Plane Waves and Spacelike Infinity

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In an earlier paper, we showed that the causal boundary of any homogeneous plane wave satisfying the weak energy condition consists of a single null line. For conformally flat plane waves such as the Penrose limit of $AdS_5 \times S^5$, all spacelike curves that reach infinity also end on this boundary and the completion is Hausdorff. However, the more generic case (including, e.g., the Penrose limits of $AdS_4 \times S^7$ and $AdS_7 \times S^4$) is more complicated. In one natural topology, not all spacelike curves have limit points in the causal completion, indicating the need to introduce additional points at ‘spacelike infinity’—the endpoints of spacelike curves. We classify the distinct ways in which spacelike curves can approach infinity, finding a two-dimensional set of distinct limits. The dimensionality of the set of points at spacelike infinity is not, however, fixed from this argument. In an alternative topology, the causal completion is already compact, but the completion is non-Hausdorff. Causal structure, conformal boundary, plane waves SUGP-03/3-2, DCTP-03/11

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Introduction

The understanding of the asymptotic structure of spacetimes plays an important role in many problems in both classical and quantum gravity. In the past, attention was primarily concentrated on the formulation of suitable notions of asymptotic flatness and the exploration of their consequences. There has, however, recently been a renewal of interest in the careful investigation of the asymptotic structure of other solutions in the context of the holographic description of string theory. In particular, the proposed duality between string theory on a plane wave background and $\mathcal{N} = 4$ SYM BMN has made it important to understand the structure of these backgrounds.

A powerful technique for studying the asymptotic structure is to construct a suitable completion of the spacetime, adjoining some ideal points representing the asymptotic behaviour. An elegant method of constructing such a completion based on the causal structure was developed in Geroch, budic, szab1, szab2, MRtop. This technique was applied to smooth homogeneous plane waves in beyond, where it was shown that the causal boundary, as defined in Geroch, budic, szab1, szab2, of any homogeneous plane wave satisfying the positive energy condition is a single null line. This generalised a result previously obtained for the special case of the maximally symmetric (attractive) plane wave in BN.

Consideration of the plane wave and other examples has exposed some defects in the approach to defining the causal completion in terms of a quotient adopted in Geroch, szab1, szab2. In MRtop, a new definition of the causal completion $\bar{M}$ for a general spacetime $M$ in terms of IP-IF pairs was proposed, and two new candidate topologies were introduced on this completion. Neither of these topologies is completely satisfactory, but they represent a net improvement on previous proposals. This new definition was applied to the homogeneous plane waves in MRtop, and we found that it reproduces the results previously announced in beyond.

In this paper, we will extend our previous investigations of the asymptotic structure, applying the topologies defined in MRtop to investigate curves that approach infinity along spacelike directions. We will show that taking limits of past and future sets along spacelike curves generically leads to complicated behavior. In addition to learning more about the asymptotic structure of the plane wave spacetime, we hope that the explicit application of the topologies may help us to better understand the differences between the two definitions advanced in MRtop.

In one of these topologies, known as $\bar{T}$, most spacelike curves in these plane waves do not have limit points, even when we attach the causal boundary to the spacetime. Hence, the causal completion of the spacetime is non-compact. If we wanted to obtain a truly compact completion of the spacetime, we would need to adjoin some additional ideal points reachable only by spacelike curves. Such points are said to constitute spacelike infinity.

This is in fact a familiar situation. In the conformal compactification of Minkowski space, there is a single point $i^0$ in the boundary which is reachable only by spacelike curves (see figure i0). If we apply the causal completion technique to Minkowski space, on the other hand, this point will not be a part of $\bar{M}$.
because no timelike curve reaches it. Hence, the causal completion is non-compact, and if we want to recover the usual conformal completion, we have to add in the point $i^0$ ‘by hand’.

figure center [width=0.4]i0.eps A conformal diagram of 1+1 Minkowski space indicating spacelike infinity ($i^0$). $i^0$