Peculiar Relics from Primordial Black Holes in the Inflationary Paradigm

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Abstract. Depending on various assumptions on the energy scale of inflation and assuming a primordial power spectrum of a Broken Scale Invariance (BSI) type, we explore the possibility for Primordial Black Holes (PBH) and Planck relics to contribute substantially to cold dark matter in the Universe. A recently proposed possibility to produce planck relics in 4-dimensional string gravity is considered. Possible experimental detection through gravitational waves is further explored. We stress that inflation with a low energy scale, and also possibly when Planck relics are produced, leads unavoidably to relics originating from PBHs that are not effectively classical during their formation, rendering the usual formalism inadequate for them.

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1. Introduction

The formation of Primordial Black Holes in the early stages of the universe is a generic feature and it is therefore interesting to study its cosmological consequences [1]. Whatever the formation model, the Primordial Black Hole (PBH) spectrum must be in agreement with two types of constraints. The first one is associated with evaporation: the density must be low enough so that physical effects due to the Hawking radiation do not contradict any observed phenomena. They are based on the entropy per baryon, the $n\bar{n}$ production at nucleosynthesis, the deuterium destruction, the Helium-4 spallation [2] [3] and, finally, on the nowadays observed gamma-ray [4] and antiproton [5] spectra. Those constraints apply for initial PBH mass between $10^9$ g and $10^{15}$ g. Using the quantity $\beta$, which gives the probability that a region has the required density contrast to form a PBH at the horizon crossing time corresponding to the considered scale, the cosmic-ray constraints are the more stringent ones, leading approximately to $\beta(M_{PBH} = 5 \times 10^{14} \text{ g}) < 10^{-26}$. The second type of constraints is associated with the normalization of the spectrum on cosmological scales probed by CMB data. Whatever the considered power spectrum to form PBHs, it must generate a correct density contrast on the COBE scale.

A pure scale-invariant Harrison-Zeldovich power spectrum from the CMB scales up to very small scales would lead to a negligible amount of PBHs. The only way to produce PBHs as a significant dark matter candidate is to increase the power on small scales without contradicting the observational data. A first attempt in this direction would be to allow for a tilt: $P(k) \propto k^n$ with $n > 1$. Even without considering possible inconsistencies with cosmic-ray data, the required value, around $n \approx 1.3$ [6] seems extremely disfavoured by the analysis of the most recent CMB experiments: between $n \approx 0.91 \pm 0.06$ [7] (WMAP measurements : CMB + running spectral index) and $n \approx 1.04 \pm 0.12$ [8] (Archeops measurements : CMB + $H_0$). A natural alternative is to boost power on small scales by means of a bump in the fluctuations power spectrum, as suggested, for example in [6]. We follow here this idea and we consider various scenarios where this is possible even for a simple step-like structure in the spectrum. This paper deals also with new models for Planck relics formation, significant when the energy scale of inflation is high, based on four dimensional string gravity. Some experimental probes through the emission of gravitational waves by coalescing PBHs are suggested. We stress that in some of these scenarios the production of PBHs from quantum fluctuations which are not highly squeezed and therefore not effectively classical [9], is unavoidable and cannot be handled with the usual formalism [10].

2. Inflation with a low energy scale

2.1. A pure step

An important consequence of low scale inflation is the decrease of the reheating temperature. Though in practice, the reheating scale can be much lower than the energy
scale at the end of inflation, it will be enough for our purposes to make the simplifying assumption that the reheating is instantaneous. A low reheating temperature is required by the possible overproduction of gravitinos \[11\] \[12\] \[13\]: \( T_{RH} < 10^8 \text{ GeV} \). This makes the horizon size at the end of inflation very large with an associated Hubble mass \( M_H > 10^{16} \text{ g} \). This point is extremely important for PBH dark matter as it allows to avoid the main problem explained in e.g. \[14\], namely the gamma-ray constraint which comes from the contribution to the \( \gamma \)-ray background of evaporated PBHs. Then only the gravitational constraint would apply for PBH masses \( M_{PBH} \) greater than \( M_{H,e} \), the Hubble mass at the end of inflation.

Previous work on the subject argued that one way to produce a significant amount of dark matter in the form of PBHs is to increase the mass variance in a well localized region so as to remain in agreement with the gamma-ray constraint. For example, using the Broken Scale Invariance (BSI) Starobinsky spectrum \[15\], it was shown that the oscillation in the power spectrum due to the jump in the derivative of the inflaton potential should produce a bump in the mass variance \[14\]. This slight increase in variance \( \sigma^2_H \) can boost the PBH formation probability \( \beta \) by more than ten orders of magnitude. The resulting bump in the probability \( \beta \) to form PBHs can yield \( \Omega_{PBH,0} \approx 0.3 \) for \( 5 \times 10^{15} \text{ g} < M_{PBH} < 10^{21} \text{ g} \) with values of \( p \approx 8 \times 10^{-4} \) \[14\] where \( p^2 \) is the ratio of the power spectrum on large scales with respect to that on small scales.

Clearly, if the horizon mass \( M_{H,e} \) at the end of inflation is larger than \( M_\ast \approx 5 \times 10^{14} \text{ g} \), the initial mass of a PBH whose lifetime is equal to the age of the Universe, then the \( \gamma \)-ray and antiproton constraints, as well as all the other ones on smaller masses associated with evaporation, are automatically evaded without any requirement about the shape of the fluctuations spectrum. PBHs with masses above \( M_\ast \) are nearly totally insensitive to the Hawking emission as the temperature \( T = h c^3/(8 \pi G k M_{PBH}) \) becomes smaller than the rest mass of any known massive field. An extremely wide mass range without constraint (except, to some extent, for microlensing upper limits) is therefore opened. To give a crude estimate, we can consider, following \[6\], that a (possibly smoothed-out) jump occurs around some characteristic mass \( M_s \). Using \[6\]

\[
\Omega_{PBH,0}(M_{PBH}) \approx 1.3 \times 10^{17} \beta(M_{PBH}) \left( \frac{10^{15} \text{ g}}{M_{PBH}} \right)^{\frac{1}{2}},
\]

(for \( h \approx 0.7 \), and \( f_0 \) stands for the quantity \( f \) today) and normalizing the mass variance with the CMB large angular scales measurements, the increase in power and therefore the amplitude \( p \) of the step can be estimated as a function of the horizon mass at the end of inflation \( M_{H,e} \). We have further

\[
\beta(M_H) = \frac{1}{\sqrt{2 \pi \sigma_H(t_k)}} \int_{\delta_{min}}^{\delta_{max}} e^{-\frac{\sigma^2_H}{2\sigma^2_H(t_k)}} d\delta \approx \frac{\sigma_H(t_k)}{\sqrt{2 \pi \delta_{min}}} e^{-\frac{\sigma^2_{min}}{2\sigma^2_H(t_k)}},
\]

where \( t_k \) is the horizon crossing time for the considered mode, \( \delta \) is the density contrast, \( M_H \) is the Hubble mass at \( t_k \) and \( \sigma^2_H(t_k) \equiv \sigma^2(R)|_{t_k} \) where \( \sigma^2(R) \equiv < \frac{\delta^2}{M^2} >_R > \) is
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computed with a Top-Hat window function $W_{TH}$ with $R = \frac{H^{-1}}{a}|t_k$, according to [16]

$$\sigma^2_H(t_k) = \frac{8}{81\pi^2} \int_0^{k_e} x^3 F(kx) W^2_{TH}(c_s x) W^2_{TH}(x) \, dx,$$

(3)

where $k_e$ corresponds to the Hubble crossing scale at the end of inflation, $c^2_s = \frac{1}{3}$ and

$$k^3 P(k, t_k) = \left(\frac{2}{3}\right)^4 F(k).$$

This leads to the result

$$p \approx \sigma^2_{COBE} \delta_{\min} \left[ \frac{1.7 \times 10^{34}}{2\pi \Omega_{PBH,0}^2 \left[ \frac{10^{15} \text{ g}}{M_{H,e}} \right]} \right] \left(\frac{LW}{\delta_{\min}}\right),$$

(4)

where $LW$ stands for the LambertW function (with $LW(xe^x) \equiv x$). With $\Omega_{PBH,0} \approx \Omega_{m,0}$, the numerical estimates are: $p \approx 6.5 \times 10^{-4}$ for $M_{H,e} = 10^{15} \text{ g}$, $p \approx 5.5 \times 10^{-4}$ for $M_{H,e} = 10^{25} \text{ g}$, $p \approx 4.1 \times 10^{-4}$ for $M_{H,e} = 10^{35} \text{ g}$. In these estimates, $\delta_{\min}$ was assumed to be around 0.7 and $\sigma^2_H \approx (10/9)\alpha^2 \delta^2_H$ with $\alpha \approx 2.5$, as computed in [16]. In principle, the reheating temperature can be as low as the MeV (the nucleosynthesis temperature), leading to huge horizon masses around $10^{38} \text{ g}$. This can be considered as the upper limit for the low-mass cutoff of PBH spectra. It is interesting that this corresponds to the highest viable PBH masses if CDM is made of PBHs [17]. It opens a very wide parameter space ($M_{H,e}, M_s$) for PBH dark matter. Furthermore, if the reheating temperature is smaller than 1 GeV ($M_{H,e} \gg 10^{16} \text{ g}$), PBHs could be one of the viable CDM candidates left, as supersymmetric dark matter cannot contribute substantially to dark matter [18].

2.2. “Quantum” relics

A generic feature of low scale inflationary models is that all PBHs that are produced will survive and not evaporate. In particular, those PBHs that would form right after inflation correspond to fluctuations that were not (long) outside the Hubble radius and are therefore not highly squeezed. Indeed for the first scales which reenter the Hubble radius, producing in these models PBHs with $M_{PBH} \gtrsim M_{H,e}$, the fluctuations are not highly squeezed. This is equivalent to saying that the decaying mode is still present and actually of the same order as the growing mode, and cannot be neglected. Hence such low scale inflation models lead to the possible production of PBHs by inflationary fluctuations which cannot be considered as stochastic classical fluctuations and that are not evaporated today, in contrast to high scale inflation where $M_{H,e} \ll M_*$. Even if there is no significant increase in power on small scales, PBHs will be produced that cannot be described as classical objects. We might therefore call them quantum relics. The intriguing point is not their abundance, which should be low, but rather the very nature of these surviving objects.

We want to illustrate that now with a concrete high energy physics inspired low scale inflationary model. This will be true, for example, with the model considered in the first part of this Section, however it is not clear whether it is possible to implement a characteristic scale in low scale inflation.

It would be interesting if the scale of inflation is the supersymmetry breaking scale or even the electroweak scale [19]. Initial conditions (through thermal effects) could
set the inflaton field $\phi$ close to the origin where some symmetry is unbroken, as in “new inflation”. Inflation then takes place at small field values. At low temperature the inflaton starts rolling away from the origin $\phi = 0$ (the effective mass term becomes negative), spontaneously breaking the underlying symmetry. The following quite general inflationary potential can be considered in the context of supergravity inflation:

$$V = \Lambda^4 \times \left[ \left( 1 - \kappa \frac{|\phi|^p}{\Lambda^p M_p^{p-q}} \right)^2 + \left( b + c \ln \left( \frac{|\phi|}{M_p} \right) \right) \left( \frac{|\phi|}{M_p} \right)^2 \right]$$  \hspace{1cm} (5)

where $\Lambda$ is the near-constant vacuum energy driving inflation, $M_p$ the planck mass, $b$ a “bare” mass term, $c$ a logarithmically varying mass term brought by radiative corrections and $\kappa$, $p$ and $q$ determine the end of inflation by inclusion of higher-order terms. It has been shown that a value for $\Lambda$ as low as $1 \text{ GeV}$ up to $\sim 10^{11} \text{ GeV}$ can be obtained for reasonable choices of $p$ and $q$ while still generating an acceptable spectrum of perturbations with a spectral index $n_s \lesssim 0.95$ close to 1 and a sufficient number of e-folds. So the Hubble mass $M_{H,e}$ at the end of inflation will satisfy $M_{H,e} > M_*$. We can quantify the degree of classicality of the formed PBHs with the ratio $D$ of the growing to the decaying mode for the scales under consideration [10]. A ratio $D \gg 1$ for a given scale corresponds to effective classicality of the fluctuations on this scale. We estimate it for adiabatic fluctuations in these models, and we find towards the end of inflation

$$D(M) \simeq \left( \frac{\Lambda}{M} \right)^4 ,$$  \hspace{1cm} (6)

where $M^4$ stands for the energy density at the time when the PBH is formed. It is related to $M_H$ through

$$M_H = 5.6 \left( \frac{10^8 \text{ GeV}}{M} \right)^2 \times 10^{16} \text{ g} .$$  \hspace{1cm} (7)

Though PBHs are here clearly irrelevant as CDM candidates, the appearance of these “quantum relics” might be one of the few ways in which the quantum nature of the inflationary fluctuations can be exhibited.

3. Inflation with a high energy scale

3.1. Two distinct inflationary stages

Another way to evade the “small scales” problems for PBH formation still in the framework of high scale inflation, is through the existence of a second inflationary stage at much lower energies. So, a first stage of inflation solves all the problems usually solved by inflation, generating the cosmological perturbations observed, and produces also PBHs including in the dangerous mass interval around $M_*$. However, thanks to the second stage of inflation, a significant PBH abundance produced during the first inflation is allowed. We give now in full generality the salient features of such a scenario.

Let us assume that the second inflation starts when the Hubble mass equals $M_{H,i}$, at a much lower energy than the first inflation scale. For simplicity we can assume
the Hubble mass is constant during the second inflation. On a large range of scales, fluctuations produced during the first inflation will reenter the Hubble radius thereby possibly producing PBHs. Part of those scales which reentered the Hubble radius between the two inflationary stages will be expelled again outside the Hubble radius during the second inflation. Let us consider the scale \( k_H \equiv a_i H_i \) that corresponds to the Hubble radius at the beginning of the second inflation. It will eventually, at the time \( t_{kh} \), reenter the Hubble radius when the Hubble mass is given by \( M_H(t_{kh}) \). It is easy to derive the following relation between \( M_H(t_{kh}) \) and \( M_{H,i} \):

\[
\frac{M_H(t_{kh})}{M_{H,i}} = e^{2N},
\]

where \( N \) is the number of e-folds during the second inflation. Due to a much lower energy scale, the amplitude of the produced fluctuations is quite negligible and will not produce PBHs. Only the fluctuations of the first, high scale, inflation will. Therefore there will be a gap in the mass range \( M_{H,i} \leq M_{PBH} \leq M_H(t_{kh}) \). In addition the density of all objects created before the second inflation will be reduced by an additional factor \( e^{-3N} \). Hence, the only significant abundance of PBHs corresponds to the range

\[
M_{PBH} \geq e^{2N} M_{H,i}.
\]

Clearly it is possible to have PBHs as CDM in this range and still evade the small scales constraint coming from the evaporated PBHs.

An interesting low scale inflationary model of that kind is thermal inflation ([20]), triggered by a scalar field termed flaton, which can appear in SUSY theories. The consequences of thermal inflation on PBH abundance was considered in [21]. By definition, a flaton has a large vacuum expectation value \( M \gg 10^3 \) GeV while having a mass of order the electroweak breaking scale \( m \sim 10^2 - 10^3 \) GeV. This leads to an almost flat potential for the flaton field \( f : V \sim V_0 - m^2 |f|^2 \) with \( V_0 \sim m^2 M^2 \). During thermal inflation the flaton field is held at the origin by finite temperature effects and the potential is dominated by the false vacuum energy \( V_0 \). Thermal inflation starts at the temperature \( T_i \sim \sqrt{mM} \) when the thermal energy density falls below \( V_0 \) and ends at \( T_e \sim m \) when the flaton can escape the false vacuum. The number of e-folds \( N \) is then immediately given as \( N = \frac{1}{2} \ln \left( \frac{M}{m} \right) \) and the density of all PBHs produced before thermal inflation is suppressed accordingly by a factor \( \left( \frac{m}{M} \right)^{3/2} \). If we take \( M \sim 10^{11} \) GeV, \( m \sim 10^3 \) GeV, thermal inflation starts at \( T_i \sim 10^7 \) GeV which corresponds to a Hubble mass \( M_{H,i} \sim 10^{18} \) g, and ends at \( T_e \sim 10^3 \) GeV. The number of e-folds is \( N \approx 9 \) and the density of PBHs with mass \( M < M_{H,i} \) is suppressed by a factor \( \sim 10^{-12} \). Clearly it is then possible to have a significant amount of PBHs and still evade the small scales constraints. Note however that with the numbers given above, PBHs as CDM can only exist in the range \( M_{PBH} \gtrsim 10^{26} \) g, starting near the edge of the range probed by the EROS data which constrain galactic dark matter in the range \( 2 \times 10^{-7} M_\odot \lesssim M_{PBH} \lesssim 1 M_\odot \), \( M_\odot \) being the solar mass [22]. In particular, this could apply to the model with a step considered in the previous chapter, followed by a stage of thermal inflation. However a step in the primordial spectrum produced during the first
period of inflation at the characteristic scale \( M_s \lesssim 2 \times 10^{-7} \, \text{M}_\odot \) leaves possibly only a tiny mass interval \( (M_H(t_{kh}) \lesssim M_{PBH} \lesssim 2 \times 10^{-7} \, \text{M}_\odot) \) where PBHs are not constrained by observations. There will be no “quantum relics” in this scenario as no PBHs are produced during the second (low-scale) inflation.

### 3.2. Planck relics

If we simply ignore moduli and gravitinos, as none of them as ever been detected, still another interesting possibility could be the production of Planck relics. Indeed, the unavoidable upper limit is imposed by gravitational waves and fixes the smallest possible horizon mass after inflation around 1 g. If the horizon mass is in this range, a natural way to produce dark matter is through PBH relics. The idea was first mentioned in [23]. Nevertheless, two critical ingredients were missing at that time: the normalization of the primordial spectrum to COBE data and a realistic (or, at least, possible) model to stop Hawking evaporation in the Planck era. The latter point received a new light in the framework of string gravity. The following 4D effective action with second order curvature corrections can be built:

\[
S = \int d^4x \sqrt{-g} \left[ -R + 2 \partial_\mu \phi \partial^\mu \phi + \lambda e^{-2\phi} S_{GB} + \ldots \right],
\]

where \( \lambda \) is the string coupling constant, \( R \) is the Ricci scalar, \( \phi \) is the dilatonic field and \( S_{GB} = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2 \) is the curvature invariant — Gauss-Bonnet term. This generalisation of the Einstein Lagrangian leads to the very important result that there is a minimal relic mass \( M_{rel} \) for such black holes [24]. Solving the equations at the first order of perturbation in the curvature gauge metric gives a minimal radius:

\[
r_{h}^{inf} = \sqrt{\lambda} \sqrt{4\sqrt{6} \phi_h(\phi_\infty)},
\]

where \( \phi_h(\phi_\infty) \) is the dilatonic value at \( r_h \). The crucial point is that this result remains true when higher order corrections or time perturbations are taken into account. The resulting value should be around \( 2M_p \). It is even increased to \( 10M_p \) if moduli fields are considered, making the conclusion very robust and conservative. The subsequent decrease of the Hawking evaporation leads to an asymptotically stable state [25] giving a quantitative argument in favour of the existence of Planck relics. For formation masses above \( 10^9 \, \text{g} \), important constraints are associated with Helium and Deuterium destruction. Actually, as we will see below, the relevant upper limit for the initial PBH mass is \( \sim 10^5 \, \text{g} \).

Once again, we consider a mass variance spectrum with a characteristic scale \( M_s \). For small initial masses \( M_{PBH} \sim 1 \, \text{g} \), the Planck relics relative density \( \Omega_{rel,0} \) can be written as

\[
\Omega_{rel,0} \approx 1.3 \times 10^{17} \beta(M_{PBH}) \frac{M_{rel}}{M_{PBH}} \left( \frac{10^{15} \, \text{g}}{M_{PBH}} \right)^{1/2}
\]

\[
\approx 2.83 \gamma \times 10^{-3} \beta(M_{PBH}) \left( \frac{10^{15} \, \text{g}}{M_{PBH}} \right)^{3/4},
\]
where we have taken $M_{\text{rel}} \equiv \gamma M_p$, while $M_{\text{PBH}}$ refers to the initial PBH mass. This leads to the following value for the step amplitude:

$$p \approx \frac{\sigma_{\text{COBE}}^{\text{H}}}{\delta_{\text{min}}} \sqrt{\frac{8.0 \times 10^{-6}}{2\pi \Omega_{\text{rel,0}}^2} \left( \frac{M_{\text{rel}}}{M_p} \right)^2 \left[ \frac{10^{15} \text{ g}}{M_{H,e}} \right]^3}. \quad (14)$$

If Planck relics are to explain $\Omega_{m,0} \approx 0.3$, $p$ varies from $7.1 \times 10^{-4}$ to $5.5 \times 10^{-4}$ for initial PBH masses between 1 g and $10^5$ g with $M_{\text{rel}} = M_p$ and between $7.3 \times 10^{-4}$ and $5.6 \times 10^{-4}$ with $M_{\text{rel}} = 10 M_p$. As for non evaporating PBHs, $p$ has a very slight dependence on $M_{H,e}$ as $\beta$ is extremely sensitive to the mass variance in this range.

Equation (13) is very accurate for small masses $M_{PBH} \approx 1$ g and its validity extends up to $M_{PBH} \approx M' \equiv 8 \gamma^{\frac{5}{6}} \times 10^5$ g, with the corresponding range $10^{-21} \gamma^{-1} \lesssim \beta(M_{PBH}) \lesssim 10^{-12} \gamma^{-1}$. We have checked this numerically using the simplifying assumption that all the evaporation produces either a relativistic, or else a non relativistic, component. For $M_{PBH}$ up to $M'$, the energy density due to evaporation is still much smaller, at the time when the relics have formed, than the preexisting radiation background. However, it should be stressed again that when $M_{PBH} \sim M_{H,e}$, the quantity $\beta$ entering (13) looses its meaning as a probability, not to mention the fact that the asymptotic domination of the growing mode is not achieved in this regime. Beyond $M'$, PBHs dominate the energy density before their evaporation is completed. This gives rise to a different expression for $\Omega_{\text{rel,0}}$ which is nearly independent of $\beta$ ([3, 26]). Interestingly, in this mass range it is possible to obtain PBHs as CDM in significant amounts only for masses $M_{PBH} \approx M'$. Thus, in the context of CDM, only equation (13) is relevant and PBHs cannot contribute significantly to CDM if their mass lies in the range $M' \lesssim M_{PBH} \lesssim 10^9$ g. Surprisingly, in such a scenario too, the notion of quantum relics resurfaces for those Planck relics originating from initial PBH masses $M_{PBH} \approx M_{H,e} \sim 1$ g. Because of their supposed large abundance, Planck relics with initial mass $M_{PBH} \sim M_{H,e} \sim 1$ g would probably be ruled out but we conjecture that a bump producing PBHs around $10^5$ g could yield a viable CDM candidate.

4. Gravitational waves

Probing PBHs experimentally is very difficult. As long as their masses are greater than $10^{15}$ g, black holes do not radiate and become really black. A decisive way to detect them, and to observationally confirm or exclude this model, could be to look for gravitational waves from coalescing PBHs. The maximum distance $R_{\text{max}}$ between the Earth and the binary system compatible with the sensitivity of a given detector for a fixed PBH mass $M_{PBH}$ is given by [27]:

$$\left( \frac{R_{\text{max}}}{20 \text{ Mpc}} \right) \approx 3.6 \cdot 10^{-21} h_{\text{SBmin}}^{-1} \left( \frac{M_{PBH}}{M_\odot} \right)^{\frac{5}{6}} \left( \frac{\nu}{100 \text{ Hz}} \right)^{-\frac{1}{6}}. \quad (15)$$

where $h_{\text{SBmin}}$ is the sensitivity of the detector, $\nu$ is the considered frequency. The number $n(M_{PBH}, R_{\text{max}})$ of PBHs inside this sphere is then estimated using an isothermal profile.
inside the Milky-Way halo for $R < 150$ kpc:

$$\rho(r, \psi) = \rho_\odot \frac{R_C^2 + R_\odot^2}{R_C^2 + R_\odot^2 - 2rR_\odot\cos\psi + r^2}$$

(16)

where $\rho_\odot$ is the local halo mass density, $R_C \approx 3$ kpc is the core radius, $R_\odot \approx 8$ kpc is the galactocentric distance, $r$ is the distance to the Earth and $\psi$ is the angle between the considered point and the galactic center seen from the Earth. This leads to:

$$n = \frac{\pi \rho_\odot}{M_{PBH}} \frac{R_C^2 + R_\odot^2}{R_\odot^2} \int_0^{R_{max}} \ln \left\{ \frac{R_C^2 + R_\odot^2 + 2rR_\odot + r^2}{R_C^2 + R_\odot^2 - 2rR_\odot + r^2} \right\} rdr.$$  (17)

For $R > 150$ kpc, an average dark matter distribution with $\rho \approx 0.3\rho_c$ is assumed. Finally the coalescence rate $f$ within this volume is computed under the natural assumption that the distribution function of the PBHs comoving separation is uniform [28]:

$$f \approx 3 \left( \frac{M_{PBH}}{M_\odot} \right)^{\frac{37}{34}} \times \frac{n(M_{PBH}, R_{max})}{t_0}$$

(18)

where $t_0$ is the age of the Universe. Gathering all those formulae and performing numerical estimates show that if PBHs have masses above $2 \times 10^{-5}M_\odot$, they should generate more than one "event" per year in the VIRGO detector. If the LISA frequencies and sensitivity are considered, the minimal mass decreases down to $10^{-11}M_\odot$. The interesting mass range probed covers nearly fifteen orders of magnitude, though it also overlaps with microlensing data which exclude a significant contribution between $2 \times 10^{-7}M_\odot$ and $1M_\odot$ [22].

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