EPR CORRELATIONS AND EPW DISTRIBUTIONS

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Dedicated to Professor E.P. Wigner

ABSTRACT

In the case of two free spin-zero particles, the wave function originally considered by Einstein, Podolsky and Rosen to exemplify EPR correlations has a non-negative Wigner distribution. This distribution gives an explicitly local account of the correlations. For an irreducible non-locality, more elaborate wave functions are required, with Wigner distributions which are not non-negative.

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It is known that with Bohm's example of EPR correlations, involving particles with spin, there is an irreducible non-locality. The non-locality cannot be removed by the introduction of hypothetical variables unknown to ordinary quantum mechanics. How is it with the original EPR example involving two particles of zero spin? Here we will see that the Wigner phase space distribution¹⁾ illuminates the problem.

Of course, if one admits "measurement" of arbitrary "observables" on arbitrary states, it is easy to mimic²⁾ the EPRB situation. Some steps have been made towards realism in that connection³⁾. Here we will consider a narrower problem, restricted to "measurement" of positions only, on two non-interacting spinless particles in free space. EPR considered "measurement" of momenta as well as positions. But the simplest way to "measure" the momenta of free particles is just to wait a long time and "measure" their positions. Here we will allow position measurements at arbitrary times t_1 and t_2 on the two particles respectively. This corresponds to "measuring" the combinations

$$\hat{q}_1 + t_1 \hat{p}_1 / m_1, \quad \hat{q}_2 + t_2 \hat{p}_2 / m_2 \quad (1)$$

at time zero, where m_1 and m_2 are the masses, and the $\hat{q}$ and $\hat{p}$ are position and momentum operators. We will be content here with just one space dimension.

The times t_1 and t_2 play the same roles here as do the two polarizer settings in the EPRB example. One can envisage then some analogue of the CHHS inequality^{4),5)} discriminating between quantum mechanics on the one hand and local causality on the other.

The QM probability of finding, at times t_1 and t_2 respectively, that particles at positions q_1 and q_2 respectively, is

$$Q(q_1, q_2, t_1, t_2)$$

with

$$\rho = |\psi(q_1, q_2, t_1, t_2)|^2 \quad (2)$$

The two-time wave function ψ satisfies the two Schrödinger equations

$$\left. \begin{aligned} i\hbar \partial \psi / \partial t_1 &= H_1 \psi = (\hat{p}_1^2 / 2m_1) \psi \\ i\hbar \partial \psi / \partial t_2 &= H_2 \psi = (\hat{p}_2^2 / 2m_2) \psi \end{aligned} \right\} \quad (3)$$

with

$$i\hat{p}_1 = \hbar \partial / \partial q_1, \quad i\hat{p}_2 = \hbar \partial / \partial q_2$$

For simplicity we will consider the case of equal masses, and take units such that

$$m_1 = m_2 = \hbar = 1$$

The same ρ , (2), can be obtained from the corresponding two-time Wigner distribution:

$$\rho = \iint \frac{dp_1}{2\pi} \frac{dp_2}{2\pi} W(q_1, q_2, p_1, p_2, t_1, t_2) \quad (4)$$

where

$$W = \iint dy_1 dy_2 e^{-i(p_1 y_1 + p_2 y_2)} \left. \begin{aligned} &\psi(q_1 + \frac{y_1}{2}, q_2 + \frac{y_2}{2}, t_1, t_2) \\ &\psi^*(q_1 - \frac{y_1}{2}, q_2 - \frac{y_2}{2}, t_1, t_2) \end{aligned} \right\} \quad (5)$$

From (3),

$$(\partial/\partial t_1 + p_1 \partial/\partial q_1) W = (\partial/\partial t_2 + p_2 \partial/\partial q_2) W = 0 \quad (6)$$

That is, W evolves exactly as does a probability distribution for a pair of freely-moving classical particles:

$$W(q_1, q_2, p_1, p_2, t_1, t_2) = W(q_1 - p_1 t_1, q_2 - p_2 t_2, p_1, p_2, t_1, t_2) \quad (7)$$

When W happens to be initially nowhere negative, the classical evolution (7) preserves the non-negativity. The original EPR wave function⁶⁾

$$\delta((q_1 + \frac{1}{2}q_0) - (q_2 - \frac{1}{2}q_0)), \quad (8)$$

assumed to hold at $t_1 = t_2 = 0$, gives

$$W(q_1, q_2, p_1, p_2, 0, 0) = \delta(q_1 - q_2 + q_0) 2\pi \delta(p_1 + p_2) \quad (9)$$

This is nowhere negative, and the evolved function (7) has the same property. Thus in this case the EPR correlations are precisely those between two classical particles in independent free classical motion.

With the wave function (8), then there is no non-locality problem when the incompleteness of the wave function description is admitted. The Wigner distribution provides a local classical model of the correlations. Since the Wigner distribution appeared in 1932, this remark could already have been made in 1935. Perhaps it was. And perhaps it was already anticipated that wave functions, other than (8), with Wigner distributions that are not non-negative, would provide a more formidable problem. We will see that this is so.

Consider, for example, the initial wave function

$$(q^2 - 2a^2) e^{-q^2/(2a^2)} \quad (10)$$

where

$$q = (q_1 + q_0/2) - (q_2 - q_0/2) \quad (11)$$

It could be made normalizable by including a factor

$$\exp -((q_1 + q_0/2) + (q_2 - q_0/2))^2/(2b^2) \quad (12)$$

But we will immediately anticipate the limit $b \rightarrow \infty$, and will consider only relative probabilities. Choosing the unit of length so that $a = 1$ gives as the initial Wigner distribution

$$W(q_1, q_2, p_1, p_2, 0, 0) = K e^{-q^2} e^{-p^2} \{(q^2 + p^2)^2 - 5q^2 + p^2 + 11/4\} \delta(p_1 + p_2) \quad (13)$$

where K is an unimportant constant, and

$$p = (p_1 - p_2) / 2 \quad (14)$$

This W , (13), is in some regions negative, for example at $(p = 0, q = 1)$. It no longer provides an explicitly local classical model of the correlations. I do not know that the failure of W to be non-negative is a sufficient condition in general for a locality paradox. But it happens that (13) implies, as well as negative regions in the Wigner distribution, a violation of the CHHS locality inequality.

To see this, first calculate the two-time position probability distribution, either from (4), (7) and (13), or from (2) and the solution of (3). The result is

$$\varrho = K'(1+\tau^2)^{-5/2} \left\{ q^4 + q^2(2\tau^2 - 4) + 3(1+\tau^2) + (1+\tau^2)^2 \right\} e^{-q^2/(1+\tau^2)} \quad (15)$$

where K' is an unimportant constant, and

$$\tau = t_1 + t_2 \quad (16)$$

Calculate then the probability D that $(q_1 + q_0/2)$ and $(q_2 - q_0/2)$ disagree in sign:

$$D(t_1, t_2) = \int_{-\infty}^{\infty} dq \, 2|q| \, \varrho \quad (17)$$

$$= K'' \left(\tau^2 + \frac{2}{5} \right) / \sqrt{\tau^2 + 1} \quad (18)$$

Consider finally the CHHS inequality

$$E(t_1, t_2) + E(t_1, t'_2) + E(t'_1, t_2) - E(t'_1, t'_2) \leq 2 \quad (19)$$

where

$$E(t_1, t_2) = \left. \begin{aligned} &(\text{probability of } (+, +)) \\ &+ (\text{probability of } (-, -)) \\ &- (\text{probability of } (+, -)) \\ &- (\text{probability of } (-, +)) \end{aligned} \right\} \quad (20)$$

(21)

Using (21), (19) becomes

$$D(t_1, t_2) + D(t_1, t'_2) + D(t'_1, t_2) - D(t'_1, t'_2) \geq 0 \quad (22)$$

With

$$t'_1 = 0, \quad t_2 = \tau, \quad t_1 = -2\tau, \quad t'_2 = 3\tau \quad (23)$$

and assuming [in view of (18)]

$$D(t_1, t_2) = F(|t_1 + t_2|) \quad (24)$$

(22) gives (for τ positive)

$$3F(\tau) - F(3\tau) \geq 0 \quad (25)$$

But this is violated by (18) when τ exceeds about 1. There is a real non-locality problem with the wave function (10).

Only some epsilonics will be added here. The essential assumption leading to 2 is (roughly speaking) that measurement on particle 1 is irrelevant for particle 2, and vice versa. This follows from local causality⁷⁾ if we look for the particles only in limited space-time regions

$$\begin{aligned} |q_1 + q_0/2| < L, \quad |t_1| < T \\ |q_2 - q_0/2| < L, \quad |t_2| < T \end{aligned} \tag{26}$$

with

$$L \ll q_0, \quad cT \ll q_0 \tag{27}$$

so that the two regions (26) have spacelike separation. We must, however, make L large enough, compared with b in (12), so that the particles are almost sure to be found in the regions in question, for in passing from (20) to (21) it was assumed that the four probabilities in (20) add to unity; and b in turn must be large compared with a , as was used to simplify the detailed calculations. So as well as (27), we specify

$$1 \gg a/b \gg (b/L) e^{-L^2/b^2} \tag{28}$$

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- 6) A. Einstein, B. Podolsky and N. Rosen, Physical Review 47 (1935) 779.
- 7) J.S. Bell, Theory of Local Beables, preprint CERN-TH.2053/75, reprinted in Epistemological Letters 9 (1976) 11 and in Dialectica 39 (1985) 86. The notion of local causality presented in this reference involves complete specification of the beables in an infinite space-time region. The following conception is more attractive in this respect: in a locally-causal theory, probabilities attached to values of local beables in one space-time region, when values are specified for all local beables in a second space-time region fully obstructing the backward light cone of the first, are unaltered by specification of values of local beables in a third region with spacelike separation from the first two.
- 8) The discussion has a new interest when the positions q_1 and q_2 are granted beable status. Then we can consider their actual values, rather than "measurement results", at arbitrary times t_1 and t_2 . External intervention by hypothetically free-willed experimenters is not involved.