DYNAMICAL ELECTROWEAK SYMMETRY BREAKING

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Dynamical symmetry breaking provides a possible solution to the electroweak hierarchy problem. It requires new strong interactions that are effective at some high-energy scale. If there is no light Higgs boson, this scale is constrained to be in the TeV range, and signals of the new interactions can be observed, directly or indirectly, in collider experiments. Even if no observable states in the Higgs sector are kinematically accessible, a Linear Collider will cover the low-energy parameter space that arises in a systematic model-independent analysis of dynamical electroweak symmetry breaking.

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1. Introduction

1.1. Particle masses

The most striking property of the particle spectrum is clearly the huge range in the fundamental scales. On a logarithmic mass scale, the known elementary particles (except for the neutrinos) populate a small region centered almost 20 orders of magnitude below the Planck mass, the fundamental scale of space-time geometry:

\[ \text{\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\nu_{1,2,3}$ & $\psi$ & $\tilde{\psi}$ & $W$ & $B$ & $Z$ & $\nu$ & $d$ & $s$ & $b$ & $u$ & $c$ & $t$ \\
\hline
10^{-12} & 10^{-9} & 10^{-6} & 10^{-3} & 10^0 & 10^3 & 10^6 & 10^9 & 10^{12} & 10^{15} & 10^{18} & \text{GeV} \\
\hline
\end{tabular}} \]

Their interactions, as far as we know, respect the gauge symmetry principle. Gauge invariance of the electroweak interactions in particular forbids mass terms for all known elementary particles (except for right-handed neutrinos). In reality, all particles but the photon are massive, so the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry is softly broken. It is manifest in the interactions, but hidden in the mass spectrum.

The masses of vector bosons are best understood as mixing terms. The electroweak gauge bosons which form a $SU(2)_L$ triplet $W^{\pm,0}$ and a $SU(2)_L$ singlet $B$, are coupled to a multiplet of scalar states in a different representation of the electroweak symmetry group. Diagonalizing the mass matrix, the resulting eigenstates consist of the massless photon and the three massive vector bosons $W^{\pm}$ and $Z$. Their longitudinal components can be identified with three components of the scalar multiplet. Similarly, the (Dirac) masses and mixings of the fermions originate from bilinear couplings of left-handed and right-handed two-component (Weyl) fermions, which are otherwise unrelated and belong to different $SU(2)_L \times U(1)_Y$ representations.

The three scalar states (i.e., the longitudinal vector bosons) in the observed spectrum do not make up a complete linear representation of $SU(2)_L$. Therefore, a field theory based just on the degrees of freedom that have been established experimentally is non-renormalizable. We can make use of such an effective theory to get a consistent low-energy expansion of the relevant physics\(^2\). This formalism will be described below in Sec. 2.
However, at high energies a non-renormalizable theory has an inherent cut-off scale where it ceases to be predictive. For instance, the scalars may turn out as composite at the cutoff scale, or there may be additional scalars which have not yet been observed. In any case, the pattern of mass generation via mixing suggests a new interaction, and the complexity of the flavor couplings indicates a similar complexity of this new Higgs sector, of which we might just be scratching the surface. In the Standard Model and its extensions the problem is solved by brute force, postulating the existence of scalar multiplets with just the appropriate quantum numbers and couplings, but this need not be the solution chosen by Nature.

Since the actual properties of the Higgs sector are unknown, in a first approach one may identify a parameter $v$ as the characteristic scale of electroweak symmetry breaking (EWSB), by convention taken as

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \approx 246 \text{ GeV}. \quad (1)$$

$v$ is an abstract quantity at this stage, and while its value is fixed by the measurement of a low-energy process (muon decay), its exact meaning depends on the underlying dynamics which is not yet accessible directly.

1.2. Exponentials

In searching for possible sources of the terms that softly break the electroweak symmetry in the fundamental Lagrangian, one may take either one of two different views:

1. The electroweak gauge symmetry is an accidental approximate symmetry.
2. The electroweak gauge symmetry is exact in the dynamics, but spontaneously broken in the low-energy states.

The first explanation is generally rejected since it implies that the smallness of the breaking terms ($v/M_{\text{Planck}} \sim 10^{-17}$) is pure coincidence. For a natural explanation of this ratio, we not only need to adopt the second scenario, but we also are led to postulate dynamical symmetry breaking. Such an extremely small number should be the result of solving fundamental dynamical equations without particularly small parameters. Ultimately, we are looking for a fundamental theory without adjustable parameters at all.

Fortunately, we know at least one solution to this problem. Strongly-coupled quantum field theories generate large scale ratios from the quantum
effect of renormalization group running. They allow the interpretation of the scale hierarchy as an exponential factor

\[ M_{\text{Planck}}/v = e^{(\text{number of order 10})} \]  

within the context of quantum field theory.\(^a\)

QCD-type condensation is the one example where this actually happens in particle physics. Neglecting all other interactions for simplicity, we could solve the renormalization group equation for the running strong coupling constant \( g_s(\mu) \) with a “natural” initial condition at the Planck scale, say \( g_s(M_{\text{Planck}}) = 1/2 \). Identifying the QCD scale \( \Lambda_{\text{QCD}} \) with the scale where \( g_s(\mu) \) becomes strong at low energies, we obtain a scale ratio \( M_{\text{Planck}}/\Lambda_{\text{QCD}} \) similar to (2). The exponent is proportional to the inverse of the initial value of \( \alpha_s = g_s^2/4\pi \) at the high scale. In the QCD case, the low-energy singularity in the perturbative evolution of the coupling is resolved by the condensation of gluons and of left-handed and right-handed fermions, breaking the chiral symmetry at the scale \( \Lambda_{\text{QCD}} \lesssim 1 \text{ GeV} \). Incidentally, this mechanism would also trigger EWSB in the GeV range, if the electroweak symmetry was not already broken at the higher scale \( v = 246 \text{ GeV} \).

In fact, dynamical symmetry breaking is the explanation for a wide variety of physical phenomena, from superconductivity to the laser effect. The overall description is simple. Out of the fundamental fields of the model one constructs scalar field multiplets \( \Phi \) with nonvanishing quantum numbers under the symmetry in question. If the effective potential for such a field has a nontrivial solution which is energetically favorable, it will get a vacuum expectation value, breaking the symmetry and allowing for new effective couplings which exact symmetry would forbid. There is no need for \( \Phi \) to correspond to independent observable degrees of freedom, since it can be a composite of other fields present in the theory (such as Cooper pairs in BCS superconductivity). Thus, while fundamental Higgs particles as \( \Phi \) quanta are not excluded, they are not necessary for the Higgs mechanism of EWSB to work.

1.3. Higgs or no Higgs?

Just as in QCD, one can associate a scale \( \Lambda \) with dynamical symmetry breaking, the compositeness scale. \( \Lambda \) might be significantly higher than the EWSB scale \( v \). Then, in the intermediate scale range \( v \ldots \Lambda \) the model will

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\(^a\)Alternatively, such an exponential could be a consequence of the space-time structure. This possibility is discussed in Ref. 7.
reduce to a renormalizable effective theory. This can only be the case if the scalars which provide the longitudinal $W$ and $Z$ states are accompanied by extra scalar states such that a complete linear representation of $SU(2)_L \times U(1)_Y$ is formed. The extra states can be observed as particles, the Higgs bosons (cf. Ref. 8).

As will be discussed below, in models without an observable Higgs state the compositeness scale cannot be higher than\(^9\)

$$\Lambda \lesssim 4\pi v \approx 3 \, \text{TeV}. \quad (3)$$

If there is a single Higgs boson with mass $m_H$, the compositeness scale is constrained instead by

$$\Lambda' \lesssim m_H \exp \frac{\Lambda^2}{12 m_H^2} \quad \text{with} \quad \Lambda = 4\pi v, \quad (4)$$

the location of the Landau pole of the Higgs self-coupling. The relation (4) implies that the concept of a Higgs boson makes sense only if $m_H \lesssim 1 \, \text{TeV}$, such that $\Lambda' > m_H^{10}$.

The ratio $\Lambda'/m_H$ could be large, but dynamical symmetry breaking by itself provides no obvious mechanism for this. One would rather expect this ratio to be of order one, so the window for composite Higgs states is narrow. On the other hand, scalar states could actually be elementary degrees of freedom. This is consistent with a dynamical solution of the hierarchy problem only if their masses are protected by a symmetry. In that case, the scalar interactions which trigger EWSB could only be generated indirectly, which shifts dynamical symmetry breaking to a new (hidden) sector of the theory. This is the way supersymmetric models are constructed, which are considered in Ref. 11.

Another possibility which opens some window for composite Higgs bosons is the identification of the Higgs multiplet with pseudo-Goldstone scalars generated by spontaneous symmetry breaking at a higher scale\(^\text{12}\). Recently, realistic models with this structure have been proposed, the so-called *Little Higgs* models\(^\text{13}\). Like supersymmetric models, they leave room for the electroweak scale being generated dynamically through strong interactions, but in these scenarios the effective theory is weakly interacting up to energies significantly beyond the TeV scale.

By contrast, in the absence of light Higgs multiplets no weakly interacting effective theory can be constructed that is valid up to that energy range. Instead, the strong interactions which accompany dynamical scale generation may be directly coupled to Standard Model particles and show
up in scattering processes once the available energies are sufficiently close to the new compositeness scale. Therefore, one can not just expect to observe the effects associated with EWSB at colliders (e.g., Higgs particles, super-partners, pseudo-Goldstone bosons), but there is hope to get a handle on those (strongly-interacting) new degrees of freedom which are responsible for EWSB.

1.4. Models of dynamical symmetry breaking

Within the context of four-dimensional field theory, dynamical symmetry breaking must be assigned to new gauge interactions. This is a common feature of all dynamical models of EWSB. The models differ in the role of fermions:

(1) Since, as in QCD, fermions which feel strong gauge interactions are confined, it is likely that the fermions (technifermions) directly associated with EWSB are unobservable at low energies. This is the original technicolor (TC) idea. Technifermion condensation is able to account for effective scalar states which provide the longitudinal components of $W$ and $Z$ bosons.

(2) Pure TC will not generate any left-right couplings for the observable fermions, just as QCD generates constituent masses for quarks, but not for leptons. However, such couplings could be due to additional dynamically broken gauge interactions at even higher energies which are felt both by ordinary and by technifermions. This mechanism is known as extended technicolor (ETC). Below the ETC scale, such interactions lead to four-fermion couplings. When technifermions condense at the compositeness scale, the desired bilinear couplings are generated.

(3) Exchanging the roles of TC and ETC, a dynamically broken gauge interaction might trigger EWSB by fermion condensation not too far above the electroweak scale. In that case, the affected fermions need not be confined. The heavy top quark is the prime candidate (topcolor) for such a strongly-interacting object. Since for this mechanism to actually work the top quark is somewhat too light, more recent models that follow this pattern implement a combined topcolor-assisted technicolor scheme. Other models of this type involve the condensation of neutrinos, which might have large Yukawa couplings despite their tiny physical masses and thus could feel new strong interactions.

(4) Alternatively, the physical top mass could be suppressed by a two-state mixing effect (top see-saw). In such models, the electroweak scale $v$ is
typically suppressed compared to the compositeness scale $\Lambda$, and there is room for composite Higgs bosons in the intermediate range.

The complicated pattern of flavor physics stands as the main obstacle for constructing a simple theory of dynamical symmetry breaking. It is difficult to simultaneously accommodate (i) very light leptons and quarks, (ii) a heavy top quark, (iii) the smallness of flavor-changing neutral currents\textsuperscript{14}. Turning the argument around, the fact that fermion mass generation cannot be separated from electroweak physics in strongly-interacting models, opens the possibility that some of the puzzles of flavor physics are resolved at future collider experiments.

2. Effective Theories of Electroweak Interactions

2.1. The bottom-up approach

While it is clearly worthwhile to search for signals of specific models in collider experiments, the limited energy range of a next-generation Linear Collider may not allow to access new states associated with the Higgs sector directly. In such a situation, a model-independent treatment of the dynamics is more appropriate. Fortunately, the formalism of effective (or phenomenological) Lagrangians\textsuperscript{2} provides a generic framework for the bottom-up description of electroweak interactions in the absence of a complete renormalizable field theory.

At energies much below the $W$ and $Z$ (and Higgs) masses, the dominant interactions of leptons and quarks are QED and QCD interactions, governed by an effective Lagrangian of operator dimension four and less:

$$\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_4$$

$$= - (Q_L M_Q Q_R + L_L M_L L_R + h.c.) - (\bar{N}_L^c M_{N_L} N_L + \bar{N}_R^c M_{N_R} N_R) - \frac{1}{4} A_{\mu\nu} A^{\mu\nu}$$

For simplicity, we ignore QCD interactions here. The building blocks consist, first of all, of left- and right-handed quark and lepton fields,

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} U_R \\ D_R \end{pmatrix}, \quad L_L = \begin{pmatrix} N_L \\ E_L \end{pmatrix}, \quad L_R = \begin{pmatrix} N_R \\ E_R \end{pmatrix}. \quad (6)$$

We omit generation and color indices, so all coupling constants should be understood as matrices. While gauge couplings are diagonal by definition, the bilinear quark and lepton couplings $M_Q$ and $M_L$ and the Majorana
mass matrices $M_{N_L}$ and $M_{N_R}$ are not.\textsuperscript{b} Upon diagonalization, from this structure one obtains the fermion masses together with the $3 \times 3$ mixing matrices which determine the weak interactions of quarks and neutrinos.

QED gauge interaction are present in the dimension-four terms which contain the electromagnetic field strength and covariant derivative

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu = \partial_\mu + ie q A_\mu,$$

where $e$ is the positron charge and $q$ the multiplying factor for the charge of a given fermion species. In the doublet notation used here, the normalized charge $q$ is a diagonal $2 \times 2$ matrix which reads

$$q_{Q,L} = \frac{1}{2} \left(y_{Q,L} + \tau_3^3\right) \quad \text{with} \quad y_Q = \frac{1}{3} \text{ and } y_L = -1. \quad (8)$$

for the quarks and leptons, respectively.

At very low energies, the full electroweak symmetry is present only in the higher-dimensional operators which induce weak interactions. While magnetic-moment type operators (dimension five)

$$\mathcal{L}_5 = \bar{Q} R \mu Q \sigma_{\mu\nu} A^{\mu\nu} Q_L + \bar{L} R \mu L \sigma_{\mu\nu} A^{\mu\nu} L_L + \text{h.c.} \quad (9)$$

are strongly suppressed, four-fermion operators (dimension six)

$$\mathcal{L}_6 = \sum_{f=Q_L, Q_R, L_L, L_R} s_{ijkl} (\bar{f}_i f_j) (\bar{f}_k f_l) + v_{ijkl} (\bar{f}_i \gamma_\mu f_j) (\bar{f}_k \gamma_\mu f_l) \quad (10)$$

are more significant. Experiment has shown that their structure is consistent with the specific factorizable form of the Fermi model,

$$\mathcal{L}_6 = -4 \sqrt{2} G_F \left(2 J^{+} J^{-} + c_w^2 J^0 J^0 \right) \quad (11)$$

with the charged and neutral currents

$$J^{\pm}_{\mu} = \frac{1}{\sqrt{2}} \left[ Q_L \gamma_\mu L_L + \bar{L}_L \gamma^\pm \gamma_\mu L_L \right] \quad (12)$$

$$J^0_\mu = \frac{1}{c_w} \left[ Q_L \left(-q_Q s_w^2 + \frac{3}{2}\right) \gamma_\mu Q_L + \bar{Q}_R (-q_Q s_w^2) \gamma_\mu Q_R + \bar{L}_L \left(-q_L s_w^2 + \frac{3}{2}\right) \gamma_\mu L_L + \bar{L}_R (-q_L s_w^2) \gamma_\mu L_R \right] \quad (13)$$

Here, $s_w \approx 0.48$ is the sine of the weak mixing angle, and $c_w = \sqrt{1 - s_w^2}$.

We do not know with certainty that the factorizable structure of four-fermion interactions is exact, but flavor-physics experiments have not yet

\textsuperscript{b}Electroweak symmetry requires $M_{N_L}$ to vanish, but $M_{N_R}$ is re-introduced in the low-energy effective theory if $M_{N_R}$ is very large and the right-handed neutrinos are integrated out.
revealed any deviations from this picture. Assuming that deviations are absent, the Fermi Lagrangian can be rewritten with the help of the vector fields $W^\pm$ and $Z$,

$$\mathcal{L}_6 = -g_W(W^+ \mu J^+_\mu + W^- \mu J^-_\mu) - g_Z(Z^\mu J^0_\mu) + \mathcal{L}_{2(W)}$$  \hspace{1cm} (14)$$

where

$$\mathcal{L}_{2(W)} = M_W^2 W^\pm \mu W^\mp_\mu + \frac{1}{2} M_Z^2 Z^\mu Z_\mu,$$  \hspace{1cm} (15)$$
such that (14) becomes equivalent to (11) when the $W$ and $Z$ fields are integrated out. After rearranging the basis,

$$W^+ = \frac{1}{\sqrt{2}}(W^1 - iW^2), \quad Z = c_w W^3 - s_w B,$$  \hspace{1cm} (16)$$

$$W^- = \frac{1}{\sqrt{2}}(W^1 + iW^2), \quad A = s_w W^3 + c_w B,$$  \hspace{1cm} (17)$$

the Fermi Lagrangian assumes the form

$$\mathcal{L} = \bar{Q}_L i D_L Q_L + \bar{Q}_R i D_R Q_R + \bar{L}_L i \bar{\Psi}_L L_L + \bar{L}_R i \bar{\Psi}_R L_R$$

$$- \frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \mathcal{L}_{2(W)} + \mathcal{L}_3$$  \hspace{1cm} (18)$$

with a local $SU(2)_L \times U(1)_Y$ symmetry manifest in the terms of the first line. With respect to this symmetry, the vector fields have the usual gauge transformation properties, and the fermions couple via covariant derivatives

$$D_{L\mu} = \partial_\mu - ig'(q + \frac{e^3}{2})B_\mu + ig^2 B^a W^a_\mu$$  \hspace{1cm} (19)$$

$$D_{R\mu} = \partial_\mu - ig' qB_\mu.$$  \hspace{1cm} (20)$$

To describe physics at and above the mass scale of the electroweak vector bosons $W^\pm$ and $Z$, we have to include kinetic terms

$$\mathcal{L}_{4(W)} = -\frac{1}{2} \text{tr} [W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} \text{tr} [B_{\mu\nu} B^{\mu\nu}],$$  \hspace{1cm} (21)$$

where the field strength tensors are defined in terms of vector fields $W^a$ ($a = 1, 2, 3$) and $B$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu]$$  \hspace{1cm} (22)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$  \hspace{1cm} (23)$$

with $W_\mu = W^a \frac{x^a}{2}$ and $B_\mu = B_\mu \frac{e}{2}$. (This is not the most general form allowed by the spontaneously broken gauge invariance, but we postpone the discussion of anomalous couplings to Sec. 2.2.)

While the dimension-four part of the effective Lagrangian exhibits full electroweak gauge symmetry, this symmetry is not manifestly present in the dimension-two and dimension-three operators, the fermion and gauge
Dynamical electroweak symmetry breaking

However, by introducing an extra field $\Sigma$ with a suitable transformation law, this problem can formally be solved without losing the universality of the effective-theory formalism. This field parameterizes our ignorance about the true nature of the Higgs sector. While the actual dynamics at high energies could be very complicated, at low energies a generic description is dictated by symmetry considerations only.

The field $\Sigma(x)$ is a $2 \times 2$ matrix which is defined to have the appropriate behavior under gauge transformations $U(x) \in SU(2)_L$ and $V(x) = \exp(i\beta \tau^3) \in U(1)_Y$:

$$
\Sigma(x) \rightarrow U(x) \Sigma(x) V^\dagger(x).
$$

Spontaneous symmetry breaking is implemented by a nonzero vacuum expectation value

$$
\langle \frac{1}{2} \text{tr} \left[ \Sigma^\dagger(x) \Sigma(x) \right] \rangle = 1
$$

which for practical purposes can be replaced by

$$
\langle \Sigma(x) \rangle \rightarrow 1 \quad \text{for} \quad x \rightarrow \infty.
$$

For perturbative calculations, it is often useful to adopt the unitary or unitarity gauge where $\Sigma(x) \equiv 1$, but when discussing the electroweak symmetry structure, we should not impose this restriction.

A unitary matrix has three degrees of freedom, therefore the minimal number of degrees of freedom parameterizing $\Sigma$ is three. A possible, but not unique, parameterization is given by

$$
\Sigma(x) = \exp \left( -\frac{i}{v} w(x) \right) \quad \text{with} \quad w(x) = w^a(x) \tau^a; \quad a = 1, 2, 3.
$$

where $v$ is the electroweak scale (1). We do not make any attempt to further constrain the dynamics associated with $\Sigma$. This question has to be solved experimentally by measuring the free parameters, looking for new states related to $\Sigma$, and comparing this to any theoretical predictions in specific models.

The fermion mass term in the effective Lagrangian is replaced by

$$
\mathcal{L}_3 = - (\bar{Q}_L \Sigma M Q_R + \bar{L}_L \Sigma M_L L_R + \text{h.c.}) - \bar{L}_R^c M_{N_R} \frac{1+\tau^3}{2} L_R
$$

which has the required $SU(2)_L \times U(1)_Y$ symmetry.

The boson mass term is replaced by a kinetic-energy term for the $\Sigma$ field. We introduce further abbreviations

$$
V_\mu = \Sigma (D_\mu \Sigma)^\dagger \quad \text{and} \quad T = \Sigma \tau^3 \Sigma^\dagger
$$

where

$$
D_\mu \Sigma = \partial_\mu \Sigma + ig W_\mu \Sigma - ig' \Sigma B_\mu
$$
to write this as

$$\mathcal{L}_{2(W)} = -\frac{\alpha^2}{4} \text{tr} [V_\mu V^\mu]$$  \hspace{1cm} (31)$$

In unitary gauge, the vector field $V_\mu$ corresponds to a particular combination of the $W$ and $B$ fields, namely

$$V_\mu = -ig W_\mu \Sigma + ig' \Sigma B_\mu$$  \hspace{1cm} (32)$$

which defines the physical massive vector bosons, $W^{\pm}_\mu$ and $Z_\mu$.

The dimension-four part of the Lagrangian does not involve the $\Sigma$ field

$$\mathcal{L}_4 = \bar{Q}_L i \not{D} Q_L + \bar{Q}_R i \not{D} Q_R + \bar{L}_L i \not{D} L_L + \bar{L}_R i \not{D} L_R - \frac{1}{2} \text{tr} [W_{\mu \nu} W^{\mu \nu}] - \frac{1}{2} \text{tr} [B_{\mu \nu} B^{\mu \nu}],$$

(33)

Allowing for $\Sigma$ self-interactions $\mathcal{L}_\Sigma$, the complete effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{2(W)} + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_\Sigma$$  \hspace{1cm} (34)$$

is invariant under the full electroweak symmetry group. In analogy with low-energy QCD, this effective theory is called the chiral Lagrangian of electroweak interactions. Its validity is not restricted to the particular scenario of dynamical symmetry breaking it is usually associated with. The minimal SM with a Higgs boson is just a special case of (34), where the field $\Sigma$ is given a definite linear representation.

### 2.2. Anomalous couplings

The guideline for constructing the effective Lagrangian (34) has been to start with the Fermi model Lagrangian and add the minimal set of fields that make the weak-interaction symmetries manifest. This requires the addition of kinetic terms for the new fields and the inclusion of $\Sigma$ factors in the boson and fermion mass terms. However, if one does one-loop calculations with (34), proper gauge-fixing and ghost terms taken into account, one will observe that additional operators are needed to make the theory finite at next-to-leading order. This is natural since the Lagrangian (34) does not yet contain all possible operators of dimension four or less consistent with electroweak symmetry.

At dimension two, there is an additional operator not yet considered, namely

$$\mathcal{L}'_{2(W)} = -\beta^2 \frac{g^2}{4} \text{tr} [TV_\mu] \text{tr} [TV^\mu].$$

(35)
Imposing CP-invariance on the effective Lagrangian\(^c\), the complete list of dimension-four operators not contained in (34) reads\(^{22}\)

\[
L_1 = \alpha_1 g' \text{tr} \left[ \Sigma B_{\mu\nu} \Sigma^\dagger W^{\mu\nu} \right]
\]

\[
L_2 = i\alpha_2 g' \text{tr} \left[ \Sigma B_{\mu\nu} \Sigma^\dagger [V^\mu, V^\nu] \right]
\]

\[
L_3 = i\alpha_3 g \text{tr} \left[ W_{\mu\nu} [V^\mu, V^\nu] \right]
\]

\[
L_4 = \alpha_4 \text{tr} [V_\mu V_\nu]^2
\]

\[
L_5 = \alpha_5 \text{tr} [V_\mu V^\mu]^2
\]

\[
L_6 = \alpha_6 \text{tr} [V_\mu V_\nu] \text{tr} [T V^\mu] \text{tr} [T V^\nu]
\]

\[
L_7 = \alpha_7 \text{tr} [V_\mu V^\mu] \text{tr} [TV_\nu] \text{tr} [TV^\nu]
\]

\[
L_8 = \frac{1}{4} \alpha_8 g^2 (\text{tr} [TW_{\mu\nu}])^2
\]

\[
L_9 = \frac{1}{2} \alpha_9 g \text{tr} [TW_{\mu\nu}] \text{tr} [TV^\mu, V^\nu]
\]

\[
L_{10} = \frac{1}{2} \alpha_{10} \text{tr} [TV_\mu] \text{tr} [TV_\nu]^2
\]

\[
L_{11} = \alpha_{11} g \epsilon_{\lambda\mu\nu\rho} \text{tr} [TV_\mu] \text{tr} [V_\rho W_{\nu\lambda}]
\]

In the general case of a nonlinear symmetry representation the Lagrangian contains terms of arbitrarily high dimension. Therefore, this list is not sufficient to make the theory finite to all orders. In each order of perturbation theory new terms are introduced with the dimension of the \(\Sigma\)-dependent terms increased by two.

This fact does not make the effective-Lagrangian approach useless. It merely implies that at each order of the perturbative expansion one should be prepared for new contributions which are generically of the order \(1/16\pi^2\) (since they are induced as loop corrections) with the operator dimension increased by two\(^9\). The two extra powers of fields or derivatives are compensated by two powers of \(1/v\), the expansion parameter of \(\Sigma\) in (27). As long as the energy is small enough, one can truncate the perturbative series to obtain an approximation of the true amplitude. In matrix elements, the loop expansion therefore becomes a low-energy expansion in terms of

\[
\frac{E^2}{(4\pi v)^2} = \frac{E^2}{\Lambda^2},
\]

where \(E\) is any linear combination of energies, masses and momenta assigned to the external particles. This sets the scale where perturbation theory breaks down in the absence of Higgs-like states:

\[
\Lambda = 4\pi v \approx 3 \text{ TeV}.
\]

\(^c\)A discussion of CP violation is beyond the scope of this review.
Of course, there may be larger contributions of the operators (36–46) than predicted by the loop expansion. Since the effect of anomalous couplings increases with energy, generically this leads to a lower cutoff scale $\Lambda' < \Lambda$.

### 2.3. Custodial symmetry

As we have seen in the previous section, there are free parameters which arise when massive vector bosons are introduced to regularize the Fermi model. One of those affects the $M_W/M_Z$ ratio which is usually referred to as the $\rho$ parameter:

$$\frac{M_W^2}{M_Z^2 c_w^2} = \rho \quad \text{where} \quad \rho = \frac{1}{1 + \beta'}.$$  

(49)

Experimentally, $\rho$ is approximately equal to unity. Hence, the coefficient $\beta'$ of the operator (35) in the chiral Lagrangian vanishes to leading order in perturbation theory.

This can be attributed to an approximate symmetry\textsuperscript{24,2}. If we take the global symmetry of the Lagrangian not to be $SU(2)_L \times U(1)_Y$ but enlarge it to $SU(2)_L \times SU(2)_R$ (where $U(1)_Y \subset SU(2)_R$) and impose the transformation law on $\Sigma$

$$\Sigma \rightarrow U \Sigma V^\dagger$$

(50)

with $U \in SU(2)_L$ and $V \in SU(2)_R$, then the term (35) is forbidden. Simultaneously, this symmetry forbids all operators in the chiral Lagrangian which contain a $T$ factor in the trace, namely $L_6$ to $L_{11}$ (41–46).

It is natural to take the right-handed fermions to be doublets under $SU(2)_R$, just as the left-handed fermions are doublets under $SU(2)_L$. However, for the fermions this is not a good symmetry. It is violated by the up-down mass differences (and by right-handed neutrino Majorana masses).

The hypercharge part of weak interactions also violates this symmetry. The $B$ vector field, in our notation, couples to $\Sigma$ by a $\tau^3$ matrix factor, which is not consistent with a right-handed $SU(2)_R$ symmetry. Nevertheless, this breaking is proportional to $s_w^2$ which is not a large parameter, and the fermion couplings affect vector and scalar interactions at the loop level only. Looking at bosonic interactions, $SU(2)_R$ invariance is a reasonable approximation.

Spontaneous symmetry breaking in this sector then takes the form

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$$

(51)
where $SU(2)_C$ is the diagonal subgroup, the custodial $SU(2)$ symmetry of electroweak interactions. Bosonic states should come as $SU(2)_C$ multiplets, and in fact, the $Z$ and $W^{\pm}$ masses are degenerate up to the factor $1/c_w^2 \approx 1 + s_w^2$. Similarly, one expects new particles associated with EWSB also to be organized as $SU(2)_C$ multiplets.$^d$

3. Goldstone boson scattering

In the chiral-Lagrangian approach described in this chapter, electroweak symmetry breaking and fermion mass generation are mediated by a matrix-valued field $\Sigma$ which parameterizes the Higgs sector dynamics. The degrees of freedom that make up $\Sigma$ are a probe for the mechanism of electroweak symmetry breaking. A complete theory would probably replace $\Sigma$ by a multitude of new (elementary or composite) fields, but in any case the scalars $w^a$ introduced in the minimal parameterization (27) must be present to serve as longitudinal $W$ and $Z$ bosons. This is the triplet of Goldstone bosons associated with spontaneous electroweak symmetry breaking.$^{25}$ Thus, the only experiment which guarantees information about electroweak symmetry breaking is a measurement of Goldstone boson interactions.

In the high-energy limit ($s, t, u$ all going to infinity) corresponding scattering amplitudes of $W_L^{\pm}, Z_L$ on the one hand and $w^{\pm}, w^0$ on the other hand become equal. This fact is known as the Equivalence Theorem.$^{26}$ We can make use of it by considering scattering amplitudes of Goldstone bosons in place of the electroweak gauge bosons that are observed in the detector (i.e., their decay products). Doing this, we should always keep in mind that for a quantitative analysis one has to compute the full electroweak amplitudes, since realistic collider energies are far from the asymptotic region.

3.1. Quasielastic scattering at leading order

The lowest-order effective Lagrangian (34) provides a unique prediction for the quasielastic $2 \rightarrow 2$ scattering amplitudes of $W$ and $Z$ bosons. In the absence of $SU(2)_C$ violation, projecting onto longitudinal states and taking

---

$^d$This argument is independent of the origin of EWSB; in Higgs models, the spectrum also tends to follow this pattern.
the high-energy limit one obtains the Low-Energy Theorem (LET)\(^{27}\)

\[
A(W_L^- W_L^- \rightarrow W_L^- W_L^-) = -s/v^2
\]
\[
A(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = -u/v^2
\]
\[
A(W_L^+ W_L^- \rightarrow Z_L Z_L) = s/v^2
\]
\[
A(Z_L Z_L \rightarrow Z_L Z_L) = 0
\]

The cross sections for on-shell scattering are calculated by squaring the amplitudes, inserting phase space factors and dividing by a symmetry factor of two for like-sign W and for ZZ final states. This symmetry factor will not be included in the amplitude in any of the relations given here.

To be precise, the LET predicts the numerical coefficient of the lowest-order term of an expansion of the scattering amplitudes in terms of \(E/v\), terms of order \(M_W/E\) neglected. Since \(M_W = g v/2\), this is in fact the limit \(g \rightarrow 0\) with \(v\) fixed. As an approximation to the exact amplitude, the LET is useful for energies larger than \(M_W\) and below the scale where either new states appear or partial-wave unitarity is saturated otherwise (see Sec. 3.4).

### 3.2. Custodial symmetry relations

The Goldstone bosons transform under \(SU(2)_C\) transformations as a triplet. Therefore, if this symmetry is exact, all quasielastic scattering amplitudes are expressible in terms of a single function \(A(s, t, u)\):

\[
A(W_L^- W_L^- \rightarrow W_L^- W_L^-) = A(t, s, u) + A(u, t, s)
\]
\[
A(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = A(s, t, u) + A(t, s, u)
\]
\[
A(W_L^+ W_L^- \rightarrow Z_L Z_L) = A(s, t, u)
\]
\[
A(Z_L Z_L \rightarrow Z_L Z_L) = A(s, t, u) + A(t, s, u) + A(u, t, s)
\]

The function \(A(s, t, u)\) satisfies

\[
A(s, u, t) = A(s, t, u).
\]

As we have seen, its Taylor expansion begins with

\[
A(s, t, u) = s/v^2.
\]

Note that these relations are strongly violated in forward scattering where photon exchange is important. There, elastic WW scattering becomes singular while WW \(\rightarrow\) ZZ stays finite. This is outside the validity region of the Equivalence Theorem.
3.3. **Next-to-leading order contributions**

Even without additional knowledge about the high-energy behavior of the theory, the next-to-leading order corrections to Goldstone scattering can be computed. Only the logarithmic terms are scheme-independent and thus physically meaningful:

\[
\text{Re } A(s, t, u) = \frac{s}{v^2} + \frac{1}{16\pi^2 v^4} \left\{ \frac{(t-u)}{6} \left[ t \ln \frac{-t}{\mu^2} - u \ln \frac{-u}{\mu^2} \right] - \frac{s^2}{2} \ln \frac{s}{\mu^2} \right\} \\
+ \alpha_4^0 \frac{4(t^2 + u^2)}{v^4} + \alpha_5^0 \frac{8s^2}{v^4}.
\]

(63)

The result depends on a renormalization scale \( \mu \). This dependence can be absorbed in a redefinition of the coefficients of the operators \( \mathcal{L}_4 \) and \( \mathcal{L}_5 \):

\[
\alpha_4(\mu) = \alpha_4^0 - \frac{1}{12} \frac{1}{16\pi^2} \ln \frac{\mu^2}{\mu_0^2}, \quad \alpha_5(\mu) = \alpha_5^0 - \frac{1}{24} \frac{1}{16\pi^2} \ln \frac{\mu^2}{\mu_0^2}
\]

(64)

Finite corrections depend on the calculational scheme, i.e., on the UV completion of the theory. They are contained in the constant coefficients \( \alpha_{4,5}^0 \), which therefore represent the relevant information. The same applies to the \( SU(2)_C \)-violating couplings \( \alpha_{5,6,10} \) which are scale-independent to this order, if the coefficient \( \beta' \) (35) is indeed zero (or \( \rho = 1 \)), as suggested by data.

3.4. **Unitarity constraints**

The optical theorem states that the total cross section for any process is equal to the imaginary part of the elastic forward scattering amplitude. If there is only elastic \( 2 \rightarrow 2 \) scattering, this can be translated into a relation for the scattering amplitude \( A(s, t, u) \). Expanding it in partial waves

\[
A(s, t, u) = 32\pi \sum \ell a_\ell(s) (2\ell + 1) P_\ell(1 + 2t/s),
\]

(65)

each partial-wave amplitude \( a_\ell \) has to satisfy

\[
|a_\ell(s) - i/2| = 1/2,
\]

(66)

i.e., as a curve in the complex plane parameterized by \( s \) it has to stay on the Argand circle, a circle with radius \( \frac{1}{2} \) around the point \( \frac{i}{2} \). In particular, the real part of the partial-wave amplitude can never exceed 1/2.
In terms of the amplitude function $A(s, t, u)$, the $SU(2)_C$ eigenamplitudes are given by

$$A(I = 0) = 3A(s, t, u) + A(t, s, u) + A(u, t, s), \quad (67)$$

$$A(I = 1) = A(t, s, u) - A(u, t, s), \quad (68)$$

$$A(I = 2) = A(t, s, u) + A(u, t, s). \quad (69)$$

Inserting the LET expressions (52–55) and expanding in terms of partial-wave amplitudes, one obtains the nonvanishing terms

$$a^I_J = 0 = \frac{s}{16\pi v^2}, \quad a^I_J = 1 = \frac{s}{96\pi v^2}, \quad a^I_J = 2 = -\frac{s}{32\pi v^2}. \quad (70)$$

There is no higher spin involved if we remain with the LET amplitudes. The critical value $a = 1/2$ is reached at the energies

$$I = 0 : \quad E = \sqrt{8\pi} v = 1.2 \text{ TeV}, \quad (71)$$

$$I = 1 : \quad E = \sqrt{48\pi} v = 3.5 \text{ TeV}, \quad (72)$$

$$I = 2 : \quad E = \sqrt{16\pi} v = 1.7 \text{ TeV}. \quad (73)$$

At these energies, perturbation theory ceases to be predictive. In order to have amplitude expressions that are at least in accord with unitarity beyond these scales, one can try to resum the perturbation series in a particular way. The result depends on the chosen resummation prescription and does not tell anything about the actual high-energy behavior. However, it can serve as a consistent implementation of particular models with distinct features at energies beyond the unitarity saturation threshold.

The idea of such unitarization models is to project each eigenamplitude function $a_\ell(s)$ onto the Argand circle. Doing this, one assumes implicitly that no new scattering channels are open, so that $2 \to 2$ quasielastic scattering dominates at all scales. Two particular models have become popular, representing extreme cases:

1. The $K$-matrix unitarization model is not limited to the perturbative expansion. Assuming that $a(s)$ is a real-valued amplitude function one starts with, the unitarized amplitude is given by

$$a_K(s) = a(s) \frac{1 + ia(s)}{1 + a(s)^2}. \quad (74)$$

Geometrically, the value $a_K(s)$ corresponds to the projection of the point $a(s)$ onto the Argand circle along the straight line connecting $z = a(s)$ with $z = i$ (Fig. 1). By construction, the $K$-matrix prescription will never generate a resonance if there is none within the function $a(s)$. 
In this respect it can be regarded as a minimal unitarization model. In particular, if the LET expression \( a(s) = s/v^2 \) is inserted, the amplitude function \( a(s) \) translates into

\[
a_K(s) = a_0 s \frac{i^2 + ia_0 s}{v^4 + a_0^2 s^2}. \tag{75}
\]

This unitarized amplitude will asymptotically approach the fixed point \( a_K(s) = i \), a resonance at infinity.

(2) To obtain the Padé unitarization model (also known as the inverse amplitude method), one separates the amplitude into two pieces. Usually, one takes the leading term \( a^{(0)}(s) \) and the real part of the next-to-leading order term \( a^{(1)}(s) \) in the chiral expansion which are proportional to \( s \) and to \( s^2 \), respectively. Then, the unitarized amplitude reads

\[
a_P(s) = \frac{a^{(0)}(s)^2}{a^{(0)}(s) - a^{(1)}(s) - ia^{(0)}(s)^2} \tag{76}
\]

If \( a^{(1)}(s) \) vanishes, this coincides with the \( K \)-matrix model. However, if \( a(s) \) has the form

\[
a(s) = a_0 \left( \frac{s}{v^2} + \frac{s^2}{v^4} \right) \tag{77}
\]

the Padé-unitarized amplitude is

\[
a_P(s) = \frac{-a_0 s/\alpha}{s - v^2/\alpha + ia_0 s/\alpha} \tag{78}
\]
This is a resonance with mass \( M = v/\sqrt{\alpha} \) and width \( \Gamma = a_0 M/\alpha \). In other words, adopting the Padé unitarization method is equivalent to the assumption that a \( 2 \to 2 \) scattering amplitude is entirely dominated by a single resonance.

Padé unitarization works remarkably well for pion-pion scattering in low-energy QCD, consistent with vector meson dominance. Unfortunately, this does not imply anything for the scattering of electroweak Goldstone bosons.

### 3.5. Resonances and new particles

A striking signature of new physics is a resonance in some scattering channel. If such resonances appear in Goldstone boson scattering, this would almost certainly give a clue in the search for the origin of electroweak symmetry breaking. While resonances are more likely to be observed at the LHC where the available energy for Goldstone boson scattering is somewhat higher than for a first-stage Linear Collider, the strongest resonance in a particular scattering channel will have a low-energy tail that contributes to the parameters in the low-energy expansion. Resonances could be elementary particles (such as a Higgs boson) or bound states of more fundamental objects yet to be discovered.

As discussed before, one expects states associated with EWSB to be grouped in multiplets of the \( SU(2)_C \) custodial symmetry. Given the \( SU(2)_C \) quantum numbers of Goldstone bosons, this leaves the following possibilities:

- **Scalar singlet** \( \sigma \):
  \[
  \mathcal{L}_\sigma = g_\sigma \sigma \frac{1}{\tau^4} \text{tr}[V_\mu V^\mu] 
  \]  
  (79)

- **Vector triplet** \( \rho^a_\mu \):
  \[
  \mathcal{L}_\rho = g_\rho \frac{\rho^a_\mu}{2} \text{tr}[\rho^a_\mu \tau^a V_\mu] 
  \]  
  (80)

- **Tensor singlet** \( \tau^{\mu\nu} \):
  \[
  \mathcal{L}_\tau = g_\tau \frac{\tau^{\mu\nu}}{2} \text{tr}[V_\mu V_\nu] 
  \]  
  (81)

\[
\vdots
\]

The amplitude functions corresponding to these couplings for the scalar and vector cases take the form

- \( A_{\sigma}(s, t, u) = g_\sigma^2 \frac{s^2}{\tau^4} \frac{1}{s - M^2} \),
  \[
  \]  
  (82)

- \( A_{\rho}(s, t, u) = g_\rho^2 \left( \frac{s - u}{t - M^2} + \frac{s - t}{u - M^2} + 3 \frac{s}{M^2} \right) \).
  \[
  \]  
  (83)

where the resonance width has to be inserted when a pole falls inside the physical region.
While scalar and tensor resonances are electrically neutral, a vector resonance multiplet $\rho$ has neutral and charged components analogous to the $W^\pm, Z$ triplet. The interaction resulting from evaluating (80) is antisymmetric, forbidding the coupling of the $\rho$ to identical particles: It can decay into $W^+W^-$ but not into $ZZ$.

If $SU(2)_C$ is violated, resonances transforming as scalar triplets ($a_0$), vector singlets ($\omega$) etc. are also accessible in Goldstone scattering, and the amplitude relations (56–60) between $Z$ and $W$ external states are lost.

A scalar resonance $\sigma$ with arbitrary coupling $g_\sigma$ is the generalization of a Higgs resonance. The width of such a state is given by

$$\Gamma_\sigma = \frac{3g_\sigma^2}{32\pi} M_\sigma.$$  

(84)

if Goldstone bosons are the only decay channels. In the SM, the coupling $g_\sigma$ itself is proportional to the mass, $g_\sigma = \sqrt{2} M/v$, and thus $\Gamma \propto M^3$. Such a state becomes very broad and loses its identity if $M \gtrsim 1$ TeV. Beyond the SM, the $\sigma$ mass and coupling need not be related, and narrow scalar resonances may be present.

A vector resonance triplet is a characteristic feature of QCD-like technicolor models\textsuperscript{15,34}. If there are no other decay channels than Goldstone bosons (i.e., longitudinal vector bosons), the resonance width is given by

$$\Gamma_\rho = \frac{9g_\rho^2}{48\pi} M_\rho.$$  

(85)

which is smaller than the width of a scalar with equal mass and coupling.

Vector resonances have the special property that they can mix with electroweak gauge bosons. This may be interpreted as a remnant of the electroweak interactions of their constituents. As a result, one expects a significant sensitivity to $\rho$ properties not just in Goldstone scattering, but also in the $e^+e^- \rightarrow W^+_L W^-_L$ scattering amplitude.

A special case are neutral pseudoscalar resonances, $\pi_T^0$ and $\eta_T^0$. While they do not couple to Goldstone pairs, they can have a coupling to pairs of transversal vector bosons which is induced by the triangle anomaly: The coupling strength is much smaller than for $\sigma$ and $\rho$ states, and one expects the coupling to $ZZ, W^+W^-, \gamma\gamma$ and even $gg$ to be similar in magnitude. This is to be contrasted with $\sigma$ resonances for which the (longitudinal) $ZZ$ and $WW$ couplings are dominant. However, this picture may be complicated by CP violation in the strong dynamics, which would induce $\eta - \sigma$ mixing. Clearly, if any resonance appears in vector boson scattering, it is
important to determine its spin and the polarization of vector bosons in the decay by angular correlation analysis.

Apart from Goldstone (vector boson) decays, all such states are likely to have a significant or even dominant fraction of heavy-quark decays: $t\bar{t}$, $t\bar{b}$, $b\bar{b}$. The channels $\tau^+\tau^-$ and $\tau^+\nu_\tau$ are also possible.

While resonant production of new states in Goldstone scattering is restricted by the symmetries of Goldstone pairs, any state associated with EWSB can in principle be pair-produced. Some particles can also be directly pair-produced in $e^+e^-$ annihilation, but Goldstone scattering gives access to additional members of new multiplets. The coupling strength extracted from the production cross section of such a particle is an independent piece of information.

In models of dynamical symmetry breaking, low-lying pseudo-Goldstone boson scalars are a common feature. Classifying them according to their $SU(2)_C$ properties, multiplets analogous to the low-energy QCD spectrum can be expected, among them $SU(2)_C$ triplets ($\pi$), doublets ($K$) and singlets ($\eta$). All can in principle be produced in Goldstone scattering, and the cross sections may be sizable. As dominant decay modes one expects longitudinally polarized vector bosons and heavy quarks, possibly accompanied by transversely polarized vector bosons (radiative decays).

Pseudo-Goldstone bosons (technipions) share quantum numbers with the $H^\pm$, $A^0$ states that are present in models with more than one Higgs doublet, e.g., the MSSM. As a consequence, the detection of Higgs-like scalars is not sufficient to establish a weakly interacting scenario of EWSB. Only by a careful analysis of the complete pattern of masses and couplings a particular model can be favored or excluded, and the measurement of the couplings to Goldstone bosons (longitudinal $W, Z$ as opposed to transversal gauge bosons) is an important ingredient. Many of the corresponding measurements are difficult or impossible at a hadron collider even if production rates are large, but are straightforward in the low-background environment of a Linear Collider.

4. Measuring Higgs sector parameters at a Linear Collider

4.1. Precision observables

The effective Lagrangian (34) and (35–46) encodes all known facts about the structure of electroweak interactions. Our current knowledge about the free parameters of this effective theory can be summarized as follows:

(1) The masses of charged fermions and vector bosons have been measured
or derived from hadronic data with high accuracy.

(2) The neutrino mass matrices are much less certain. We know about three different eigenstates with very low mass and considerable mixing, but nothing about any other eigenstates.

(3) The current-current structure of weak interactions is well established for the first two generations of quark and leptons. For the third generation, there is still some room for contributions that do not fit in this picture.

(4) Dimension-five operators (magnetic moments, flavor-changing penguin operators, etc.) are suppressed, and all measurements and limits are consistent with loop effects of the known particles.

In short, the gauge symmetry structure is extremely well tested, at least for the first two fermion families. This makes the chiral-Lagrangian approach a meaningful parameterization. Any anomalous effect can consistently be parameterized by the coefficients of gauge-invariant (higher-dimensional) operators. In the bosonic sector in particular, a basis of the CP-conserving dimension-four operators is given by (35–46). These terms are sensitive to Higgs sector physics since they contain factors of the symmetry-breaking $\Sigma$ field, and therefore carry the information about EWSB that is available at low energies until new degrees of freedom are observed directly.

Only three parameters in this list are significantly constrained by the electroweak precision data gathered during the last decade. In the analysis of $Z$ pole observables, of the $W$ mass and of the flavor-independent low-energy data, all deviations from the SM prediction can, to leading nontrivial order, be parameterized by the coefficients of the three operators that are bilinear in the vector fields. In our terminology, these are $\alpha_1$, $\beta'$ and $\alpha_8$ (36, 35, 43). Another parameterization has been introduced by Peskin and Takeuchi, who considered the Taylor expansion of vector boson propagators. The leading deviations from the SM relations are given by three parameters $\Delta S$, $\Delta T$ and $\Delta U$, which are related to the chiral Lagrangian coefficients by

$$
\Delta S = -16\pi\alpha_1, \quad \alpha\Delta T = -\beta', \quad \Delta U = -16\pi\alpha_8. \quad (86)
$$

$\Delta T$ parameterizes custodial $SU(2)_C$ violation in the Higgs sector, analogous to the $\rho$ parameter which is related to it (49) by

$$
\Delta \rho = \alpha \Delta T. \quad (87)
$$

$\Delta S$ describes anomalous mixing of weak and hypercharge bosons and thus affects the measured value of the weak mixing angle, while $\Delta U$ parame-
terizes $SU(2)_C$ violation in the left-handed gauge sector. In most models, there is little room for the latter effect, so $\Delta U$ is usually not considered.

It should be emphasized that, in the absence of a Higgs boson, the SM radiative corrections to the interactions parameterized by $\Delta S$ and $\Delta T$ are logarithmically divergent. To obtain numeric values for them, one has to introduce a cutoff in the low-energy effective theory. It is customary to introduce a (fictitious) Higgs boson for that purpose, so that values for $\Delta S$ and $\Delta T$ have to be understood in reference to some fixed Higgs boson mass.

The experimental constraints on $S$ and $T$ are summarized in Fig. 2, where the reference Higgs boson mass has been set to 100 GeV. For this value, the exclusion contour encloses the origin, and $\Delta S$ and $\Delta T$ are consistent with zero. Incidentally, this is the prediction of the minimal SM with a light Higgs boson, which is therefore consistent with the precision data. If the reference Higgs mass $m_H$ is changed to higher values, the origin of the $ST$ plane moves into the lower right direction, as indicated in the plot. Then, the Higgs sector has to contribute nonzero shifts $\Delta S$ and $\Delta T$. In fact, the actual location of the exclusion contour requires a positive value of $\Delta T$ to make the data consistent with a high effective Higgs mass. In the absence of a light Higgs boson one should therefore expect new physics to provide a certain amount of custodial $SU(2)_C$ violation.

In models of dynamical symmetry breaking, $S$ and $T$ (or $\alpha_1$ and $\beta'$) receive contributions from the compositeness scale $\Lambda$ which are of the order $\Delta \alpha_1 \sim v^2/\Lambda^2$. If $\Lambda \sim 4\pi v$, the natural upper limit for $\Lambda$ in the absence of a Higgs resonance, this correction is of the same order as the shift from a light to a heavy Higgs boson in Fig. 2. Of course, the sign and precise value of the new contributions is model-dependent. While QCD-like technicolor models are disfavored by the data since they typically have $\Delta S > 0$ and $\Delta T \approx 0$ (conserved $SU(2)_C$), topcolor models, for instance, predict custodial symmetry violation and positive $\Delta T$, which makes them consistent with data for a larger range of physical Higgs masses.

The bands in Fig. 2 indicate the areas in the $ST$ plane allowed by individual observables which depend on $\Delta S$ and $\Delta T$. It is important that they all intersect in the same region, so that a meaningful exclusion contour can be drawn. If this were not the case, one would have to include operators of dimension six in the analysis. The magnitude of their contribution is parameterically of the same order as two-loop radiative corrections which, however, are strongly scheme-dependent in the absence of a physical Higgs boson.
Fig. 2. Exclusion contour in the $ST$ plane allowed by the electroweak precision data. The arrows denote the directions of increasing top mass and increasing Higgs mass $^{37}$.

The consistency of the individual bands in the present plot indicates that the inclusion of higher-order effects is not yet necessary, but significant improvements in the experimental sensitivity (as expected from a Giga-Z experiment, cf. Ref. $^{38}$) would provide such a level of precision that a two-parameter analysis in terms of $S$ and $T$ becomes obsolete. This is obviously desirable in the context of weakly interacting models where all observables are computable to higher order in perturbation theory, and information on additional parameters in the theory can be gained in this way. However, since it is unlikely that for strong interactions a reliable calculation of next-to-next-to-leading order effects is possible, in the context of dynamical EWSB the experimental coverage of a larger subset of the operator coefficients (36–46) will be more valuable.

4.2. Triple gauge couplings

In the operator basis (36–46), the operators $L_2$, $L_3$, $L_9$ and $L_{11}$ do not contribute to vector boson two-point functions, but modify the trilinear couplings of the photon and of the $W$ and $Z$ bosons. A standard parameterization of a CP-conserving triple gauge boson vertex has been introduced
\[ \mathcal{L}_{WWV} = g_{WWV} \left[ i g_1^V V_\mu (W^{\nu} W_{\mu}^{\nu} - W_{\mu}^{\nu} W^{\nu}) + i \kappa V W_{\mu}^{\nu} W^{\mu \nu} + \left( \frac{\lambda V}{M_W^2} W_{\mu}^{\nu} W_{\nu}^{\mu} + g_5^V \epsilon^{\mu \nu \rho \sigma} (W_{\mu}^{- \rho} W_{\nu}^{+} - \partial_{\rho} W_{\nu}^{+} - \partial_{\mu} W_{\nu}^{+} W_{\sigma} \right) \right], \]

where \( V \) denotes the photon (\( V = \gamma \) or \( A_\mu \)) and the \( Z \) boson interactions with prefactors \( g_{WW\gamma} = e \) and \( g_{WWZ} = g_{\gamma w} \), respectively. If all anomalous operator coefficients vanish, we have

\[
\begin{align*}
 g_1^\gamma &= g_5^\gamma = \kappa_\gamma = \kappa_Z = 1, \\
 g_5^Z &= g_5^\gamma = \lambda_\gamma = \lambda_Z = 0.
\end{align*}
\]

Nonzero coefficients of the dimension-four operators contribute the following shifts:\(^{40e}\)

\[
\begin{align*}
 \Delta \kappa_\gamma &= g^2 \alpha_2 + g^2 \alpha_3 + g^2 \alpha_9, \\
 \Delta \kappa_Z &= -g^2 \alpha_2 + g^2 \alpha_3 + g^2 \alpha_9, \\
 \Delta g_1^Z &= \frac{\epsilon}{\epsilon} g^2 \alpha_3, \\
 \Delta g_5^Z &= \frac{\epsilon}{\epsilon} g^2 \alpha_{11}, \\
 \Delta g_1^\gamma &= \Delta g_5^\gamma = 0, \\
 \Delta \lambda_\gamma &= \Delta \lambda_Z = 0.
\end{align*}
\]

Due to electromagnetic gauge invariance, corrections to \( g_1^\gamma \) and \( g_5^\gamma \) have to vanish at zero momentum transfer. However, the absence of corrections to the \( \lambda \) couplings up to this order is a characteristic feature of the strongly-interacting scenario. Nonzero values for these coefficients are only introduced at higher order in the chiral expansion, i.e., by dimension-six operators. (In the weakly-interacting scenario where a light Higgs boson is present, all anomalous terms scale as dimension six.) The reason is that \( \lambda \) multiplies a term that involves transversal vector fields only and thus does not probe the Higgs sector directly. By contrast, the operators \( \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_9 \) and \( \mathcal{L}_{11} \) (37, 38, 44, 46) involve Goldstone scalars, visible as the longitudinal components of vector bosons. Hence, the strongly interacting scenario predicts the transversal couplings \( \lambda_\gamma \) and \( \lambda_Z \) to be significantly suppressed.

\(^{40e}\)There are also shifts due to \( \alpha_1, \beta' \) and \( \alpha_8 \). They are constrained already now by the existing data as discussed in the previous section. These contributions, together with the one-loop radiative corrections, have to be included, but can be assumed to be known in a complete triple gauge boson coupling analysis.
compared to possible deviations in the $g$ and $\kappa$ parameters. Projecting onto longitudinal polarization states of the vector bosons by exploiting angular correlations of their decay products will enhance the relevant contributions.

With exact custodial $SU(2)_C$ symmetry we have $\alpha_9 = \alpha_{11} = 0$ and get the additional relations

$$\Delta \kappa_2 = -\frac{c^2_w}{s^2_w}(\Delta \kappa_Z - \Delta g^Z_1) \quad \text{and} \quad g^Z_5 = 0,$$

If this symmetry assumption is valid, the leading anomalous effect on the couplings depends on just two parameters, $\alpha_2$ and $\alpha_3$, which could be measured by considering $Z$ observables only, $g_Z$ and $\kappa_Z$.

The measurement of $W$ and $Z$ pair production at LEP2 has provided the first meaningful bounds on anomalous triple gauge boson couplings. The current precision is still low compared to that one already achieved for $\Delta S$ and $\Delta T$. This situation will change when a Linear Collider is available. As discussed in Ref. 38, the experimental accuracy on the vector boson self-interactions will then become competitive, so the indirect sensitivity to the Higgs sector structure can be considerably improved by the complete coverage of the operator basis.

4.3. $W$ and $Z$ scattering amplitudes

The remaining parameters $\alpha_{4,5,6,7,10}$ (39–42, 45) in the CP-conserving chiral Lagrangian do not contribute at tree-level to bilinear or trilinear vector boson couplings. Rather, they affect Goldstone boson scattering and thus show up as anomalous quartic couplings of (longitudinally polarized) $W$ and $Z$ bosons.

From the list of operators introducing quartic couplings, only $L_4$ and $L_5$ conserve custodial $SU(2)_C$, so in the symmetric case the quartic vector boson couplings depend on just two new parameters, $\alpha_4$ and $\alpha_5$. The other terms, $L_6$, $L_7$ and $L_{10}$, describe new sources of $SU(2)_C$ violation. This counting does not include the effect of the parameters $\alpha_{3,8,9,11}$ which also contribute anomalous quartic vector-boson interactions. As discussed before, by combining low-energy and high-energy Linear Collider data the latter will be sufficiently constrained to be considered fixed in the analysis of quartic couplings, which must also include the (calculable) one-loop SM corrections.

Concentrating on the genuine quartic couplings, in the $SU(2)_C$ symmetric limit the contribution of $\alpha_4$ and $\alpha_5$ to the Goldstone scattering amplitude is given by (63). However, this amplitude is not accessible directly, but is embedded in multi-fermion processes involving intermediate gauge
boson production and decay. There are three different classes of processes which probe Goldstone scattering at lepton colliders:

(1) The one-loop amplitude for the production of a final state $VV'$ (where $V, V' = W^\pm, Z$) is affected by a rescattering correction:

$$Z/\gamma e^- e^+ \to W/Z$$

In this process rescattering of the vector bosons takes place at the full c.m. energy of the annihilating fermions. To make use of this fact, in a global fit of the parameters describing pair production their imaginary part has to be extracted. Only the channel with spin and isospin 1 can be accessed.

(2) The second process class which is sensitive to the symmetry-breaking sector is triple vector boson production:

$$e^- e^+ \to ZW^+ W^-$$  \hspace{1cm} (98)
$$e^- e^+ \to ZZZ$$  \hspace{1cm} (99)

Gauge invariance makes the cross section for all processes of this type fall off with $1/s$. Therefore, the best measurements are not necessarily done at the highest energy but somewhat above threshold, and the sensitivity is limited by the available luminosity. The variable to project out resonances or to observe the effect of anomalous couplings is the invariant mass of vector boson pairs.

(3) The obvious place to look for Goldstone boson scattering amplitudes is vector boson fusion:

$$e^- e^+ \to e^- e^+ VV'$$  \hspace{1cm} (100)
$$e^- e^+ \to \nu \bar{\nu} VV'$$  \hspace{1cm} (101)
This class of processes has a cross section that rises logarithmically with energy, so increasing the energy as well as the luminosity will improve the experimental sensitivity. In the asymptotic limit where masses can be neglected, the intermediate vector bosons are essentially on-shell and the relevant Goldstone scattering amplitudes are probed directly, albeit at an effective c.m. energy which is significantly lower than the full collider energy.

Which process is actually most sensitive depends on machine parameters and experimental details (cf. Ref. 38). Concerning rescattering corrections, disentangling the imaginary parts of all form factors in vector boson pair production (i.e., four-fermion production) is a nontrivial task. Collider runs with different combinations of electron and positron polarization are needed for a clean separation of all contributions.\(^{(41)}\)

The other processes involve six-fermion production in \(e^+e^-\) collisions, where in the case of triple vector boson production all three fermion pairs originate from vector boson decays, such that the signal can be isolated by invariant mass constraints. In the case of vector boson fusion, the spectator neutrinos (electron/positron) go predominantly into the forward direction, hence the characteristic signature of this process is a large missing invariant mass (\(e^+e^-\) invariant mass).

For realistic Linear Collider parameters, triple vector boson production\(^{(42)}\) appears to be less sensitive to the parameters of Goldstone scattering than vector boson fusion\(^{(43)}\) once the collider energy is sufficient for the latter process to have a significant rate. In the asymptotic high-energy limit, this rate can be approximated by the cross section for the on-shell subprocess of \(2 \rightarrow 2\) vector boson scattering, folded by the splitting probabilities \(e \rightarrow W\nu\) (or \(e \rightarrow Z\nu\)). These structure functions are plotted in Fig. 3.

The figure shows that the emission of longitudinally polarized \(W\) bosons is significantly suppressed compared to transversally polarized ones, in particular towards the high-energy end of the spectrum. Only the former probe
the anomalous quartic interactions we are interested in, so there is a significant $W_T$-induced background which has to be reduced by a suitable experimental strategy\cite{44,43,45}. Since the longitudinal spectrum drops sharply near $x = 1$, the energies that can be reached for Goldstone boson scattering are considerably lower than the collider energy. For that reason, a Linear Collider with an energy of at least 0.8 to 1 TeV is necessary to achieve a reasonable precision in the determination of the quartic couplings. At these energies, unitary constraints on the amplitudes (Sec. 3.4) are not yet an issue, and the chiral Lagrangian parameterization describes the scattering processes in a model-independent way.

Anomalous quartic couplings involving photons will not be induced by the dimension-four operators $L_4$ to $L_{10}$. Therefore, in $e\gamma$ and $\gamma\gamma$ collisions vector boson pair production does not provide independent information on a strongly interacting Higgs sector at this level, and in $e^+e^-$ collisions photon-induced processes should be considered as a background to vector boson fusion. Dimension-six operators, however, allow for anomalous quartic couplings involving photons and can be probed independently in these channels.

A complete coverage of the parameter space, which requires also the inclusion of the $SU(2)_C$ violating operators in the analysis, will be possible only by combining all available channels and including, in particular, results for the analogous processes at the LHC. Nevertheless, the results presented
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in Ref. 38 show that by considering only the dominant vector boson fusion channels at a Linear Collider, the experimental precision on the quartic couplings $\alpha_4$ and $\alpha_5$ will be in the percent range, not much worse that the expected accuracy in determining the bilinear and trilinear couplings discussed before.\(^1\)

5. Conclusions

Dynamical symmetry breaking provides a natural explanation for the electroweak scale. If there is no weakly interacting effective theory which describes physics beyond the TeV range, as it might be the case if a light Higgs boson exists, the dynamics responsible for electroweak symmetry breaking could be directly accessible at future colliders. The study of Goldstone-boson scattering amplitudes is then the key for accessing the (strongly-interacting) Higgs sector of the Standard Model.

The inherent scale of new strong interactions is likely beyond 1 TeV, therefore in this chapter we have considered mainly the indirect effects on precision observables at a Linear Collider where initially the energy may not be sufficient to produce new states. However, compared to existing precision data the set of observables that can be measured at a Linear Collider with at least percent accuracy is greatly enlarged, and essentially all interactions in the leading nontrivial order of the low-energy expansion are covered. Together with hadron collider data, this will allow to significantly constrain the possible scenarios and open the path towards a satisfactory theory of electroweak symmetry breaking and mass generation.

References


\(^1\)In comparing numerical values, one should take into account that some authors extract a factor $16\pi^2$ from the $\alpha$ parameters, such that their natural values are $\mathcal{O}(1)$. 


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37. J. Bagger, talk at the 2001 Snowmass Summer Study, figure by M. Swartz.