Abstract

The gauge-fixed action of a ‘spacetime-filling’ D3-brane with dilaton-axion coupling is formulated in N=1 superspace. We investigate its symmetries by paying special attention to a possible non-linearly realized extra supersymmetry, and emphasize the need of a linear superfield coupled to an abelian Chern-Simons superfield to represent a dilaton-axion supermultiplet in the off-shell manifestly supersymmetric approach.
1 Introduction

The supersymmetric D-brane actions with local fermionic kappa symmetry were constructed in ref. [1]. When the kappa-symmetry is fixed, half of supersymmetry is spontaneously broken, whereas the fermionic superpartner (with respect to unbroken half of supersymmetry) of the $U(1)$ gauge field in the D-brane worldvolume can be identified with the Goldstone fermion. The most relevant part of the gauge-fixed D-brane action is given by a supersymmetric Born-Infeld (BI) action [1]. Gauge-fixing results in the D-brane actions whose all supersymmetries are non-linearly realized, i.e. non-manifest. Unbroken supersymmetries can sometimes be made manifest by using superspace [2, 3].

The electric-magnetic self-duality of the BI action can be extended to a full $SL(2, \mathbb{Z})$ duality in the case of a gauge-fixed ‘spacetime-filling’ D3-brane with axion-dilaton coupling [4]. This feature can be made manifest when considering the D3-brane action as the double dimensionally reduced M5-brane action on a 2-torus [5]. The dilaton-axion can be identified with the complex structure of the torus, while the $SL(2, \mathbb{Z})$ self-duality of a D3-brane is then nothing but the modular group of the torus [5]. In this Letter we make manifest the unbroken N=1 supersymmetry of the spacetime-filling D3-brane action with dilaton-axion coupling, and investigate its other relevant symmetries in flat N=1 superspace.

2 N=1 BI action in superspace

In this section we briefly describe the N=1 BI action is superspace, which is the prerequisite to our investigation in sect. 3. The BI action in Minkowski spacetime of signature $\eta = \text{diag}(+, -, -, -)$ is [6]

$$S_{\text{BI}} = \frac{1}{\kappa^2} \int d^4x \sqrt{-\det(\eta_{mn} + \kappa F_{mn})}, \quad (1)$$

where $F_{mn} = \partial_mA_n - \partial_nA_m$, $m,n = 0,1,2,3$, and $\kappa$ is the dimensional coupling constant ($\kappa = 2\pi\alpha'$ in string theory). The N=1 supersymmetric extension of the action (1) can be interpreted as the Goldstone-Maxwell action associated with partial $(1/2)$ spontaneous supersymmetry breaking, N=2 to N=1, whose Goldstone fermion is photino of a Maxwell (vector) N=1 multiplet with respect to unbroken N=1 supersymmetry [2, 3]. Manifest supersymmetry does not respect the standard form (1) of the BI action. The complex bosonic variable, having the most natural supersymmetric
extention, is given by
\[ \omega = \alpha + i\beta , \quad \alpha = \frac{1}{4} F_{mn} F_{mn} , \quad \beta = \frac{1}{4} \tilde{F}_{mn} \tilde{F}_{mn} , \quad \tilde{F}_{mn} = \frac{1}{2} \epsilon^{mnpq} F_{pq} . \] (2)

The BI Lagrangian (1) can be rewritten in terms of \( \omega \) and \( \bar{\omega} \) as
\[ L_{BI}(\omega, \bar{\omega}) = L_{\text{free}} + L_{\text{int}}, \]
where the particular structure function \( Y(\omega, \bar{\omega}) \) has been introduced,
\[ Y(\omega, \bar{\omega}) = \frac{1}{1 + \frac{\kappa^2}{2} (\omega + \bar{\omega}) + \sqrt{1 + \kappa^2 (\omega + \bar{\omega}) + \frac{\kappa^2}{4} (\omega - \bar{\omega})^2}} . \] (4)

A supersymmetrization of the bosonic BI theory (1) in the form (3) amounts to replacing the field strength \( F_{mn} \) by the N=1 chiral spinor superfield strength \( \tilde{W}_\alpha \), and \( \omega \) by the N=1 chiral scalar superfield \( K = \frac{1}{8} \bar{D}^2 \bar{W}^2 \), viz.
\[ S_{sBI} = \frac{1}{4} \left( \int d^4 x d^2 \theta W^2 + \text{h.c.} \right) + \frac{\kappa^2}{8} \int d^4 x d^4 \theta W^2 \bar{W}^2 Y(K, \bar{K}) \] (5)
with the same structure function (4), so that the bosonic terms of eq. (5) exactly reproduce eq. (1). We use the standard notation, \( W^2 = W^\alpha W_\alpha \) and \( \bar{W}^2 = \bar{W}^\alpha \bar{W}_\alpha \), and similarly for the N=1 flat superspace covariant derivatives \( D^\alpha \) and \( \bar{D}_\dot{\alpha} \) with \( \alpha = 1, 2 \) and \( \dot{\alpha} = \dot{1}, \dot{2} \). The gauge superfield strength \( W_\alpha \) obeys the superfield Bianchi identities
\[ D_\dot{\alpha} W_\alpha = 0 \quad \text{and} \quad D^\alpha W_\alpha = D^\alpha W_\dot{\alpha} . \] (6)

In the chiral basis the gauge superfield strength reads
\[ W_\alpha(x, \theta) = -i \psi_\alpha(x) + \left[ \delta_\alpha^\beta D(x) - i (\sigma^{mn})_\alpha^\beta F_{mn}(x) \right] \theta_\beta + \theta^2 (\sigma^m \partial_m)_{\alpha\beta} \bar{\psi}_\beta(x) , \] (7)
where \( \psi_\alpha(x) \) is the fermionic superpartner (photino) of the abelian BI vector field \( A_m \), and \( D \) is the real auxiliary field. In the N=1 super-BI theory (5) setting \( D = 0 \) is consistent with its equations of motion (this is called the ‘auxiliary freedom’ [7]).

The action (5) can be put into the simple ‘non-linear sigma-model’ form [2, 3]
\[ S_{sBI} = \int d^4 x d^2 \theta X + \text{h.c.} , \] (8)
whose chiral superfield Lagrangian \( X \) is determined via the recursive relation [2, 3]
\[ X + \frac{\kappa^2}{4} X \bar{D}^2 \bar{X} = \frac{1}{4} W^\alpha W_\alpha . \] (9)

The BI action (1) is well-known to be invariant under non-trivial electric-magnetic duality [8]. This means that treating \( F \) as a generic two-form, enforcing the Bianchi
identity, \( dF = 0 \), by means of a Lagrange multiplier (= dual vector potential) in the first-order action, and integrating out \( F \) in favor of the Lagrange multiplier yield the dual action having the same form as eq. (1) in terms of the dual vector potential. The same is true in N=1 superspace for the action (5) when introducing the dual N=1 superfield strength as an N=1 Lagrange multiplier, and integrating over \( W \) in the corresponding first-order action, i.e. after the N=1 superfield Legendre transform [3].

Another highly non-trivial property of eq. (5) is its invariance under the (non-linearly realized and spontaneously broken) second N=1 supersymmetry with rigid spinor parameter \( \eta_\alpha \) [2],

\[
\delta_\eta W_\alpha = \frac{1}{\kappa} \eta_\alpha + \frac{\kappa}{4} D^2 \bar{X} \eta_\alpha + i \kappa (\sigma^m \bar{\eta})_\alpha \partial_m X .
\] (10)
The transformations (10) are consistent with the N=1 Bianchi identities (6), and they realize a supersymmetry algebra. The invariance of the action (5) under the transformations (10) follows from

\[
\delta_\eta X = \frac{1}{\kappa} W^\alpha \eta_\alpha = \text{total derivative}.
\]

To make manifest the hidden second supersymmetry of the the N=1 BI theory, one can reformulate it in the formalism of non-linear realizations [9]. The Goldstone superfield \( \Psi \) having the standard transformation law in the chiral version of the non-linearly realized N=2 supersymmetry [10], \( \delta_\eta \Psi = \frac{1}{\kappa} \eta - 2i \kappa (\Psi \sigma^m \bar{\eta}) \partial_m \Psi \), is given by

\[
\Psi_\alpha = \frac{W_\alpha}{1 + \frac{\kappa^2}{4} D^2 \bar{X}} + \ldots ,
\] (11)
where the dots stand for the higher-order fermionic terms [9]. The new Goldstone superfield \( \Psi \) obeys the non-linear N=1 superspace constraints

\[
\mathcal{D}^\alpha \Psi_\alpha = \mathcal{D}^\alpha \bar{\Psi} = 0
\] (12)
that are also covariant under the second non-linearly realized supersymmetry. The N=2 covariant derivatives in N=1 superspace [2]

\[
\mathcal{D}_\alpha = D_\alpha + i \kappa^2 (D_\alpha \Psi \sigma^m \bar{\Psi} + D_\alpha \bar{\Psi} \bar{\sigma}^m \Psi) D_m \quad \text{and} \quad D_m = (\omega^{-1})_m^n \partial_n ,
\] (13)
where \( \omega_m^n = \delta_m^n - i \kappa^2 (\partial_m \Psi \sigma^n \bar{\Psi} + \partial_m \bar{\Psi} \bar{\sigma}^n \Psi) \), form a closed algebra. The action (5) may be rewritten in terms of \( \Psi \) and the N=2 covariant derivatives (13) as

\[
S_{sBI} = \frac{1}{4} \int d^4 x d^2 \theta \mathcal{E}^{-1} \Psi^2 + \text{h.c.} ,
\] (14)
whose N=1 chiral superfield \( \mathcal{E}^{-1} = 1 + \frac{\kappa^2}{4} D^2 \bar{X} + \ldots \), should transform as a density under the second supersymmetry, \( \delta_\eta \mathcal{E}^{-1} = -2i \kappa \partial_m (\mathcal{E}^{-1} \Psi \sigma^m \bar{\eta}) \).
Both the electric-magnetic self-duality and the second non-linearly realized supersymmetry of the N=1 BI action may have been expected from its anticipated connection to the D3-brane action. It is just these key properties that allow one to identify the N=1 BI action with the low-energy effective action of the spacetime-filling D3-brane in the case of slowly varying fields. Any direct gauge-fixing of the kappa-symmetric D3-brane action [1] would yield highly involved supersymmetry transformations, whose precise relation to the standard N=1 superspace transformations implies complicated field redefinitions. We didn’t attempt to establish this connection explicitly.

3 N=1 BI action with dilaton-axion coupling

The bosonic BI action coupled to a background dilaton $\phi$ and axion $C$ reads

$$S_{\text{bosonic}} = \frac{1}{4\pi} \int d^4x \sqrt{-\det(\eta_{mn} + e^{-\phi/2} F_{mn})} + \frac{1}{32\pi} \epsilon^{mnpq} C F_{mn} F_{pq} .$$

(15)

The dilaton-axion background now plays the role of the effective coupling constant, so that we chose $\kappa = 1$ for simplicity. We also rescaled the BI action by a factor of $4\pi$, in order to make it invariant under the T-duality transformations, $C \to C + n$, where $n \in \mathbb{Z}$, because $C$ multiplies the topological density in eq. (15).

It is not difficult to supersymmetrize eq. (15) in N=1 superspace, by using the results of sect. 2. First, let’s define a complex scalar

$$\rho = e^{-\phi} + iC ,$$

(16)

and assume that it belongs to an N=1 chiral superfield,

$$\Phi = \rho + \theta^\alpha \lambda_\alpha + \theta^2 F ,$$

(17)

where we have introduced the physical dilatino $\lambda_\alpha$ and the ‘auxiliary’ field $F$. This is not quite innocent procedure in the theories with higher derivatives, because the field $F$ should be truly auxiliary or, at least, $F = 0$ should be a solution to the equations of motion (the auxiliary freedom). Equation (5) implies the N=1 supersymmetric extension of eq. (15) in the form

$$4\pi S = \frac{1}{4} \left( \int d^4xd^2\theta \Phi W^2 + \text{h.c.} \right)$$

$$+ \frac{1}{32} \int d^4xd^4\theta (\Phi + \bar{\Phi})^2 W^2 \bar{W}^2 \mathcal{V} \left( \frac{1}{2}(\Phi + \bar{\Phi}) K, \frac{1}{2}(\Phi + \bar{\Phi}) \bar{K} \right) ,$$

(18)
with \textit{the same} function $\mathcal{Y}$ defined by eq. (4) at $\kappa = 1$, where we have used the identity
\begin{equation}
D^2 W^2 - \bar{D}^2 \bar{W}^2 = i \varepsilon^{mnpq} F_{mn} F_{pq} .
\end{equation}

The N=1 Legendre transform of the action (18) with respect to the gauge superfield $W$ yields the dual N=1 superspace action that has \textit{the same} form (18) in terms of the dual N=1 superfield strength \textit{and} the dual coupling
\begin{equation}
\tilde{\Phi} = \frac{1}{\Phi} .
\end{equation}
Together with imaginary shifts of $\Phi$ by integers the S-duality transformation (20) generates the full $SL(2, \mathbb{Z})$ duality, as required. In fact, the action (18) is invariant under the continuous $SL(2, \mathbb{R})$ duality, as it belongs to the class of the $SL(2, \mathbb{R})$ duality invariant actions constructed in ref. [11]. Of course, in quantum theory only $SL(2, \mathbb{Z})$ survives.

The $SL(2, \mathbb{R})$ duality invariant dilaton and axion kinetic terms to be added to eq. (18),
\begin{equation}
\mathcal{L}(\phi, C) = \frac{1}{2} (\partial_m \phi)^2 + \frac{1}{2} e^{2\phi} (\partial_m C)^2 ,
\end{equation}
are given by the Kähler non-linear sigma-model with a Kähler potential
\begin{equation}
K(S, \bar{S}) = - \ln (S + \bar{S}) .
\end{equation}
The N=1 supersymmetrization of eq. (21) in superspace is straightforward,
\begin{equation}
S_{\text{kin.}} = - \int d^4 x d^4 \theta \ln (S + \bar{S}) .
\end{equation}

There is, however, a problem with another (non-linearly realized) supersymmetry. A variation of the leading terms in eq. (18) yields
\begin{equation}
\delta_\eta \mathcal{L} = \frac{1}{2} \int d^2 \theta \, \Phi W^\alpha \eta_\alpha + \text{h.c.} ,
\end{equation}
which is a total derivative only for a \textit{constant} dilaton-axion background $\Phi$. Yet another problem is the auxiliary freedom of $F$.

The way out of both problems may be the assignment of dilaton and axion to an N=1 \textit{linear} multiplet $G$, instead of the N=1 chiral multiplet $\Phi$. As regards the bosonic action (15), this means trading $C$ against a gauge two-form $B$, at the expense of giving up the manifest $U(1)$ gauge invariance, viz.
\begin{equation}
\int CF \wedge F = - \int dC \wedge (A \wedge F) = \int ^* dB \wedge \Theta ,
\end{equation}
where the star denotes the Poincaré dual, \( * (dC) = dB \) and \( \Theta = A \wedge F \) is the abelian Chern-Simons three-form. In N=1 superspace a real linear superfield \( G \) is defined by the constraints

\[
D^2 G = \bar{D}^2 G = 0.
\]

(26)

It consists of a real scalar (dilaton), an antisymmetric tensor \( (B) \) subject to the gauge transformation \( \delta B = d\xi \) with the one-form gauge parameter \( \xi \), a dilatino \( \lambda \), and no auxiliary fields. The two-form \( B \) enters the superfield \( G \) only via its field strength \( dB \).

The leading term in eq. (18) can then be rewritten to the form

\[
\frac{1}{4} \left( \int d^4 x d^2 \theta \Phi W^2 + \text{h.c.} \right) = \frac{1}{4} \int d^4 x d^4 \theta (\Phi + \bar{\Phi}) \Omega,
\]

(27)

where we have introduced the Chern-Simons superfield \( \Omega \) via the equations

\[
W^2 = \frac{1}{2} \bar{D}^2 \Omega, \quad \bar{W}^2 = \frac{1}{2} D^2 \Omega.
\]

(28)

By using a solution \( W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V \) to the Bianchi identities (6), in terms of the real gauge scalar superfield \( V \) subject to the gauge transformations \( V \to V + i(\Lambda - \bar{\Lambda}) \), with \( \bar{D}_\alpha \Lambda = 0 \), we easily find \( \Omega = -\frac{1}{4} (D^\alpha V) W_\alpha + \text{h.c.} \).

The full action given by a sum of eqs. (18) and (23) is now dependent upon the chiral superfields \( \Phi \) and \( \bar{\Phi} \) only through their linear combination \( \frac{1}{2} (\Phi + \bar{\Phi}) \), so that it is possible to dualize this action in terms of the linear superfield \( G \) by Legendre transform. \(^2\) We replace in eqs. (18) and (23) the combination \( \frac{1}{2} (\Phi + \bar{\Phi}) \) by a general real superfield \( U \), and add extra term

\[
\int d^4 x d^4 \theta U G
\]

(29)

to the action (18). On the one hand side, varying eq. (26) with respect to \( G \) (in fact, with respect to a potential \( J_\alpha \) in the general solution \( G = D^\alpha \bar{D}^2 J_\alpha + \bar{D}_\alpha D^2 \bar{J}^\alpha \) to the defining constraints (26)), we get \( U = \frac{1}{2} (\Phi + \bar{\Phi}) \) back. On the other hand side, varying with respect to \( U \) in the action

\[
S = \int d^4 x d^4 \theta \left[ -\ln U + U G + \frac{1}{8\pi} U \Omega + \frac{1}{32\pi} U^2 W^2 \bar{W}^2 Y(UK, U\bar{K}) \right]
\]

(30)

we find an algebraic equation on \( U \):

\[
\frac{1}{U} = \left( G + \frac{1}{8\pi} \Omega \right) + \frac{1}{32\pi} W^2 \bar{W}^2 \left( 2U Y(UK, U\bar{K}) + U^2 \frac{\partial Y(UK, U\bar{K})}{\partial U} \right).
\]

(31)

\(^2\)The possibility of such transformation was noticed in ref. [11].

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Since $W_\alpha W_\beta W_\gamma = 0$ due to the anti-commutativity of $W_\alpha$, the second term on the right-hand-side of recursive relation (31) can be considered as an ‘exact’ perturbation. This leads to a complete solution to eq. (31) in the form

$$U^{-1} = G_{\text{mod}} + \frac{1}{32\pi} W^2 \bar{W}^2 \left( \frac{2 \mathcal{Y}(G_{\text{mod}}^{-1} K, G_{\text{mod}}^{-1} \bar{K})}{G_{\text{mod}}} - \frac{\partial \mathcal{Y}(G_{\text{mod}}^{-1} K, G_{\text{mod}}^{-1} \bar{K})}{\partial G_{\text{mod}}} \right),$$  

where we have introduced the ‘modified’ N=1 linear multiplet $G_{\text{mod}}$ as

$$G_{\text{mod}} = G + \frac{1}{8\pi} \Omega. \quad (33)$$

The appearance of the N=1 Chern-Simons superfield $\Omega$ is quite natural from the point of view of string theory and D-branes, where Chern-Simons-type couplings (in components) are known to appear in the famous Green-Schwarz anomaly cancellation mechanism and in the (dual) D-brane actions. In particular, the dilaton superfield $G$ must transform under the $U(1)$ gauge transformations as

$$\delta G = \frac{i}{32\pi} (D^\alpha \Lambda) W_\alpha + \text{h.c.} \quad (34)$$

in order to make $G_{\text{mod}}$ gauge-invariant. Equations (26) and (28) lead to the manifestly gauge-invariant constraints on $G_{\text{mod}},$

$$\bar{D}^2 G_{\text{mod}} = \frac{1}{4\pi} W^2, \quad D^2 G_{\text{mod}} = \frac{1}{4\pi} \bar{W}^2. \quad (35)$$

Such couplings were extensively studied in superspace (see e.g., ref. [12] for a recent review), while the relevant superspace geometry appears to be closely related to a three-form N=1 multiplet introduced in ref. [13].

Substituting the solution (32) into the action (30) yields the dual action in the form

$$S = \int d^4 x d^4 \theta \left\{ \ln G_{\text{mod}} + \frac{1}{32\pi} W^2 \bar{W}^2 G_{\text{mod}}^{-2} \mathcal{Y}(G_{\text{mod}}^{-1} K, G_{\text{mod}}^{-1} \bar{K}) \right\}. \quad (36)$$

The existence of another non-linearly realized and spontaneously broken supersymmetry with the transformation law $\delta W_\alpha = \eta_\alpha + \ldots$ implies a non-trivial transformation law of $G_{\text{mod}}$ as well, because of the constraint (35),

$$\delta_\eta G_{\text{mod}} = -\frac{1}{8\pi} (\eta^\alpha D_\alpha V + \bar{\eta}_\dot{\alpha} \bar{D}^\dot{\alpha} \dot{V}) + \ldots. \quad (37)$$

The natural (minimal) manifestly N=2 covariant version of the constraints (35) is given by

$$\bar{D}^2 \tilde{G}_{\text{mod}} = \frac{1}{4\pi} \Psi^2, \quad D^2 \tilde{G}_{\text{mod}} = \frac{1}{4\pi} \bar{\Psi}^2, \quad (38)$$

where we have substituted the Maxwell-Goldstone N=1 superfield $W$ by the N=1 Goldstone superfield $\Psi$, and the N=1 linear (dilaton-axion) superfield $G_{\text{mod}}$ by its
fully covariant counterpart $\tilde{G}_{\text{mod}}$. The superfield $W$ obeys the ‘canonical’ constraints (6) but it has the complicated transformation law (10), whereas the $\mathbb{N}=1$ Goldstone superfield $\Psi$ has the ‘canonical’ transformation law under the second supersymmetry but it obeys the complicated constraints (12). The same remarks also apply to $G_{\text{mod}}$ and $\tilde{G}_{\text{mod}}$, respectively.

The defining constraints (38) on $\tilde{G}_{\text{mod}}$ are apparently consistent with the constraints (12) due to the identities

$$\mathcal{D}_\alpha \mathcal{D}_\beta \mathcal{D}_\gamma = \bar{\mathcal{D}}_\alpha \bar{\mathcal{D}}_\beta \bar{\mathcal{D}}_\gamma = 0 \quad (39)$$

that follow from the definitions (13). It may be useful to rewrite the action (36) into the covariant form

$$S = \int d^4x d^4\theta E^{-1} \ln \tilde{G}_{\text{mod}} \quad (40)$$

with respect to the hidden (extra) supersymmetry, where we have introduced a density $E^{-1}(\Psi, \bar{\Psi}, \tilde{G}_{\text{mod}})$ in the full $\mathbb{N}=1$ superspace.

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