetry today, which is usually expressed as the baryon-to-entropy ratio \cite{2}, \( n_B/\gamma \simeq n_B/7n_\gamma = 7.0 \pm 1.5 \times 10^{-11}. \) This is consistent both, with the nucleosynthesis, as determined by the observed D/H ratio, and with recent cosmic microwave background observations \cite{3}.

Over the years a large number of scenarios have been proposed to explain the observed matter-antimatter asymmetry of the Universe. Broadly speaking, they can be divided into two classes: the models based on the grand unification of forces, and the models based on the electroweak symmetry breaking. In grand unified models \cite{4, 5} one uses the B-L violation by the gauge and/or Higgs sectors of the GUT model. The particular realisations include baryogenesis at preheating through the inflaton decay into heavy GUT particles \cite{6, 7}, whose out-of-equilibrium decays can produce a nonzero B-L; leptogenesis \cite{8, 9}, which involves the physics of heavy majorana neutrinos; the Affleck-Dine mechanism \cite{10}, which involves the dynamics of (B-L)-violating flat directions of supersymmetric models \cite{11} (which may, but need not, be embedded in a grand-unified model), remnants of which may survive as B-balls \cite{12}.

Concerning the electroweak scale baryogenesis, the Standard Model (SM) cannot alone be responsible for the observed matter-antimatter asymmetry, primarily because the LEP bound on the Higgs mass \( m_H \geq 110 \) GeV \cite{13} is inconsistent with the requirement that the transition be strongly first order, in order for the baryons produced in the symmetric phase be not washed-out by the sphaleron transitions in the Higgs (‘broken’) phase.

Supersymmetric extensions of the Standard Model, on the other hand, may result in a strong first order transition. Indeed, in the Minimal Supersymmetric Standard Model the sphaleron bound can be satisfied, provided the stop and the lightest Higgs particles are not too heavy, \( m_\tilde{t} \leq 170 \) GeV and \( m_H \leq 120 \) GeV \cite{14}. Non-minimal supersymmetric extensions can typically provide stronger phase transitions, and are hence less constrained \cite{15}.

An efficient mechanism for baryon production at the electroweak phase transition is the charge transport mechanism \cite{16}, which works as follows: At a first order transition, when the Universe supercools, the bubbles of the Higgs phase nucleate and grow. In presence of a CP-violating condensate at the bubble interface, as a consequence of collisions of chiral fermions with scalar particles in presence of a scalar field condensate, CP-violating currents are created and transported into the symmetric phase, where they bias baryon number production. The baryons thus produced are transported back into the Higgs phase where they are frozen-in. The main unsolved problem of electroweak baryogenesis is the systematic computation of the relevant CP-violating currents generated at the bubble interface. Here we shall reformulate this problem in terms of calculating CP-violating sources in the kinetic
Boltzmann equations for fermions.

The techniques we report here are relevant for calculation of sources in the limit of thick phase boundaries and a weak coupling to the Higgs condensate, which are both generically realised in supersymmetric models. In this case one can show that, to linear order in the Planck constant \( \bar{\hbar} \), the quasiparticle picture for fermions survives \([17, 18]\). In presence of a CP-violating condensate there are two types of sources: the semiclassical force in the flow term of the kinetic Boltzmann equation, and the collisional sources. The semiclassical force was originally introduced for baryogenesis in two-Higgs doublet models \([19]\), and subsequently adapted to the chargino baryogenesis in the Minimal Supersymmetric Standard Model (MSSM) \([20]\). The semiclassical force corresponds to tree-level interactions with a semiclassical background field, and it is universal in that its form is independent on interactions. The collisional sources, on the other hand, arise when fermions in the loop diagrams interact with scalar background fields. These sources arise first from the one-loop diagrams, in which fermions interact with a CP-violating scalar background. When viewed in the kinetic Boltzmann equation, these processes correspond to tree-level interactions in which fermions absorb or emit scalar particles, whilst interacting in a CP-violating manner with the scalar background. The precise form of the collisional sources depends on the form of the interaction. In the following sections we discuss how one can study the CP-violating collisional sources induced by a typical Yukawa interaction term.

2 Kinetic equations and the quasiparticle picture

Here we work in the simple model of chiral fermions coupled to a complex scalar field via the Yukawa interaction \([17, 18]\)

\[
\mathcal{L}_{Yu} = -y\phi\bar{\psi}_L\psi_R - y\phi^*\bar{\psi}_R\psi_L, \tag{1}
\]

which, at a phase transition, may give rise to a complex, spatially varying, mass term

\[
m(u) \equiv y'\Phi_0(u) = m_R(u) + im_I(u) = |m(u)|e^{i\theta(u)}. \tag{2}
\]

Here \( \Phi_0(u) = \langle \Omega|\hat{\Phi}(u)|\Omega \rangle \), and \( |\Omega \rangle \) is the physical state. Such a situation is realised, for example, by the Higgs field at a first order electroweak phase transition.

The dynamics of quantum fields can be studied by considering the equations of motion arising from the two-particle irreducible (2PI) effective action \([21, 22]\) in the Schwinger-Keldysh closed-time-path formalism \([23, 24]\). This formalism is suitable for studying the dynamics of the non-equilibrium fermionic and bosonic two-point functions

\[
iS_{\alpha\beta}(u,v) = \langle \Omega|T_C[\psi_\alpha(u)\bar{\psi}_\beta(v)]|\Omega \rangle \tag{3}
\]

\[
i\Delta(u,v) = \langle \Omega|T_C[\phi(u)\phi^+(v)]|\Omega \rangle, \tag{4}
\]

where the time ordering \( T_C \) is along the Schwinger contour shown in figure 1. The \( C \)-time ordering can be conveniently represented in the Keldysh component formalism. For example, for nonequilibrium dynamics of quantum fields the following Wightman propagators are of
particular relevance,

\[ iS^<(u, v) = -\langle \Omega|\bar{\psi}(v)\psi(u)|\Omega \rangle \]
\[ i\Delta^<(u, v) = \langle \Omega|\bar{\phi}(v)\phi(u)|\Omega \rangle. \]  
(5)

Figure 1: The closed time contour for the Schwinger-Keldysh nonequilibrium formalism.

For thick walls, i.e. when the de Broglie wavelength \( \ell_{\text{dB}} \) of typical plasma excitations is small in comparison to the phase interface thickness \( L_w \), it is suitable to work in the Wigner representation for the propagators, which corresponds to the Fourier transform with respect to the relative coordinate \( r = u - v \), and expand in the gradients of average coordinate \( x = (u + v)/2 \). This then represents an expansion in powers of \( \ell_{\text{dB}}/L_w \). When written in this Wigner representation, the kinetic equations for fermions become [25]

\[ DS^< \equiv \left( \frac{i}{2} \partial^2 - \left( mP_R - m^*P_L \right)e^{-\frac{i}{2}\nabla \cdot \partial_k} \right) S^< = C_\psi, \]  
(6)

where for simplicity we neglected the contributions from self-energy corrections to the mass and the collisional broadening term, which is of the form \( -e^{-i\gamma}\{\Sigma^<\}\{S_h\} \), where \( S_h = (S^r + S^a)/2 \) is the hermitean part of the propagator, and \( S^r \) and \( S^a \) denote the retarded and advanced propagators, respectively. By considering, as an example, the scalar field theory, we have been able to show that, to first order in gradients, the flow term can be rewritten as [27]

\[ A_s \diamond \{\Omega^2\}\{n_\phi\} - 2\Gamma_\phi A_s \Delta_h \diamond \{\Gamma_\phi\}\{n_\phi\} = C_\phi, \]  
(7)

where \( A_s \propto \delta(k^2 - m^2_\phi - \Sigma_h) \) denotes the on-shell spectral function, \( n_\phi \) the bosonic occupation number, \( \Delta_h = (\Delta^r + \Delta^a)/2 \) and \( C_\phi \) the scalar collision term. This implies that, when working to linear order in the width \( \Gamma_\phi = (i/2)(\Pi^r - \Pi^<) \) and self-energy \( \Pi_h = (\Pi^r + \Pi^a)/2 \), one can include the self-energy and collision term, while the effect of the collisional broadening, described by the second term in (7), can be consistently neglected. While we focus here on the effects of a pseudoscalar mass, one should keep in mind that one can include the effect of the self-energy within the on-shell approximation, provided one appropriately modifies the spectral condition.

When the collision term \( C_\psi \) is approximated at the one-loop (with the resummed propagators), equation (6) corresponds to the nonequilibrium fermionic Schwinger-Dyson equation shown in figure 2. Since the flow term of the scalar equation (also shown in figure 2) does not yield CP-violating sources at first order in gradients [17, 27], we do not discuss it here.

As the bubbles grow large, they tend to become more and more planar. Hence, it suffices to consider the limit of a planar phase interface, in which the mass condensate in the wall frame becomes a function of one coordinate only, \( m = m(z) \). Further, we keep only the
Figure 2: The one-loop Schwinger-Dyson equations for the out-of-equilibrium fermionic ($S$) and scalar ($\Delta$) propagators. When projected on-shell and expanded in gradients, these equations reduce to the kinetic Boltzmann equations.

terms that contribute at order $\hbar$ to Eq. (6), which implies that we need to keep second order gradients of the mass term

$$me^{-\frac{1}{2}\vec{\alpha} \cdot \partial_k} = m + \frac{i}{2} m' \partial_{k_z} - \frac{1}{8} m'' \partial_{k_z}^2 + o(\partial_{k_z}^3), \quad (8)$$

where $m = m(z)$, $m' \equiv \partial_z m$ and $m'' \equiv \partial_{k_z}^2 m$. On the other hand, in the collision term $C_\psi$ we need to consider terms only up to linear order in derivatives

$$C_\psi = C_{\psi 0} + C_{\psi 1} + ..$$

$$C_{\psi 0} = -\frac{1}{2} \left( \Sigma^> S^< - \Sigma^< S^> \right)$$

$$C_{\psi 1} = -\frac{i}{4} \left( \partial^{(1)}_{k_z} \partial^{(2)}_{k_z} - \partial^{(1)}_{k_z} \partial^{(2)}_{k_z} \right) \left( \Sigma^> S^< - \Sigma^< S^> \right), \quad (9)$$

where $\Sigma^< \text{ and } \Sigma^>$ represent the fermionic self-energies, and the derivatives $\partial^{(1)}_{k_z}$, $\partial^{(1)}_{k_z}$ ($\partial^{(2)}_{k_z}$, $\partial^{(2)}_{k_z}$) act on the first (second) factor in the parentheses.

An important observation is that, when $G = G(k_\mu, t - \vec{x}_\parallel, \vec{k}_\parallel, z)$, the spin

$$S_z \equiv L^{-1}(\Lambda) \tilde{S}_z L(\Lambda) = \gamma^\parallel (\tilde{S}_z - i(\vec{v}_\parallel \times \vec{\alpha})_z) \quad (10)$$

is conserved

$$[D, S_z] S^< = 0, \quad (11)$$

where $D$ is the differential operator in Eq. (6), $\vec{\alpha} = \gamma^0 \vec{\gamma}, \tilde{S}_z = \gamma^0 \gamma^3 \gamma^5$, and $\gamma^\parallel = 1/(1 - \vec{v}_\parallel^2)^{1/2}$. The spin operator corresponds to the boosted spin in the $z$-direction (of the interface motion), and when written as the Pauli-Lubanski spin operator,

$$S_{PL}(k, s) \equiv -\frac{1}{e_0} \vec{k} \times \gamma^5, \quad e_0 \equiv (k^2)^{1/2}, \quad s^2 = -1, \quad s \cdot k = 0, \quad (12)$$
the spin 4-vector corresponds to

$$ s^\mu = \frac{1}{k_0 e_0} \begin{pmatrix} k_0 k_z \\ k_z k_z \\ k_y k_z \\ k_z k_0 \end{pmatrix}. $$

(13)

In the highly relativistic limit we have $\tilde{k}_0^2 \approx k_z^2$, and the spin vector $\vec{s} \propto \vec{k}$, such that the spin operator $S_z$ approaches the helicity operator,

$$ \hat{H}(\vec{k}) = -\frac{1}{e_0} \kappa \gamma^5 $$

$$ = \vec{k} \cdot \gamma^0 \gamma^5, \quad h^\mu = \frac{1}{e_0} \left( \frac{\vec{k}}{k_0} \right), $$

(14)

as one would expect. (As usually, the helicity operator measures spin in the direction of particle’s motion, $\vec{k} = \vec{k}/|\vec{k}|$.) As a consequence, for light particles with momenta of order the temperature, $k \sim T \gg m$, the spin states we consider here can be approximated by the helicity states, which are often used in literature for baryogenesis calculations [19, 20]. To answer the question to what extent is this fulfilled, requires a detailed quantitative study, which is beyond the scope of this talk [27].

This discussion implies that, without loss of generality, the fermionic Wigner function can be written in the following block-diagonal form

$$ S^\prec = \sum_{s=\pm} S^\prec_s $$

$$ S^\prec_s = L(\Lambda)^{-1} S^\prec L(\Lambda) $$

$$ -i\gamma^0 \tilde{S}^\prec_s = \frac{1}{4} (1 + s\sigma^3) \otimes \rho^a \tilde{g}^s_a, $$

(15)

where $\sigma^3$ and $\rho^i$ ($i = 1, 2, 3$) are the Pauli matrices, $\rho^0 = 1$ is the $2 \times 2$ unity matrix, and $L(\Lambda)$ is the following Lorentz boost operator

$$ L(\Lambda) = \frac{k_0 + \tilde{k}_0 - \gamma^0 \gamma \cdot \tilde{k}_\parallel}{\sqrt{2k_0(k_0 + \tilde{k}_0)}}, $$

(16)

with $\tilde{k}_0 = \text{sign}(k_0)(k_0^2 - \tilde{k}_\parallel^2)^{1/2}$, and $\Lambda$ corresponds to the Lorentz boost that transforms away $\tilde{k}_\parallel$.

With the decomposition (15) the trace of the antihermitean part of Eq. (6) can be written as the following algebraic constraint equation [18]

$$ \left( k^2 - |m|^2 + \frac{s}{k_0} |m|^2 \theta' \right) g^s_{00} = 0, $$

(17)

where $g^s_{00} = \gamma_{\parallel} \tilde{g}^s_{0}$ denotes the particle density on phase space $\{k_\mu, x_\nu\}$. Equation (17) has a spectral solution
\[ g_{00}^s \equiv \sum_{\pm} \frac{2\pi}{Z_{s\pm}} n_s \delta(k_0 \mp \omega_{s\pm}), \]  

where \( \omega_{s\pm} \) denotes the dispersion relation

\[ \omega_{s\pm} = \omega_0 \mp s \frac{|m|^2 \theta'}{2\omega_0 \omega_0}, \quad \omega_0 = \sqrt{k^2 + |m|^2}, \quad \tilde{\omega}_0 = \sqrt{\omega_0^2 - \vec{k}_\parallel^2} \]  

and \( Z_{s\pm} = 1 \mp s|m|^2 \theta'/2\tilde{\omega}_0^3 \). The delta functions in (18) project \( n_s(k_\mu, t - \vec{x}_\parallel \cdot \vec{k}_\parallel, z) \) on-shell, thus yielding the distribution functions \( f_{s+} \) and \( f_{s-} \) for particles and antiparticles with spin \( s \), respectively, defined by

\[ f_{s+} \equiv n_s(\omega_{s+}, k_z, t - \vec{x}_\parallel \cdot \vec{k}_\parallel, z) \]
\[ f_{s-} \equiv 1 - n_s(-\omega_{s-}, -k_z, t + \vec{x}_\parallel \cdot \vec{k}_\parallel, z). \]

This on-shell projection proves the implicit assumption underlying the semiclassical WKB-methods, that the plasma can be described as a collection of single-particle excitations with a nontrivial space-dependent dispersion relation. In fact, the decomposition (15), Eq. (17) and the subsequent discussion imply that the physical states that correspond to the quasiparticle plasma excitations are the eigenstates of the spin operator (10).

Taking the trace of the Hermitian part of Eq. (6), integrating over the positive and negative frequencies and taking account of (18) and (20), one obtains the following on-shell kinetic equations

\[ \partial_t f_{s\pm} + \vec{v}_{s\pm} \cdot \nabla f_{s\pm} + v_{s\pm} \partial_z f_{s\pm} + F_{s\pm} \partial_{k_z} f_{s\pm} = C_{s\pm}[f_{s\pm}], \]  

where \( f_{s\pm} = f_{s\pm}(\vec{k}, z, t - \vec{x}_\parallel \cdot \vec{k}_\parallel) \), \( C_{s\pm}[f_{s\pm}] \) is the collision term obtained by integrating (9) over the positive and negative frequencies, respectively, the quasiparticle group velocity \( v_{s\pm} \equiv k_z/\omega_{s\pm} \) is expressed in terms of the kinetic momentum \( k_z \) and the quasiparticle energy \( \omega_{s\pm} \) (19), and the semiclassical force

\[ F_{s\pm} = -\frac{|m|^2 \theta'}{2\omega_{s\pm}} \pm \frac{s|m|^2 \theta'}{2\tilde{\omega}_0 \omega_0}. \]

In the stationary limit in the wall frame the distribution function simplifies to \( f_{s\pm} = f_{s\pm}(\vec{k}, z) \). When compared with the 1+1 dimensional case [17], the sole, but significant, difference in the force (22) is that the CP-violating \( |m|^2 \theta' \)-term is enhanced by the boost-factor \( \gamma_\parallel = \omega_0/\tilde{\omega}_0 \), \( \tilde{\omega}_0 = (\omega_0^2 - \vec{k}_\parallel^2)^{1/2} \), which, when integrated over the momenta, leads to an enhancement by about a factor two in the CP-violating source from the semiclassical force.

3 Sources for baryogenesis in the fluid equations

Fluid transport equations are usually obtained by taking the first two moments of the Boltzmann transport equation (21): integrating (21) over the spatial momenta results in the
Figure 3: The flow term sources (25)-(26) characterised by the integrals $x_i^2 J_a(x_i)$ (red solid) and $x_i^3 J_b(x_i)$ (green dashed) as a function of the rescaled mass $x_i = |m_i|/T$. The sum of the two sources (dotted blue) is also shown.

continuity equation for the vector current, while multiplying by the velocity and integrating over the momenta yields the Euler equation. The physical content of these equations can be summarized as the particle number and fluid momentum density conservation laws for fluids, respectively. This procedure is necessarily approximate simply because the fluid equations describe only very roughly the rich momentum dependence described by the distribution functions of the Boltzmann equation (22). The fluid equations can be easily reduced to the diffusion equation which has so far being used almost exclusively for electroweak baryogenesis calculations at a first order electroweak phase transition. A useful intermediate step in the derivation of the fluid equations is rewriting Eq. (21) for the CP-violating departure from equilibrium $\delta f_{si} = \delta f_{si+} - \delta f_{si-}$ as follows
\[
\left( \partial_t + \frac{k_z}{\omega_{0i}} \partial_z - \frac{|m_i|^2}{2\omega_{0i}} \partial_{k_z} \right) \delta f_{si} + v_w \delta F_{si} (\partial_\omega f_\omega)_{\omega_{0i}}
+ v_w F_{0i} \delta \omega_{si} \left[ \left( \frac{\partial_\omega f_\omega}{\omega_\omega} \right)_{\omega_{0i}} - (\partial_\omega^2 f_\omega)_{\omega_{0i}} \right] = C_{\psi si},
\]
where $i$ is the species (flavour) index, $f_\omega = 1/(e^{\beta \omega} + 1)$, and
\[
\begin{align*}
F_{0i} &= -\frac{|m_i|^2}{2\omega_{0i}} \\
\delta \omega_{si} &= s \frac{(|m_i|^2 \theta_i)'}{\omega_{0i} \tilde{\omega}_{0i}} \\
\delta F_{si} &= F_{si+} - F_{si-} = s \frac{(|m_i|^2 \theta_i)'}{\omega_{0i} \tilde{\omega}_{0i}} \\
C_{\psi si} &= C_{\psi si+} - C_{\psi si-}.
\end{align*}
\]
Figure 4: The semiclassical force baryogenesis mediated by charginos of the MSSM calculated in the helicity basis. The figure shows contours for the baryon-to-entropy ratio in the units of $10^{-11}$ for two wall velocities $v_w = 0.01$ and $v_w = 0.03$ as a function of the soft susy breaking parameters $\mu$ and $m_2$. A maximal CP violation in the chargino sector is assumed. The shaded (yellow) regions are ruled out by the LEP measurements. The observed baryon asymmetry is in these units $5 - 9$. (The figure is taken from the latter reference in [20]).

The total source is simply the sum of the two, $S_{si} = S^a_{si} + S^b_{si}$. To get a more quantitative understanding of these sources, in figure 3 we plot the integrals $J_a$ and $J_b$ in equations (25) and (26). A closer inspection of the sources $S^a_{si}$ and $S^b_{si}$ indicates that the total source $S_{si}$ can be also rewritten as the sum of two sources: the source $\propto |m_i|^2 \theta'^i$, characterized by $x_i^2 J_a + x_i^3 J_b$, and the source $\propto |m_i|^2 \theta''^i$, characterized by $x_i^3 J_a$. We note that, in the spin state quasiparticle basis the flow term sources appear in the continuity equation for the vector current, while in the helicity basis, which is usually used in literature [19, 20], the flow term sources appear in the Euler equation. In figure 4 we show recent results of baryogenesis calculations of Ref. [20] based on the CP-violating contribution to the semiclassical...
force in the chargino sector of the Minimal Supersymmetric Standard Model (MSSM). This
calculation is based on the quasiparticle helicity states picture, and it suggests that one
can dynamically obtain baryon production marginally consistent with the observed value,
\( n_B/s = 7.0 \pm 1.5 \times 10^{-11} \), provided \( m_2 \sim \mu \sim 150 \text{ GeV} \) and \( v_w \sim 0.03 \) (c = 1).

At this moment, the question of baryogenesis mediated via the charginos of the MSSM is
not completely resolved. Indeed, more recently the results, which we summarise in figure 5,
have been reported [28], where the relevant CP-violating sources were computed in the
flavour basis. Flavour mixing was, however, not taken account of, which is, on the wall,
formally of the order \( h^0 \), and hence cannot be neglected.

In conclusion we note that, even though we have recently witnessed important progress
modelling dynamical baryon production in supersymmetric models, some important ques-
tions remain unresolved which may have quantitative impact on the final results for baryon
production.

\[ \begin{align*}
\text{Figure 5: Baryon production mediated by charginos of the MSSM from Ref. [28], expressed as the multiple of the observed value, } \eta_{BBN} &= n_B/s \simeq 6 \times 10^{-11}. \text{ The figures show contours for the}\n\text{baryon-to-photon ratio in the units of } 10^{-10} \text{ as a function of } \mu \text{ and } m_2 \text{ for } \tan \beta = 10, (a) } m_2 = \mu \text{ and (b) } m_2 = 200 \text{ GeV. The wall velocity is taken to be } v_w = 0.01, \text{ and the maximum CP violation in the chargino sector is assumed.} \\
\end{align*} \]

We now turn to discussion of the collision term sources in Eqs. (21) and (23). We assume
that the self-energies \( \Sigma^{>,<} \) are approximated by the one-loop expressions (cf. figure 2)
\[ \Sigma^{<,>}(k, x) = iy^2 \int \frac{d^4k'}{(2\pi)^4} \left[ (2\pi)^4 \delta(k - k' + k'') P_L S^{<,>}(k', x) P_R \Delta^{<,>}(k'', x) + (2\pi)^4 \delta(k - k' - k'') P_R S^{<,>}(k', x) P_L \Delta^{<,>}(k'', x) \right], \tag{27} \]
where \( \Delta^{<} \) and \( \Delta^{>} \) denote the bosonic Wigner functions. This expression contains both
the CP-violating sources and relaxation towards equilibrium. The CP-violating sources can
be evaluated by approximating the Wigner functions $S^{>,<}$ and $\Delta^{>,<}$ by the equilibrium expressions accurate to first order in derivatives. The results of the investigation are as follows. There is no source contributing to the continuity equation, while the source arising in the Euler equation is of the form [25]

$$2 \int_{\pm} \frac{d^4k}{(2\pi)^4} \frac{k_z}{\omega_0} C_{\psi si} = v_w y^s \frac{s|m|^2\theta'}{32\pi^3T} I_f(|m|, m_\phi),$$

where the function $I_f(|m|, m_\phi)$ is plotted in figure 6. It is encouraging that the source vanishes for small values of the mass parameters, which suggests that the expansion in gradients we used here yields the dominant sources. Note that the source is nonvanishing only in the kinematically allowed region, $m_\phi \geq 2|m|$. When the masses are large, $|m|, m_\phi \gg T$, the source is, as expected, Boltzmann-suppressed. It would be of interest to make a comparison between the sources in the flow term and those in the collision term, and apply our methods to realistic models [27].

![Figure 6:](image)

Figure 6: The collisional source contributing to the fermionic kinetic equation at one loop for the mass ratios $m_\phi/|m| = 2.1, 2.5, 3, 4, 10$ and 20, respectively. The source peaks for $|m| \approx 0.7T$ and $m_\phi \approx 4|m|$.

References


