Relativistic predictions of spin observables for exclusive proton knockout reactions

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Abstract

Within the framework of the relativistic distorted wave impulse approximation (DWIA), we investigate the sensitivity of complete sets of polarization transfer observables \( (P, A_y, D_{nn}, D_{s's}, D_{\ell'\ell}, D_{s'\ell}, \text{and } D_{\ell's}) \), for exclusive proton knockout from the \( 3s_{1/2}, 2d_{3/2} \) and \( 2d_{5/2} \) states in \( ^{208}\text{Pb} \), at an incident laboratory kinetic energy of 202 MeV, and for coincident coplanar scattering angles \( (28.0^\circ, -54.6^\circ) \), to different distorting optical potentials, finite-range (FR) versus zero-range (ZR) approximations to the DWIA, as well as medium-modified meson-nucleon coupling constants and meson masses. Results are also compared to the nonrelativistic DWIA predictions based on the Schrödinger equation.

For knockout from the \( 3s_{1/2} \) state, \( A_y, D_{nn} \) and \( D_{s's} \) exhibit large differences between Dirac and Schrödinger-based DWIA models. On the other hand, for knockout from the \( 2d_{3/2} \) state, the most sensitive observables to the latter are \( D_{nn}, D_{\ell's} \) and \( D_{\ell'\ell} \), whereas the corresponding observables for \( 2d_{5/2} \) knockout are \( D_{s'\ell} \) and \( D_{\ell's} \). The most sensitive observables to ZR versus FR approximations to the DWIA are the induced polarization \( (P) \), \( D_{\ell's} \) and \( D_{s'\ell} \) for knockout from the \( 3s_{1/2}, 2d_{3/2} \) and \( 2d_{5/2} \) states, respectively.

Although polarization transfer observables are relatively insensitive to different global Dirac optical potential parameter sets, distorting optical potentials are crucial for describing the oscillatory behavior of spin observables. In addition, it is seen that \( A_y, P \) and \( D_{s's} \) are very sensitive to reductions in the meson-coupling constants and meson masses by the nuclear medium for proton knockout from all three states.

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I. INTRODUCTION

It is now well established that spin observables are more appropriate than unpolarized cross sections for discriminating between different physical processes partaking in nuclear reactions \[1\]. Different spin observables usually exhibit selective sensitivity to different physical effects and, hence, in order to test the validity of a theoretical model, it is advisable to measure as many independent spin observables as possible or, at the very least one needs to identify (via model predictions) specific observables which can potentially address the physical problem of interest.

One of the most challenging problems in nuclear physics is to understand how the properties of the strong interaction are modified inside nuclear matter. Various theoretical models \[2, 3, 4\] predict the modification of meson-nucleon coupling constants as well as nucleon and meson masses by normal nuclear matter. At present there is no overwhelming experimental evidence supporting these predictions. However, we believe that exclusive \((\vec{p}, 2\vec{p})\) reactions - whereby an incident polarized proton knocks out a bound proton from a specific orbital in the nucleus and the two scattered protons, one of which is polarized, are detected in coincidence - are ideally suited for studying the behavior of the nucleon-nucleon (NN) interaction in the nuclear medium. By exploiting the discriminatory nature of independent spin observables for the knockout protons from deep to low-lying single particle states in nuclei, one can in principle extract information about the density dependence of the NN interaction in a model-dependent fashion. Indeed, with the recent developments in the production of polarized proton beams and the construction of high resolution spectrometers with focal plane polarimeters, it is possible to measure complete sets of polarization transfer observables which relate the components of a scattered polarized proton beam to the corresponding components of an incident proton beam which is polarized in an arbitrary direction [see Sec. (IV)].

To date most exclusive proton knockout data have been analyzed within the framework of the distorted wave impulse approximation (DWIA), the main ingredients of which are the scattering wave functions for the incoming and two outgoing protons, the boundstate wave function of the struck proton in the target nucleus, and the interaction between the incident proton and bound proton. Furthermore, the impulse approximation assumes that the form of the NN scattering matrix in the nuclear medium is the same as that for free NN scattering.
The DWIA also assumes that the main influence of the nuclear medium is to modify (distort) the scattering wave functions relative to their corresponding plane wave values for scattering in free space: nuclear distortion effects are incorporated via the inclusion of appropriate optical potentials, gauged by elastic scattering data, in the underlying equations of motion.

Since the tremendous success of the relativistic mean-field theory \([2]\) for describing nuclear reactions and nuclear structure, there are serious concerns regarding the validity of nonrelativistic Schrödinger-equation-based models in nuclear physics. In this paper, we focus on a relativistic description of exclusive \((\vec{p},2\vec{p})\) spin observables. Conventional wisdom claims that, since the binding energy of a nucleon in a nucleus is relatively small compared to the rest mass of a nucleon, relativistic effects are unimportant for nuclear structure problems, and hence the nonrelativistic Schrödinger equation should provide an appropriate dynamical basis for nuclear physics studies. In recent years, however, the ability of quantum hadrodynamics, an effective relativistic field theory, to provide a mechanism for nuclear saturation and spin-orbit splitting in nuclei, has led to growing evidence that the relativistic Dirac equation is the correct underlying dynamical equation. In particular, the small nuclear binding energy and the strength of the spin-orbit interaction both result from the subtle interplay between an attractive Lorentz scalar (attributed to the exchange of sigma mesons), with a strength of approximately \(-400\) MeV, and a repulsive vector potential (attributed to the exchange of omega mesons) with a strength of approximately \(+350\) MeV \([2]\).

Recently, we demonstrated that the relativistic DWIA provides an excellent description of analyzing power data for the knockout of protons from the \(3s_{1/2},2d_{3/2}\) and \(2d_{3/2}\) and states in \(^{208}\)Pb at an incident energy of 202 MeV and for coincident coplanar scattering angles \((28.0^\circ,-54.6^\circ)\) \([5]\). Our motivation for choosing the \(^{208}\)Pb target and a relatively low incident energy of 202 MeV was to maximize the influence of distortion effects, while still maintaining the validity of the impulse approximation, and also avoiding complications associated with the inclusion of recoil corrections in the relativistic Dirac equation \([6, 7]\). In particular, we studied the effect of medium-modified coupling constants and meson masses on the above analyzing powers for both zero-range (ZR) and finite-range (FR) approximations to the relativistic DWIA. On one hand, the relativistic ZR predictions suggested that the scattering matrix for NN scattering in the nuclear medium is adequately represented by the corresponding matrix for free NN scattering, without nuclear medium corrections. On the other hand, the relativistic FR results imply that a 10% to 20% reduction of meson-coupling...
constants and meson masses by the nuclear medium is essential for providing a consistent
description of the 3s_{1/2}, 2d_{3/2} and 2d_{5/2} analyzing powers. Hence, within the context of
the relativistic DWIA, it is not clear whether nuclear-medium modifications are important
or not. In addition, one needs to fully understand whether the differences between ZR
and FR calculations are attributed to essential physics or numerical errors due to extensive
computational procedures associated with FR predictions (compared to ZR calculations).
In Refs. [5, 8] it was also reported that the nonrelativistic Schrödinger-equation DWIA
predictions completely fail to reproduce the 3s_{1/2} and 2d_{3/2} analyzing powers. Systematic
corrections to the nonrelativistic model - such as different kinematic prescriptions for the
NN amplitudes, non-local corrections to the scattering wave functions, density-dependent
modifications to the free NN scattering amplitudes, as well as the influence of different
scattering and boundstate potentials - failed to remedy the nonrelativistic dilemma [8].
Although the analyzing power results seem to suggest that the Dirac equation is the preferred
dynamical equation, a more definite statement regarding the role of dynamics can only be
made after comparing model predictions to complete sets of polarization transfer observables
for proton knockout from a variety of states in nuclei. In addition, such a comparison
will deepen our understanding of the influence of the nuclear medium effects on the NN
interaction as well as shed light on the role of FR versus ZR effects in exclusive proton
knockout reactions.

Unfortunately, there are no published spin observable data, other than the analyzing
power, for the reaction kinematics of interest. In an effort to demonstrate the unique abil-
ity of polarization data, and in particular data on complete sets of polarization transfer
observables, to selectively address many of the above-mentioned physics issues, we present
the first relativistic and nonrelativistic predictions of complete sets of polarization transfer
observables for exclusive proton knockout from the 3s_{1/2}, 2d_{3/2} and 2d_{5/2} states in \(^{208}\)Pb,
at an incident laboratory kinetic energy of 202 MeV, and for coincident coplanar scattering
angles (28.0°, −54.6°). More specifically we investigate the sensitivity of these observables to
FR versus ZR approximations to the relativistic DWIA as well as medium-modified meson-
nucleon coupling constants and meson masses. In order to reliably extract information on
the latter it is necessary to minimize model-input uncertainties. The most likely source
of uncertainty could be related to ambiguities associated with the choice of global optical
potential parameters for generating the incident and outgoing scattering wave functions:
different global parameter sets are constrained by different sets of experimental data for elastic proton-nucleus scattering. For a heavy target nucleus such as $^{208}$Pb the effect of nuclear distortion is to reduce the unpolarized triple differential cross section to about 5% of its plane wave value: differences in optical potential parameter sets translate to an uncertainty of 10% in the latter cross sections $[9]$. The question arises as to how sensitive polarization transfer observables are to nuclear distortion and, in particular, to different optical potential parameter sets. Current qualitative arguments suggest that, since polarization transfer observables are ratios of polarized cross sections, distortion effects on the scattering wave functions effectively cancel, and hence simple plane wave models (ignoring nuclear distortion) should be appropriate for studying polarization phenomena $[10, 11]$. Recently we demonstrated that, contrary to intuition, the ($p, 2p$) analyzing power is extremely sensitive to nuclear distortion within the context of the relativistic DWIA $[3]$. The analyzing power is, however, relatively insensitive to different global Dirac optical potential parameter sets. In this paper we extend the latter investigation to study, for the first time, the effect of nuclear distortion on complete sets of polarization transfer observables for exclusive ($p, 2p$) reactions.

In Sec. (II), we briefly describe the essential ingredients underlying the relativistic DWIA for both ZR and FR approximations to the NN interaction. Thereafter, in Sec. (III), we discuss our prescription for invoking nuclear medium modifications of the NN interaction. The formalism for calculating complete sets of spin observables is presented in Sec. (IV). Results are presented in Sec. (V), and we summarize and draw conclusions in Sec. (VI).

II. RELATIVISTIC DISTORTED WAVE IMPULSE APPROXIMATION

Both ZR and FR approximations to the relativistic DWIA have been discussed in detail in Refs. [12] and [13, 14], respectively. In this section, we briefly describe the main ingredients of these models. The exclusive ($p, 2p$) reaction of interest is schematically depicted in Fig. (II), whereby an incident proton, $a$, knocks out a bound proton, $b$, from a specific orbital in the target nucleus $A$, resulting in three particles in the final state, namely the recoil residual nucleus, $C$, and two outgoing protons, $a'$ and $b$, which are detected in coincidence at coplanar laboratory scattering angles, $\theta_a'$ and $\theta_b$, respectively. All kinematic quantities are completely determined by specifying the rest masses, $m_i$, of particles, where $i = (a, A, a', b, C)$, the
FIG. 1: Schematic representation for the coplanar \((p,2p)\) reaction of interest.

laboratory kinetic energy, \(T_a\), of incident particle \(a\), the laboratory kinetic energy, \(T_{a'}\), of scattered particle \(a'\), the laboratory scattering angles \(\theta_a\) and \(\theta_{a'}\), and also the binding energy of the proton that is to be knocked out of the target nucleus, \(A\).

For a finite-range (FR) NN interaction, the relativistic distorted wave transition matrix element is given by

\[
T_{LJM_J}(s_a, s_{a'}, s_b) = \int d\vec{r} d\vec{r}' [\bar{\psi}^{(-)}(\vec{r}, \vec{k}_{a'C}, s_{a'}) \otimes \bar{\psi}^{(-)}(\vec{r}', \vec{k}_{bC}, s_b)] \times \hat{t}_{NN}(|\vec{r} - \vec{r}'|) \left[\psi^{(+)}(\vec{r}, \vec{k}_{aA}, s_a) \otimes \phi_{LJM_J}(\vec{r}')\right]
\]  

(1)

where \(\otimes\) denotes the Kronecker product. The four-component scattering wave functions, \(\psi(\vec{r}, \vec{k}_{ij}, s_i)\), are solutions to the fixed-energy Dirac equation with spherical scalar, \(S(r)\), and time-like vector, \(V(r)\), nuclear optical potentials: \(\psi^{(+)}(\vec{r}, \vec{k}_{aA}, s_a)\) is the relativistic scattering wave function of the incident particle, \(a\), with outgoing boundary conditions [indicated by the superscript \((+)]\), where \(\vec{k}_{aA}\) is the momentum of particle \(a\) in the \((a + A)\) center-of-mass system, and \(s_a\) is the spin projection of particle \(a\) with respect to \(\vec{k}_{aA}\) as the \(\hat{z}\)-quantization axis; \(\bar{\psi}^{(-)}(\vec{r}, \vec{k}_{jC}, s_j)\) is the adjoint relativistic scattering wave function for particle \(j\) \([j = (a', b)]\) with incoming boundary conditions [indicated by the superscript \((-)]\), where \(\vec{k}_{jC}\) is the momentum of particle \(j\) in the \((j + C)\) center-of-mass system, and \(s_j\) is the spin projec-
tion of particle $j$ with respect to $\vec{k}_{jC}$ as the $\hat{z}$-quantization axis. The boundstate proton wave function, $\phi_{LJM_j}^B(\vec{r})$, labeled by single-particle quantum numbers $L$, $J$, and $M_j$, is obtained via selfconsistent solution to the Dirac-Hartree field equations of quantum hadrodynamics [15]. In addition, we adopt the impulse approximation which assumes that the form of the NN scattering matrix in the nuclear medium is the same as that for free NN scattering. Furthermore, we assume that the antisymmetrized NN scattering matrix, $\hat{t}_{NN}(|\vec{r} - \vec{r}'|)$, is parameterized in terms of the five Fermi covariants [16], the so-called IA1 representation of the NN scattering amplitudes. In principle, the NN $t$-matrix can be obtained via solution of the Bethe-Salpeter equation, where the on-shell NN amplitudes are matrix elements of this $t$-matrix. However, the complexity of this approach gives limited physical insight into the resulting amplitudes. An alternative approach is to fit the amplitudes directly with some phenomenological form, rather than generating the $t$-matrix from a microscopic interaction. Although the microscopic approach is certainly more fundamental, the advantage of phenomenological fits lies in their simple analytical form, which allows them to be conveniently incorporated in calculations requiring the NN $t$-matrix as input. The NN $t$-matrix employed in this paper is based on the relativistic meson-exchange model described in Ref. [17], the so-called relativistic Horowitz-Love-Franey (HLF) model, where the direct and exchange contributions to the IA1 amplitudes are parameterized separately in terms of a number of Yukawa-type meson exchanges in first-order Born approximation. The parameters of this interaction, namely the meson masses, meson-nucleon coupling constants and the cutoff parameters, have been adjusted to reproduce the free NN elastic scattering observables.

Adopting a much simpler zero-range (ZR) approximation for the NN interaction, namely

$$\hat{t}_{NN}(|\vec{r} - \vec{r}'|) = \hat{t}_{NN}(T_{eff}^{ab}, \theta_{eff}^{cm}) \delta(\vec{r} - \vec{r}')$$  \hspace{1cm} (2)$$

the relativistic distorted wave transition matrix element in Eq. (1) reduces to

$$T_{LJM_j}(s_a, s_{a'}, s_b) = \int d\vec{r} [ \bar{\psi}^{(-)}(\vec{r}, \vec{k}_{a'C}, s_{a'}) \otimes \psi^{(-)}(\vec{r}, \vec{k}_{b'C}, s_b) ]$$

$$\hat{t}_{NN}(T_{eff}^{ab}, \theta_{eff}^{cm}) \left[ \bar{\psi}^{(+)}(\vec{r}, \vec{k}_{a'A}, s_a) \otimes \phi_{LJM_j}^B(\vec{r}) \right]$$ \hspace{1cm} (3)$$

where $T_{eff}^{ab}$ and $\theta_{eff}^{cm}$ represent the effective two-body laboratory kinetic energy and center-of-mass scattering angles, respectively.

As already mentioned, a FR approximation to the DWIA is inherently more sophisticated than a ZR approximation. However, in practice, the numerical evaluation of the
six-dimensional FR transition matrix elements, given by Eq. (1), is nontrivial and subject to numerical uncertainties. On the other hand, for the ZR approximation, the three-dimensional integral given by Eq. (3), ensures numerical stability and rapid convergence (and hence faster computational time). Another advantage of the ZR approximation is that one can directly employ experimental NN scattering amplitudes, rather than rely on a relativistic meson-exchange model, and hence, one is insensitive to uncertainties associated with interpolations and/or extrapolations of the limited meson-exchange parameter sets. In this paper, we compare FR and ZR predictions for complete sets of polarization transfer observables.

In principle, one could employ the HLF model for also generating microscopic relativistic scalar and vector optical potentials by folding the NN $t$-matrix with the appropriate Lorentz densities via the $t\rho$ approximation. An attractive feature of the $t\rho$ approximation is selfconsistency, that is, the HLF model is used for generating both NN scattering amplitudes and optical potentials. However, for the kinematic region of interest to this paper, we consider it inappropriate to employ microscopic $t\rho$ optical potentials, the reason being that HLF parameter sets only exist at 135 MeV and 200 MeV, whereas optical potentials for the outgoing protons are required at energies ranging between 24 and 170 MeV. Thus, enforcing selfconsistency would involve large, and relatively crude, interpolations/extrapolations, leading to inaccurate predictions of the spin observables. Furthermore, the validity of the impulse approximation, to generate microscopic $t\rho$ optical potentials at energies lower than 100 MeV, is questionable. Hence, in this paper we consider only global Dirac optical potentials \[20\], as opposed to microscopic $t\rho$ optical potentials, for obtaining the scattering wavefunctions of the Dirac equation.

III. NUCLEAR MEDIUM EFFECTS

For estimating the influence of nuclear medium modifications of the NN interaction on spin observables, we adopt the Brown-Rho scaling conjecture[3] which attributes nuclear-medium modifications of meson-nucleon coupling constants, as well as nucleon- and meson-masses, to partial restoration of chiral symmetry. In particular, we invoke the scaling relations proposed by Brown and Rho[3], and also applied by Krein et al.\[18\] to $(p,2p)$
reactions, namely

$$\frac{m_\sigma^*}{m_\sigma} \approx \frac{m_\rho^*}{m_\rho} \approx \frac{m_\omega^*}{m_\omega} \equiv \xi,$$  \hspace{1cm} (4)$$

$$\frac{g_{jN}^*}{g_{\sigma N}} \approx \frac{g_{\omega N}^*}{g_{\omega N}} \equiv \chi,$$  \hspace{1cm} (5)$$

where the medium-modified and free meson masses are denoted by $m_i^*$ and $m_i$, with $i \in (\sigma, \rho, \omega)$, respectively. Meson-nucleon coupling constants, with and without nuclear medium modifications, are denoted by $g_{jN}^*$ and $g_{jN}$, where $j \in (\sigma, \omega)$, respectively: see Sec. (V) for typical values of $\xi$ and $\chi$.

IV. SPIN OBSERVABLES

The spin observables of interest are denoted by $D_{ij}$ and are related to the probability that an incident beam of particles, $a$, with spin-polarization $j$ induces a spin-polarization $i'$ for the scattered beam of particles, $a'$: the subscript $j = (0, \ell, n, s)$ is used to specify the polarization of the incident beam, $a$, along any of the orthogonal directions

$$\hat{\ell} = \hat{z} = \hat{k}_{\alpha A},$$

$$\hat{n} = \hat{y} = \hat{k}_{\alpha A} \times \hat{k}_{\alpha C},$$

$$\hat{s} = \hat{x} = \hat{n} \times \hat{\ell},$$  \hspace{1cm} (6)$$

and the subscript $i' = (0, \ell', n', s')$ denotes the polarization of the scattered beam, $a'$, along any of the orthogonal directions:

$$\hat{\ell}' = \hat{z}' = \hat{k}_{\alpha'C},$$

$$\hat{n}' = \hat{n} = \hat{y},$$

$$\hat{s}' = \hat{x}' = \hat{n} \times \hat{\ell}'.$$  \hspace{1cm} (7)$$

The choice $j (i') = 0$ is used to denote an unpolarized incident (scattered) beam. With the above coordinate axes in the initial and final channels, the spin observables, $D_{ij}$, are defined by

$$D_{ij} = \frac{\text{Tr}(T \sigma_j T^\dagger \sigma_{i'}^* \sigma_{i'} \sigma_{i'})}{\text{Tr}(T T^\dagger)},$$  \hspace{1cm} (8)$$
where $D_{n0} = P$ refers to the induced polarization, $D_{0n} = A_y$ denotes the analyzing power, and the other polarization transfer observables of interest are $D_{nn}$, $D_{s's}$, $D_{s'l'}$, $D_{s'l}$, and $D_{l's}$. The denominator of Eq. 8 is related to the unpolarized triple differential cross section, i.e.,

$$\frac{d^3\sigma}{dT_a' d\Omega_a' d\Omega_b} \propto \text{Tr}(TT^\dagger).$$

(9)

In Eq. 8, the symbols $\sigma_i'$ and $\sigma_j$ denote the usual $2 \times 2$ Pauli spin matrices, namely,

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\sigma_{s'} = \sigma_s = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\sigma_n = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

$$\sigma_{l'} = \sigma_\ell = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(10)

and the $2 \times 2$ matrix $T$ is given by

$$T = \begin{pmatrix} T_{LJ}^{s_a=+\frac{1}{2}, s_a'=+\frac{1}{2}} & T_{LJ}^{s_a=-\frac{1}{2}, s_a'=+\frac{1}{2}} \\ T_{LJ}^{s_a=+\frac{1}{2}, s_a'=-\frac{1}{2}} & T_{LJ}^{s_a=-\frac{1}{2}, s_a'=-\frac{1}{2}} \end{pmatrix}.$$

(11)

where $s_a = \pm \frac{1}{2}$ and $s_{a'} = \pm \frac{1}{2}$ refer to the spin projections of particles $a$ and $a'$ along the $\mathbf{z}$ and $\mathbf{z}'$ axes, defined in Eqs. (3) and (7), respectively; the matrix $T_{s_a,s_a'}^{s_{a'},s_{a'}}$ is related to the relativistic $(p, 2p)$ transition matrix element $T_{LJ,MJ}(s_a, s_{a'}, s_b)$, defined in Eqs. (11) and (13), via

$$T_{s_a,s_{a'}}^{s_{a'},s_{a'}} = \sum_{MJ,s_b} T_{LJ,MJ}(s_a, s_{a'}, s_b).$$

(12)

V. RESULTS

In this section we study the sensitivity of complete sets of exclusive $(\vec{p}, 2\vec{p})$ spin observables, for the knockout of protons from the $3s_{1/2}$, $2d_{3/2}$ and $2d_{5/2}$ states in $^{208}$Pb, at an incident energy of 202 MeV, and for coincident coplanar scattering angles $(28.0^\circ, -54.6^\circ)$, to distorting optical potentials, FR versus ZR approximations to the relativistic DWIA, as
well as to medium-modified meson-nucleon coupling constants and meson masses. We also compare our relativistic results to corresponding nonrelativistic Schrödinger-based predictions based on the computer code THREEDEE of Chant and Roos [19]. Our aim is to identify specific observables which can be measured in order to unravel and understand the role of the above approximations, model ingredients and different dynamical models. Unless otherwise specified, all DWIA predictions employ the energy-dependent global Dirac optical potential parameter set constrained by $^{208}\text{Pb}(p,p)$ elastic scattering data for incident proton energies between 21 MeV and 1040 MeV [20].

First, we display the influence of relativistic nuclear distortion effects on complete sets of spin observables by comparing relativistic ZR-DWIA to relativistic plane wave predictions (with zero scattering potentials) for knockout from the $3s_{1/2}$, $2d_{3/2}$ and $2d_{5/2}$ states in Figs. (2), (3) and (4), respectively: the solid lines indicate the relativistic ZR distorted wave result and the dotted lines represent the relativistic plane wave result. For completeness we include the analyzing power calculations reported in Ref. [5]. As already mentioned in the latter publication, we see that the prominent oscillatory structure of the analyzing powers is mostly attributed to distortions of the scattering wave functions. Similarly, for the other spin observables there are large differences between the relativistic distorted wave and plane wave results. The above observations clearly illustrate the importance of including nuclear distorting optical potentials for calculating spin observables, thus refuting previous qualitative claims that spin observables, being ratios of cross sections, are insensitive to nuclear distortion effects. In addition, we have also investigated the sensitivity of all spin observables to a variety of different global Dirac optical potential parameter sets [20]. Although these results are not displayed, we find that all spin observables are relatively insensitive to different global optical potentials, with differences between parameter sets being smaller than the experimental statistical error indicated on the analyzing powers.

Next, we compare relativistic FR-DWIA (dot-dashed line) to relativistic ZR-DWIA (solid line) predictions. In general, it is seen that most spin observables are relatively sensitive to differences in ZR and FR predictions. For knockout from the $3s_{1/2}$ state [Fig. (2)], the induced polarization, $P$, is the most sensitive observable to differences between FR and ZR predictions, whereas $D_{s's's'}$ is relatively insensitive. For knockout from the $2d_{3/2}$ state [Fig. (3)], on the other hand, $D_{\ell's's}$ displays large differences between ZR and FR calculations, whereas $A_y$, $P$ and $D_{s's'}$ display small differences. For the $2d_{5/2}$ state [Fig. (4)], $D_{s'\ell}$ and $A_y$ are
FIG. 2: Polarization transfer observables plotted as a function of the kinetic energy, $T_{a'}$, for the knockout of protons from the $3s_{1/2}$ state in $^{208}\text{Pb}$, at an incident energy of 202 MeV, and for coincident coplanar scattering angles ($28.0^\circ, -54.6^\circ$). The different line types represent the following calculations: relativistic ZR-DWIA (solid line), relativistic plane wave (dotted line), nonrelativistic DWIA (dashed line), and relativistic FR-DWIA (dot-dashed line). The analyzing power data are from Ref. [8].

the most and least sensitive observables to FR versus ZR differences, respectively. When comparing ZR and FR predictions to the only existing proton knockout spin observable data on $^{208}\text{Pb}$, namely the analyzing power, we generally see that the ZR predictions provide an excellent description for knockout from all three states. Note, however, that although the relativistic FR (dot-dashed line) predictions are not as spectacular as the corresponding ZR calculations, they still provide a reasonable qualitative description of the data. The measurement of observables which display large differences between ZR and FR predictions will check the consistency of the analyzing power results and serve to further constrain
FIG. 3: Polarization transfer observables plotted as a function of the kinetic energy, $T_{a'}$, for the knockout of protons from the $2d_{3/2}$ state in $^{208}$Pb, at an incident energy of 202 MeV, and for coincident coplanar scattering angles $(28.0^\circ, -54.6^\circ)$. The different line types represent the following calculations: relativistic ZR-DWIA (solid line), relativistic plane wave (dotted line), nonrelativistic DWIA (dashed line), and relativistic FR-DWIA (dot-dashed line). The analyzing power data are from Ref. [8].

We also compare our relativistic ZR and FR calculations to nonrelativistic [dashed line in Figs. (2), (3) and (4)] DWIA predictions based on the commonly-used computer code THREEDEEE of Chant and Roos [19]. First, we mention the analyzing power results reported in Ref. [5]. With the exception of the $2d_{5/2}$, it is clearly seen that the relativistic ZR (solid line) and FR (dot-dashed line) predictions are consistently superior compared to the corresponding nonrelativistic calculations. This suggests that the Dirac equation is the most appropriate dynamical equation for the description of analyzing powers. Moreover, these
results represent the clearest signatures to date for the evidence of relativistic dynamics in polarization phenomena. However, before claiming with certainty that the relativistic equation is the most appropriate dynamical equation, it is necessary to identify additional observables which should be measured in order to further study the question of dynamics. In this respect, we generally see that all spin observables are relatively sensitive to Dirac- versus Schrödinger-based DWIA models. In particular, for knockout from the $3s_{1/2}$ state the spin observables $A_y$, $D_{nn}$ and $D_{s's's}$ exhibit large differences between Dirac and Schrödinger-based DWIA models. On the other hand, for knockout from the $2d_{3/2}$ state the most sensitive observables to dynamical differences are $D_{nn}$, $D_{ℓ's'}$ and $D_{ℓ'ℓ}$. For $2d_{5/2}$ knockout the most sensitive observables are $D_{s's}$ and $D_{ℓ'ℓ}$. A number of interesting observations are made at the point $T_{d'} ≈ 145 \text{ MeV}$ corresponding to minimum recoil momentum. First of all, we see that for $3s_{1/2}$ knockout the induced polarization ($P$), $D_{s'ℓ}$ and $D_{ℓ's'}$, the relativistic plane wave, ZR-DWIA and nonrelativistic DWIA predictions are virtually identical at this point; ZR-DWIA and FR-DWIA predictions are nearly identical for both $D_{nn}$ and $D_{s'ℓ}$. For knockout from the $2d_{3/2}$ state, both FR-DWIA and ZR-DWIA yield similar results for $P$, $D_{s's}$ and $D_{s'ℓ}$; $P$ and $D_{s's}$ are insensitive to nuclear distortion at the point in question. Finally, we see that for $2d_{5/2}$ knockout, relativistic plane wave, FR-DWIA and nonrelativistic DWIA predictions are virtually identical for $D_{s'ℓ}$ and $D_{ℓ's'}$. Hence, by measuring spin observables at minimum recoil momentum one can eliminate differences between different dynamical models and model parameters and focus on a specific issue of interest.

Next we study the sensitivity of spin observables to the nuclear medium modifications of the NN interaction [discussed in Sec. (III)] within the context of the relativistic DWIA. In Ref. [5] we studied the sensitivity of analyzing powers to 20% reductions of meson-nucleon coupling constants and meson masses by the nuclear medium relative to the values for free NN scattering. More specifically we chose $ξ = χ$ and varied these values between 1.0 and 0.8 for knockout from all three states of interest. The latter equality is only assumed for simplicity, so as to get a feeling for the sensitivity of observables to changes in the relevant meson-nucleon coupling constants and meson masses. The choice of values for $ξ$ and $χ$ is motivated by the fact that the proton-knockout reactions of interest are mainly localized in the nuclear surface and, hence, the nuclear medium modifications are expected to play a relatively minor role. Actually, using the procedure proposed in Ref. [21], the effective mean densities are estimated to be between 0.08 and 0.15 of the saturation density. In
FIG. 4: Polarization transfer observables plotted as a function of the kinetic energy, $T_{a'}$, for the knockout of protons from the 2d$_{5/2}$ state in $^{208}$Pb, at an incident energy of 202 MeV, and for coincident coplanar scattering angles ($28.0^\circ$, $-54.6^\circ$). The different line types represent the following calculations: relativistic ZR-DWIA (solid line), relativistic plane wave (dotted line), nonrelativistic DWIA (dashed line), and relativistic FR-DWIA (dot-dashed line). The analyzing power data are from Ref. [8].

In particular, for the analyzing powers in question we established that for values of $\xi = \chi < 0.8$ both FR-DWIA and ZR-DWIA models fail to reproduce the experimental analyzing power data. Regarding nuclear medium effects, for the ZR predictions we concluded in Ref. [5] that the inclusion of medium-modified meson-nucleon coupling constants and meson masses successfully described the analyzing power data, whereas the ZR predictions suggest that the scattering matrix for NN scattering in the nuclear medium is adequately represented by the corresponding matrix for free NN scattering, excluding corrections for the nuclear medium. It is important to measure other spin observable data in order to check the consistency.
of the conclusion based on only the analyzing power data. In this paper we investigate
the sensitivity of complete sets of polarization transfer observables to reductions of the
meson masses and meson-nucleon coupling constants varying from 0% to 20%: the vertically
hatched and dotted bands in Figs. 5, 6 and 7 represent the sensitivity of a particular
spin observable to reductions of coupling constants and meson masses ranging from 0% to
20% for both FR-DWIA and ZR-DWIA models, respectively.

FIG. 5: Polarization transfer observables plotted as a function of the kinetic energy, \( T_{a'} \), for
the knockout of protons from the 3s_{1/2} state in \(^{208}\text{Pb}\), at an incident energy of 202 MeV, and
for coincident coplanar scattering angles (28.0°, −54.6°). The vertically hatched band represents
the sensitivity of a particular FR-DWIA spin observables to a reduction of coupling constants
and meson masses ranging from 0% to 20% of the free values. The dotted band represents the
corresponding ZR-DWIA predictions.

For the knockout from all three states, we see that \( A_y \), P and \( D_{s's} \) are very sensitive to
reductions in the meson-coupling constants and meson masses. On the other hand, the spin
observables $D_{s'\ell}$ and $D_{\ell'\ell}$ exhibit minimal sensitivity to nuclear medium effects. Note that at the point corresponding to minimum recoil momentum, the spin observables $D_{nn}$, $D_{s'\ell}$ and $D_{\ell'\ell}$ are insensitive to nuclear medium effects for $3s_{1/2}$ knockout. On the other hand, for both $2d_{3/2}$ and $2d_{5/2}$ states, $D_{s'\ell}$ and $D_{\ell'\ell}$ also exhibit minimal sensitivity to nuclear medium corrections at minimum recoil. This is also the case for $D_{nn}$ and $D_{\ell'\ell}$ for the $2d_{5/2}$ state.

FIG. 6: Polarization transfer observables plotted as a function of the kinetic energy, $T_{a'}$, for the knockout of protons from the $2d_{3/2}$ state in $^{208}$Pb, at an incident energy of 202 MeV, and for coincident coplanar scattering angles $(28.0^\circ, -54.6^\circ)$. The vertically hatched band represents the sensitivity of a particular FR-DWIA spin observables to a reduction of meson-coupling constants and meson masses ranging from 0% to 20% of the free values. The dotted band represents the corresponding ZR-DWIA predictions.
FIG. 7: Polarization transfer observables plotted as a function of the kinetic energy, $T_{a'}$, for the knockout of protons from the $2d_{5/2}$ state in $^{208}$Pb, at an incident energy of 202 MeV, and for coincident coplanar scattering angles $(28.0^\circ, -54.6^\circ)$. The vertically hatched band represents the sensitivity of a particular FR-DWIA spin observables to a reduction of meson-coupling constants and meson masses ranging from 0% to 20% of the free values. The dotted band represents the corresponding ZR-DWIA predictions.

VI. SUMMARY AND CONCLUSIONS

In this paper we have exploited the discriminatory nature of complete sets of polarization transfer observables ($P, A_y, D_{nn}, D_{s's}, D_{s'\ell}, D_{\ell's}$ and $D_{\ell\ell}$) for exclusive ($\vec{p}, 2\vec{p}$) reactions to address a number of important physics issues. One of our aims was to identify specific observables which can yield information on whether the relativistic Dirac equation or the nonrelativistic Schrödinger equation is the more appropriate dynamical equation for the description of polarization phenomena within the framework of distorted wave impulse-
approximation models. In addition, we also studied the sensitivity of spin observables to nuclear distortion effects, finite-range (FR) versus zero-range (ZR) approximations of the relativistic DWIA, as well as to reductions of meson-nucleon coupling constants and meson masses by the surrounding nuclear medium in which the NN interaction occurs. In particular, we focused on proton knockout from the $3s_{1/2}$, $2d_{3/2}$ and $2d_{5/2}$ states in $^{208}$Pb, at an incident laboratory kinetic energy of 202 MeV, and for coincident coplanar scattering angles ($28.0^\circ, -54.6^\circ$). The motivation for choosing a heavy target nucleus, $^{208}$Pb, and a relatively low incident energy of 202 MeV is to maximize the influence of distortion effects as well as maximize differences between FR and ZR approximations to the relativistic DWIA, while still maintaining the validity of the impulse approximation, and also avoiding complications associated with the inclusion of recoil corrections in the relativistic Dirac equation. Another important consideration for our choice of reaction kinematics is the availability of analyzing power data to provide initial constraints on current distorted wave models. Unfortunately, there are no published data on other spin observables for the reaction kinematics of interest.

Previously, we established the clear superiority of relativistic DWIA models, compared to the nonrelativistic DWIA models, for describing exclusive ($\vec{p}, 2p$) analyzing powers. In this paper, we identify additional observables which display large differences to Dirac-versus Schrödinger-equation-based models and which need to be measured in order to check the consistency of the analyzing power predictions regarding the role of different dynamical models. In particular, for knockout from the $3s_{1/2}$ state the spin observables $A_y$, $D_{nn}$ and $D_{s's}$ exhibit large differences between Dirac and Schrödinger-based DWIA models. On the other hand, for knockout from the $2d_{3/2}$ state the most sensitive observables to dynamical differences are $D_{nn}$, $D_{\ell'\ell'}$ and $D_{\ell'\ell'}$, whereas for $2d_{5/2}$ knockout the corresponding observables are $D_{s's}$ and $D_{\ell'\ell'}$.

Regarding observables which display large differences between FR and ZR approximations to the relativistic DWIA, we see that for knockout from the $3s_{1/2}$ state, the induced polarization, $P$, is the most sensitive, whereas $D_{\ell'\ell'}$ and $D_{s'\ell}$ are the most sensitive observables for $2d_{3/2}$ and $2d_{5/2}$ knockout, respectively.

We have also established that all polarization transfer observables are relatively insensitive to different global Dirac optical potential parameter sets. In addition, by comparing relativistic DWIA predictions to corresponding plane wave predictions, we also demonstrated the importance of distorting potentials for describing the oscillatory behavior of spin ob-
servables thus refuting, for the first time, qualitative arguments that spin observables are insensitive to nuclear distortion effects.

We have also shown that the analyzing power data alone are unable to establish whether the nuclear medium does indeed reduce meson-nucleon coupling constants and meson masses: on one hand, the relativistic ZR predictions suggest that the scattering matrix for NN scattering in the nuclear medium is adequately represented by the corresponding matrix for free NN scattering. On the other hand, the relativistic FR results suggest that a 10% to 20% reduction of meson-nucleon coupling constants and meson masses by the nuclear medium is essential for providing a consistent description of the $3s_{1/2}$, $2d_{3/2}$ and $2d_{5/2}$ analyzing powers. In this paper we studied the sensitivity of the other polarization transfer observables to reductions in these parameters varying between 0% and 20%. For the knockout from all three states we see that $A_y$, $P$ and $D_{s's}$ are very sensitive to reductions in the meson-coupling constants and meson masses.

We also established a number of interesting model predictions for spin observables at the kinematic point corresponding to minimum recoil momentum. For $3s_{1/2}$ knockout the relativistic plane wave, ZR-DWIA and nonrelativistic DWIA predictions are virtually identical for the induced polarization ($P$), $D_{s'\ell}$ and $D_{\ell's}$; ZR-DWIA and FR-DWIA predictions are nearly identical for both $D_{nn}$ and $D_{s'\ell}$. For knockout from the $2d_{3/2}$ state both FR-DWIA and ZR-DWIA yield similar results for $P$, $D_{s's}$ and $D_{s'\ell}$; $P$ and $D_{s's}$ are insensitive to relativistic nuclear distortion at the point in question. We also observe that for $2d_{5/2}$ knockout, relativistic plane wave, FR-DWIA and nonrelativistic DWIA predictions are virtually identical for $D_{s'\ell}$ and $D_{\ell's}$. Regarding the influence of nuclear medium effects, the spin observables $D_{nn}$, $D_{s'\ell}$ and $D_{\ell'\ell}$ are insensitive for $3s_{1/2}$ knockout. On the other hand, for the $2d_{3/2}$ and $2d_{5/2}$ states, $D_{s'\ell}$ and $D_{\ell'\ell}$ also exhibit minimal sensitivity to nuclear medium corrections at minimum recoil. This is also the case for $D_{nn}$ and $D_{\ell'\ell}$ for the $2d_{5/2}$ state. Hence, by measuring spin observables at minimum recoil momentum one can eliminate differences between different dynamical models and model parameters and focus on a specific issue of interest.

Once again, we stress the urgent need for experimental data on polarization observables, in addition to the commonly-measured analyzing power, in order to resolve issues concerning the role of relativistic versus nonrelativistic dynamics in nuclear physics, as well as study the influence of the nuclear medium on the strong interaction.
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