Entropy and similarity in galaxy systems

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The Birmingham-CfA cluster scaling project - III: entropy and similarity in galaxy systems

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ABSTRACT

We examine profiles and scaling properties of the entropy of the intergalactic gas in a sample of 66 virialized systems, ranging in mass from single elliptical galaxies to rich clusters, for which we have resolved X-ray temperature profiles. Some of the properties we derive appear to be inconsistent with any of the models put forward to explain the breaking of self-similarity in the baryon content of clusters. In particular, the entropy profiles, scaled to the virial radius, are broadly similar in form across the sample, apart from a normalization factor which differs from the simple self-similar scaling with temperature. Low mass systems do not show the large isentropic cores predicted by preheating models, and the high entropy excesses reported at large radii in groups by ?? are confirmed, and found to extend even to moderately rich clusters. We discuss the implications of these results for the evolutionary history of the hot gas in clusters, and suggest that preheating may affect the entropy of intracluster gas primarily by reducing the density of material accreting into groups and clusters along cosmic filaments.

Key words: galaxies: clusters: general – intergalactic medium – X-rays: galaxies: clusters

1 INTRODUCTION

In the widely accepted standard model for cosmic structure formation, the Universe evolves hierarchically, as primordial density fluctuations, amplified by gravity, collapse and merge to form progressively larger systems. This hierarchical development leads to the prediction of self-similar scalings between systems of different masses and at different epochs. These scalings are also seen in cosmological simulations involving only gravitationally driven evolution, including compression and shock heating of the baryonic matter. Such simulations (??, e.g.,) show that the density profiles of both dark matter and baryonic material, when radially scaled to the virial radius (which we define here to be the radius \(R_{200}\), within which the mean density of a system is 200 times the critical density of the Universe) are almost identical in virialized systems covering a wide range of masses, from individual galaxies to rich clusters.

Given self-similar scalings of gas temperature and density, scaled X-ray surface brightness profiles are also expected to be similar. Furthermore, a simple scaling is expected between X-ray luminosity, \(L_X\), and temperature \(T\). Assuming that the emission is dominated by bremsstrahlung, \(L_X \propto M_{\text{gas}}^2 R_{200}^{-3} T^{1/2}\), or \(L_X \propto f_{\text{gas}} T^2\), where \(M_{\text{gas}}\) is the gas mass within radius \(R\), and \(f_{\text{gas}} = M_{\text{gas}}/M\) is the gas mass fraction. X-ray properties of clusters deviate substantially from this simple scaling, and the observed \(L_X:T\) relation (??) is considerably steeper than \(T^2\) in the cluster regime, and steepens further (??) in galaxy groups. It showed that the latter effect is due to the suppression of \(L_X\) in galaxy groups, arising from a reduction in gas density in the inner regions of poor systems, relative to richer clusters.

It is instructive to view this in terms of the entropy of the intergalactic medium (IGM), which in the self-similar case should increase in a very simple scaling with the mean temperature of virialized systems. In practice, it is found (??) that an excess in the entropy, above the self-similar prediction, is apparent in the inner regions of poor clusters and groups, outside the dense central regions in which cooling is expected to radiate away entropy within the age of the Uni-

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verse. This effect has been referred to as the ‘entropy floor’, with the implication that additional physical processes, beyond gravity and resulting compression and shock heating, have acted to set a lower limit to the entropy which the gas in collapsed haloes can have.

A great deal of theoretical work has been devoted to explaining this phenomenon over the past few years. As we will discuss in some detail later, the explanations proposed fall into three main classes: the gas has been heated either at an early epoch, before clusters were assembled, or it has been heated in situ by star formation and/or energy input from active galactic nuclei (AGN). Alternatively, some authors have argued that cooling alone will remove low entropy gas from the centres of haloes, producing a very similar effect to non-gravitational heating.

In the present paper we aim to confront these models with the observed properties of the hot gas in a large sample of galaxy systems, spanning a wide range in total mass. In the fullness of time, high quality X-ray observations of the density and temperature structure of the IGM will be available from XMM-Newton and Chandra. However, at present, such observations are sparse, and it is essential to have a broadly representative and wide-ranging sample of virialized systems in order to study scaling properties. The value of this in the context of similarity-breaking in clusters has already been shown by a number of earlier studies.

In the present, and companion papers, ? (Paper I) and ? (Paper II), we examine scaling properties derived from the largest sample of virialized systems with resolved X-ray temperature profiles yet assembled. Following a brief description of the sample and our analysis in section 2 (details are given in Paper I), we present the profiles and scaling properties for the entropy and temperature across our sample in sections 3 and 4. These results are used in section 5, along with relevant results from Papers I and II, to test the various models proposed to account for the entropy floor, and finally in section 6 we draw our conclusions from this study, and propose a new model to explain the behaviour of the entropy of intracluster gas.

2 SAMPLE AND ANALYSIS

Our sample comprises 66 virialized systems, from rich clusters of galaxies, through groups and down to the level of individual galaxy-sized haloes. In ?)[hereafter san03]san03, we reported a detailed study of the 3-dimensional X-ray properties of this sample, based on data from the ROSAT and ASCA observatories, which we assembled from the work of three separate investigators (?????????), combined with a number of cool groups analysed specially to provide better coverage of the crucial low end of the mass range. To each system we fitted analytical functions, describing the gas density and temperature variation with radius, outside any central cooling region, which was excised or fitted separately. This approach allows us to put the X-ray data from the three earlier studies on a unified footing, and gives us the freedom to extrapolate the gas properties to arbitrary radius. We used a beta model to parametrize the density and specified the temperature variation with either a linear ramp or a polytropic IGM. We have used these data to determine the gravitating mass profile and thus to calculate radii of overdensity in a self-consistent manner. Similarly, we have derived mean temperatures for each system, by averaging the gas temperature within 0.3R200, weighted by its luminosity san03.

3 ENTROPY AND TEMPERATURE DISTRIBUTIONS

For convenience, we define ‘entropy’ in the present paper as

$$S = T/n_e^{2/3} \text{ keV cm}^2,$$

(1)

which relates directly to observations. This has been referred to by a number of authors as the ‘adiabat’, since (apart from a constant relating to mean particle mass) it is the coefficient relating pressure and density in the adiabatic relationship $P = K \rho^{\gamma}$. Hence $S$ is conserved in any adiabatic process. Note that the true thermodynamic entropy is related to our definition via a logarithm and additive constant.

In Fig. 1, we overlay the scaled entropy profiles for all 66 systems in our sample. Under the assumption that all these systems form at the same redshift, their mean mass densities should be identical. Hence in the simple self-similar case, where all have similar profiles and identical gas fractions, $S$ will simply scale with mean system temperature $T$. We apply this scaling, and scale the radial coordinate to $R_{200}$ for each system, derived from our fitted models (Paper I). It can be seen that the entropy profiles of the cooler systems, scaled in this way, tend to be significantly higher than those of richer clusters. To see this more clearly, in the right panel of Fig. 1, we show the profiles grouped into bands of mean system temperature. In this grouped plot we have excluded the two galaxies in our sample, whose profiles may be dominated by stellar wind losses, rather than by the processes operating in groups and clusters.

It can be seen that there is a strong tendency for the scaled entropy to be higher, at a given scaled radius, in cooler systems. Simulations and analytical models of cluster formation involving only gravity and shock heating, produce entropy profiles with logarithmic slopes of approximately 1.1 (?), which agrees rather well with the slope of our profiles outside 0.1R200. The mean profile in the 2.9–4.6 keV band is significantly affected by two systems, visible in the left hand panel, which have strongly rising profiles. One of these is AWM7, for which the temperature profile is subject to especially large systematic uncertainties, as a result of the strong cooling flow, as discussed in Paper I. The temperature model adopted, from the analysis of ?), with a temperature profile that is subject to especially large systematic uncertainties, as a result of the strong cooling flow, as discussed in Paper I. The temperature model adopted, from the analysis of ?), rises strongly with radius. As a result, the slope of $S(r)$ in this band is almost certainly overestimated in the figure.

One possibility which we can discard right away, is that the observed scaling results from systematic differences in the formation epochs of low and high mass systems. In hierarchical models, low mass systems are expected to form, on average, earlier than high mass ones. Hence they should tend to have higher mean densities, and therefore lower gas entropy – the opposite of what we observe. It is true that, due to the usual selection effects, the rarer and more luminous massive systems in our sample tend to be at higher redshifts than the groups. Hence if, contrary to expectations, all systems virialized at the redshift we observe them, then the more massive systems would be more dense, and have lower
entropies. However, this effect is considerably smaller than the trend which we observe. The highest redshift clusters in our sample are at $z < 0.2$, hence their mean densities, scaling as $(1+z)^3$, could be 70% higher than the lowest redshift systems, and their entropies (scaling as $\propto T^2$) consequently lower by a factor of 1.44. All but four members of our sample lie at $z < 0.1$, and hence would have entropies changed by less than 20% as a result of such a redshift-dependent density scaling. In contrast, Fig. 1 shows that the scaled entropy actually differs by a factor 3 between the high and low temperature bins for our sample.

At small radii, our fitted models exclude the effects of cooled gas, since any central cooling region is excised, or represented by a separate component in our analysis (sect. 2). This central cooling region is present in 54 of our 66 systems (see Table 1 in Paper I), and in these cases its radius ($r_{\text{cool}}$) ranges from 0.03$r_{200}$ to 0.2$r_{200}$, with a median value of $r_{\text{cool}} = 0.06r_{200}$. Apart from the coolest systems, our entropy profiles generally flatten inside 0.1$r_{200}$. This effect is also seen in many cosmological simulations (7) even in the absence of non-gravitational heating and cooling processes, and appears to result from the introduction of a core into the gas density distribution, due to transfer of energy between baryonic and dark matter during merger events (7). Preheating models generally predict large isentropic central regions in low mass systems, which are not seen in our data. We will return to this point in section 5 below.

The preheating model of (7) predicts that entropy should scale according to $(1 + T/T_0)^2$, rather than $T$, where $T_0$ is a constant related to the degree of preheating, which they estimate as $T_0 = 2$ keV to provide a best fit to the entropy floor data of (7). In Fig. 2, we show the effect of this scaling on our temperature-grouped entropy profiles. This scaling does indeed bring the profiles into good agreement, apart from the fact that our coolest systems show little sign of any central entropy core. Note from the left hand panel in Fig. 1, that this is a general feature of almost all the entropy profiles for cool systems, rather than a result of averaging together systems with cores and others with strongly dropping central entropies.

It is also instructive to compare temperature profiles across the range of masses in our sample. We use the virial radii and masses calculated from our fitted models to compute the virial temperature

$$T_{200} = \frac{GM\mu m_p}{2R_{200}},$$

expressed in keV. This is used to normalize each temperature profile, and the scaled profiles are grouped in temperature bands, to reduce the large scatter and make trends more obvious. The result is shown in Fig. 3. As with the grouped entropy plots, we have excluded the two galaxies from these profiles. We have also omitted AWM7 from the 2.9–4.6 keV band, since its highly uncertain (see discussion above) and strongly rising $T(r)$ profile distorts the mean profile for the whole band.

Raising the entropy of the IGM results in increased temperatures as well as lower densities, and for a given level of entropy increase this effect will be most prominent in low mass systems, where the ‘natural’ shock-generated entropy is lower. As can be seen, our observations do indeed show such an effect at large radii – at $R_{200}$ the scaled temperatures are ordered in precisely this way. However, at smaller radii, this is not the case, and in particular, the coolest systems display very flat temperature profiles, whilst the most massive clusters show the largest rise in temperature between $R_{200}$ and the centre. This behaviour is not what is predicted by most models, as we will discuss later.

### 4 SCALING PROPERTIES

The claim of the discovery of an entropy floor in galaxy systems (7) was based on the measurement of gas entropy at a scaled radius of 0.1$r_{200}$, in systems spanning a wide temperature range. This radius was chosen to lie close to the centre,
where shock-generated entropy should be a minimum, hence maximising the sensitivity to any additional entropy, whilst lying outside the region where the cooling time is less than the age of the Universe, and hence the entropy may be reduced. This initial study was improved by (Paper II) to explore the interpretation of a relation which followed the self-similar line down from hot systems, flattening towards a temperature floor value of $139 h_{50}^{-1/3} \text{ keV cm}^2$ from a sample of 20 systems, which is essentially a subset of the present work.

In Fig. 4, we show the corresponding plot from our much larger sample. With the benefit of this increase in sample size, the trend looks rather different from its appearance in (Paper I), where a dearth of systems in the 1.5–3.5 keV band led to the interpretation of a relation which followed the self-similar line down from hot systems, flattening towards a floor value of $139 h_{50}^{-1/3} \text{ keV cm}^2$, corresponding, with our value of $H_0$, to $124 h_{70}^{-1/3} \text{ keV cm}^2$. However, it appears from our data that the behaviour is rather that of a slope in $S(T)$ which is significantly shallower than the self-similar relation $S \propto T^{2/3}$. The trend looks rather different from its appearance in (Paper I), where a dearth of systems in the 1.5–3.5 keV band led to the interpretation of a relation which followed the self-similar line down from hot systems, flattening towards a floor value of $139 h_{50}^{-1/3} \text{ keV cm}^2$, corresponding, with our value of $H_0$, to $124 h_{70}^{-1/3} \text{ keV cm}^2$. However, it appears from our data that the behaviour is rather that of a slope in $S(T)$ which is significantly shallower than the self-similar relation $S \propto T^{2/3}$.

Using unweighted orthogonal regression (Paper I), we obtain a relation $S(0.1R_{200}) \propto T^{0.65 \pm 0.05}$, which is marked in Fig. 4. The $? h_{50}^{-1/3}$ floor value lies at the bottom of this trend, but it is not clear whether it sets a lower limit to the entropy in galaxy groups, since no groups with $T \lesssim 0.6 \text{ keV}$ have been bright enough for detailed study to date. The two galaxies in our sample do clearly have entropies which lie below the floor level, however much or all of the hot gas in these systems may have its origin in stellar mass loss, rather than retained primordial material (Paper II), so it may well be fortuitous that they fall close to the trend set by the groups and clusters.

Grouping the points into temperature bins (Fig. 5) makes the unbroken nature of the trend very clear. An unweighted orthogonal fit to these grouped points gives a logarithmic slope of $0.57 \pm 0.04$, slightly flatter than that derived from Fig. 4. Note that these results suggest the presence of excess entropy of $\sim 100 \text{ keV cm}^2$ in all temperature bands, relative to the hottest systems. Fig. 5 also compares the effect on the relation of calculating $R_{200}$ from a measurement of the mass profile, compared to assuming a scaling $R_{200} \propto T^{1/2}$, as was done (employing the formula derived from simulations by (Paper I)) by (Paper II) and (Paper I). As discussed in Paper I, the $? h_{50}^{-1/3}$ formula agrees reasonably well with our measurements in rich clusters, but in cool systems, the upward bias in temperature, relative to simulations such as those of (Paper I), which include only gravitational physics, causes the $R_{200} \propto T^{1/2}$ scaling to overestimate $R_{200}$ leading in turn to an upward bias in the entropy (since $S$ rises with radius). This may have contributed, in previous studies, to the appearance of flattening in the $S(T)$ relation towards low $T$.

Whilst measurements close to the cluster centre provide the most sensitive probe of excess entropy, detection of additional entropy at large radii is especially interesting, since many preheating models predict that the rise in entropy is essentially restricted to those central regions where shock-generated entropy is less than the floor value set by preheating. (Paper I) were the first to find evidence for excess entropy at a much larger radius, $R_{300}$, corresponding (Paper II) to $\sim 2.3 R_{200}$. They argued that the high excess entropy observed at large radii in groups and poor clusters indicates that their IGM is dominated throughout by the effects of preheating, with shocks playing little or no role. We aim, in this paper, to check this result with our larger sample.

Our data, shown in Fig. 6, confirm the existence of substantial excess entropy at $R_{300}$, above a self-similar extrapolation from the values seen in rich clusters (dotted line). The trend is more clearly seen in temperature-grouped data.
shown in the right hand panel, and the logarithmic slope of \( S(T) \) is similar to that seen at 0.1\( R_{200} \). The departure from self-similar entropy values is apparent not only in the coolest systems, but also in quite rich clusters. In fact, the largest absolute values of excess entropy (\( \sim 1000 \text{ keV cm}^2 \)) are seen in clusters with \( T \sim 3-4 \text{ keV} \).

For direct comparison with \( ? \), we show in Fig. 7 the entropy at \( R_{200} \) scaled by \( M_{200}^{-2/3} \), and grouped into mass bins to suppress fluctuations. For a set of self-similar systems, virialising at the same epoch (and hence having the same mean density), this scaling should renormalize the entropy to a constant value, independent of system temperature (since \( S \propto T \propto M^{2/3} \)). Clearly real clusters do not follow this self-similar law. Whilst our plot is broadly consistent in shape with Fig. 3 of \( ? \), our larger sample again reveals important additional features. \( ? \) concluded that excess entropy was only present in systems with \( M_{200} \lesssim 10^{14} h^{-1}_{70} M_\odot \) (\( 7 \times 10^{13} M_\odot \) for our choice of \( H_0 \)), whereas it is clear from our data that a trend in scaled entropy is present across the full mass range of clusters and groups.

\( ? \) also examined the entropy at a fixed value of enclosed gravitating mass (\( 3 \times 10^{13} h^{-1}_{70} M_\odot \)) over their sample of systems. The infall velocity of gas into an accretion shock should be similar to the free fall velocity at the shock radius, which depends upon the enclosed mass and mean density of the Universe at the epoch in question. In clusters, an enclosed mass of \( 3 \times 10^{13} h^{-1}_{70} M_\odot \) lies deep within the system, whilst in small groups it can lie close to \( R_{200} \). Since cluster cores are generally assembled at higher redshifts than the outskirts of groups, one expects the gas density in the shell under consideration to be higher, and the entropy generated by the thermalised kinetic energy is therefore lower, by a factor \( (1+z_f) \), where \( z_f \) is the redshift at which the shell was accreted. In practice, \( ? \) found the entropy of this shell to be bimodal across their sample, with a typical value of \( \sim 300 \text{ keV cm}^2 \) for systems with \( T \lesssim 3 \text{ keV} \), and with considerably lower values (scattered over the range 100-300 \text{ keV cm}^2) in hotter systems. Such a distribution cannot be accounted for on the basis of a \( 1+z_f \) scaling, and \( ? \) suggested instead that the entropy in cool systems is set by preheating, taking place at \( z \sim 3 \), after many cluster cores had already collapsed.

The corresponding plot for our sample, which is almost double the size of that of \( ? \), is shown in Fig. 8. For direct comparison we have derived entropies at an enclosed mass of \( 2.14 \times 10^{13} h^{-1}_{70} M_\odot \), allowing for our different choice of Hubble constant. Note that in some of the most massive systems, the radius enclosing \( 2.14 \times 10^{13} h^{-1}_{70} M_\odot \) lies (just) within the cooling region, and hence our derived entropy value is effectively extrapolated, using the model for the non-cooling component (see section 3). This affects six of the clusters which contribute to the highest temperature bin in the right panel of Fig. 8, and a handful of cooler clusters. Our results look significantly different from those of \( ? \). The entropy appears to be non-monotonic, with a minimum value in systems with \( T \sim 3-4 \text{ keV} \). From a purely phenomenological perspective, this non-monotonic behaviour can be understood in terms of mass profiles which take the NFW form (3)

\[
\rho(r) = \frac{\rho_0}{(r/r_s)(1+r/r_s)^2},
\]

combined with entropy profiles which rise as \( S \propto r^{1+1} \), outside some flatter central core. In groups, the mass shell under consideration here lies well outside \( r_s \), where enclosed mass grows roughly linearly with radius.
Figure 4. Gas entropy at 0.1\(R_{200}\) as a function of system temperature. The diamonds represent the two galaxies. The solid line is the best fit to the barred points. The dotted line has a self-similar slope of 1 and is normalized to the mean of the hottest 8 clusters. The dashed line is the entropy floor of 124\(h_{70}^{-1/3}\) keV cm\(^2\) from ?).

Figure 5. Gas entropy at 0.1\(R_{200}\) as a function of system temperature, excluding the two galaxies and grouped to a minimum of 8 points per bin. Barred crosses are based our measured values of \(R_{200}\), whilst diamonds result from calculation of \(R_{200}\) from mean system temperature using the \(T^{1/2}\) scaling of ?). The dotted line shows the self-similar slope of 1, normalized to the 8 hottest clusters.
Figure 6. Entropy at $R_{500}$ as a function of system temperature for each of the 66 systems, with the galaxies marked as diamonds (left panel) and grouped to a minimum of 8 points per bin, excluding the galaxies (right panel). The dotted line shows the self-similar slope of 1, normalized to the 8 hottest clusters.

Figure 7. Gas entropy at $R_{500}$, normalized by $M_{500}^{2/3}$, as a function of the total mass within $R_{500}$ (excluding the two galaxies) and grouped to a minimum of 8 points per bin.

\begin{equation}
M(<r) \sim C r T_{200}^{3/2},
\end{equation}

so that the radius at which an enclosed mass of $2.14 \times 10^{13} h_{70}^{-1} M_\odot$ is achieved scales as

\begin{equation}
r_* \propto T_{200}^{-3/2}.
\end{equation}

Since the entropy at this radius scales as

\begin{equation}
S(r_*) \propto T_{200} r_*^{1.1},
\end{equation}

we expect

\begin{equation}
S(r_*) \propto T_{200}^{-0.65},
\end{equation}

which gives a reasonable match to the slope at $T < 3$ keV, as shown in the right hand panel of Fig. 8.

In more massive systems ($M_{200} > 2 \times 10^{14} h_{70}^{-1} M_\odot$, $T > 3$ keV), the mass shell we are studying moves inside the scale radius $r_*$, so that the enclosed mass grows as $r^2$. An analysis similar to that above then results in

\begin{equation}
S(r_*) \propto T_{200}^{0.175},
\end{equation}

provided that $S(r) \propto r^{1.1}$ at these small radii. However, we saw in Fig. 1 that for all but the coolest systems, $S(r)$ flattens within the cluster core, in which case the positive
slope of the $S(r_\ast)T$ relation will be steeper than 0.175, as is observed.

The above arguments seem able to explain the general form of the relation in Fig.8, using just the shape of the NFW profile, coupled with entropy profiles similar to those seen in simulations incorporating only gravity and shock heating. We therefore conclude that there may be no need to invoke modification of the entropy profiles by preheating to explain this particular result, despite the initially surprising, non-monotonic trend.

5 DISCUSSION

A substantial amount of theoretical and computational effort has been directed towards the problem of similarity-breaking in groups and clusters, especially over the past four years. We now compare the various models with the results above, and with additional information from the companion papers to this (Paper I and Paper II), to see how they fare.

5.1 Cooling models

A number of authors (?????) have suggested that it may be unnecessary to invoke additional heating to explain the entropy floor, since cooling will remove low entropy gas from clusters. The counterintuitive fact that cooling can have similar effects to heating in this regard was first noted by ?), who found, on the basis of 1D hydrodynamical simulations of cluster growth, that cooling acted to flatten the gas density profiles, especially in low mass systems, and to steepen the $L:T$ relation, but not sufficiently to match observations. More sophisticated 3D simulations of cluster evolution by ?) and ?) have shown that larger effects can be produced, depending upon the spatial resolution of the simulations. ?) show that their simulations have converged in this respect, although their cooling function does not incorporate emission from metal lines, which becomes very significant at $T < 2$ keV.

Cooling achieves its effect of breaking the similarity between low and high mass clusters, since, at a given gas density, the cooling time is shorter at lower temperatures. Hence a larger fraction of the hot baryons cool in groups, compared to richer clusters (?). ?) show that this is able to reproduce the cluster-group $L:T$ relation quite well, except that the predicted steepening in the relation falls at a slightly lower temperature than that observed. They also compare the predicted gas entropies at $0.1R_{200}$ with the results of ?) at $T < 4$ keV, and find reasonable consistency. However, agreement with the results from the present study (Figs. 4 and 5), which is superior to that of ?) in terms of sample size and allowance for non-isothermality, is much less good. ? find that $S(0.1R_{200})$ is raised in low temperature systems, but converges with the self-similar trend through hot clusters for systems with $T > 3$ keV. In contrast, our results show a relation which is flatter than the self-similar line across the full temperature range. ?) and ?) produce results in better agreement with our observations, but their analytical model is based on an assumed variation in star formation efficiency with system mass which is much stronger than that which we derive from the subsample of our systems for which we have optical luminosities (Paper II).

However, the most fundamental problem faced by cooling-only models, as many of their proponents have acknowledged, is that cosmological simulations without some form of effective ‘feedback’ of energy into the baryonic component as stars form, have a serious ‘overcooling’ problem, which has been recognised for many years. At high redshift, the Universe is dense, and a large fraction of the baryons cool and form stars within small collapsed haloes. For example, ?) find that in the largest systems in their simulation (with $T \sim 4$ keV) almost half of the baryons have cooled out of the hot phase, whilst in small groups over 80% of the gas has cooled. Unless this cooled gas does not form stars, the very high star formation efficiencies implied are far above those observed in clusters (c.f. discussion in Paper II) or inferred globally from the K-band luminosity function of galaxies (?). Furthermore, recent X-ray spectral observations from

Figure 8. Left: entropy at the radius enclosing a mass of $3 \times 10^{13} h^{-1}_{50} M_\odot$ as a function of system temperature (excluding the two galaxies). Right: the same data grouped into bins containing 8 points or more, with simple limiting trends, discussed in the text, marked for comparison.
5.2 Preheating models

Since less energy is required to raise the entropy of gas by a given amount when its density is lower, it is energetically favourable to ‘preheat’ gas before it is concentrated into a cluster or group potential. A comparison of the excess entropy required, with that potentially available from supernova explosions associated with star formation (see e.g. [val99]) shows that a high efficiency of heating of the intergalactic gas is required even in the most favourable circumstances – i.e. only a modest fraction of the supernova energy must be radiated away. However, such high efficiencies have been inferred on the basis of observations and modelling in the case of starburst winds (see e.g.), and so are not necessarily unreasonable during the epoch of galaxy formation.

A large reservoir of energy at high redshift is potentially available from the formation of massive black holes in galaxies (see e.g.) fab01. It seems likely that cooling rates have been seriously overestimated, for reasons which are still under very active debate, but may be related to the effects of feedback from AGN, which probably exist at the centres of all cooling flows.

XMM-Newton and Chandra have made it clear that our understanding of cooling gas in the local Universe, on scales ranging from clusters to elliptical galaxies, is deficient (see e.g.). fab01. It seems likely that cooling rates have been seriously overestimated, for reasons which are still under very active debate, but may be related to the effects of feedback from AGN, which probably exist at the centres of all cooling flows.

\( M_{200} < 8 \times 10^{13} M_{\odot} \) (corresponding to \( \sim 2 \) keV, from our \( M/T \) relation in Paper I) is entirely isentropic in these models, in strong conflict with our results (Fig. 1).

\( \) and (?) have attempted to explore the effects of shock heating on the preheated accreting gas, ? concentrate on the scaling properties of the gas entropy immediately inside the accretion shock. They find that this is expected to scale with mean system temperature according to \( (1+T/T_0) \), where \( T_0 \) is an adjustable parameter related to the initial adiabat on which the preheated gas lies. By choosing the preheating entropy to be \( 120 \) keV cm\(^2\), corresponding to \( T_0 = 2 \) keV, they obtain a good match to the \( S(0.1R_{200}):T \) plot of ?, though their curve is rather too concave in comparison with the trend seen in Fig. 5. In order to calculate the expected \( L/T \) relation from their model, ? make two key additional assumptions: the IGM is assumed to be isothermal, and entropy profiles, \( S(r/R_{200}) \), are assumed to have the same shape in all systems. Their resulting \( L/T \) model steepens rather too gently to match the group data (which show a very steep slope – e.g. \( L \propto T^{4.3} \) from ?)) entirely satisfactorily. The isothermal assumption is in conflict with our data (Fig. 3) and with the results of cosmological simulations, which generally show a decline in temperature by about a factor 3 from the inner regions to the virial radius (see e.g. [bor01]). A picture involving self-similar entropy profiles, with a scale factor given by \( (1+B/T_0) \), is in remarkably good agreement with our results, as shown in Fig 2. However, it should be noted that the (?) model predicts only the way in which entropy should scale just inside the shock – the self-similarity in the radial distribution of entropy inside the shock is an assumption, rather than an output of their model.

A more comprehensive treatment is provided by (?), who calculate entropy profiles based on solving the shock jump conditions over typical accretion histories, for systems spanning a range of final mass. They find that shock heating generates entropy profiles with a characteristic logarithmic slope \( S \propto r^{-1.1} \), outside a central isentropic core, which, for a given preheating level, occupies a larger fraction of \( R_{200} \) in lower mass systems. The slope of 1.1 agrees well with our results (Fig. 1), and with numerical simulations (see e.g. [bor01]). However, as can be seen from Fig. 1, we do not see the larger isentropic cores in \( S(r/R_{200}) \) which appear to be a feature of all preheating models.

?) explore the energetics and evolutionary history of preheating the IGM through outflows from radio loud, and broad absorption line quasars. Their conclusion is that AGN could provide the energy input required to account for the entropy floor. However, in the context of our results, their most interesting result is that the specific energy injected by AGN into the gas, is expected to be higher in low mass systems, for two reasons: the incidence of powerful AGN is expected to be higher in smaller clusters, and the fraction of their power expended in \( P \delta^2 \) work is larger in smaller systems. This appears to be contrary to our entropy scaling results, shown in Figs. 5 and 6, which show that at small radii the excess entropy is similar (\( \sim 100 \) keV cm\(^2\)) across
a wide range of system masses, whilst at larger radii it is actually highest in moderately rich \((T \sim 3-4\text{ keV})\) clusters.

5.3 Star formation models

A natural development of cooling models, which can help to address the overcooling problem, is the incorporation of feedback due to star formation. In numerical studies, the successful implementation of such a scheme is extremely challenging, and has been the Holy Grail of those engaged in simulations of galaxy formation for some years. 7) introduced an interesting perspective on this issue, in the context of similarity breaking in clusters, by noting that it makes rather little difference quite how effective feedback in is in heating the IGM, since any gas within virialized systems which has low entropy will have a short cooling time, and so is removed from its location near the centre of a dark halo either by dropping out of the hot phase, or being heated by star formation in its vicinity, such that it escapes to large radii. In particular, they noted that the entropy of gas with a cooling time equal to the age of the Universe is \(\sim 100\text{ keV cm}^{-2}\), in striking agreement with the entropy floor reported in groups and clusters. Since the cooling time scales as \(T^{1/2}n\) in systems with \(T > 2\text{ keV}\), where bremsstrahlung dominates cooling, whilst \(S = T/n^{2/3}\), it follows that a given cooling time is achieved in gas with an entropy which scales as

\[
S_{\text{cool}} \propto (t_{\text{cool}}T)^{2/3},
\]

so that the entropy floor generated in this way is expected to be higher in hotter systems (7). This feature may help to explain the flat trend in entropy with temperature apparent in Fig. 5 – note the similarity between the dependence of \(S_{\text{cool}}\) on \(T\), and the slope seen in Figs. 4,5 and 6.

This approach was developed further by 7), who constructed models in which the entropy distribution is either truncated below some critical entropy (corresponding to gas cooling out, or being ejected as a result of vigorous heating), or shifted by the addition of an entropy boost throughout (corresponding to preheating of the entire IGM). The two different prescriptions for entropy modification produce only subtle differences in observable properties: shifted entropy models have rather flatter gas density profiles at large radii in poor groups, and also slightly higher gas temperatures. These differences are not distinguishable with data of the quality used in the present study, and may well be too challenging even for future studies with XMM-Newton and Chandra. One impressive feature of these models is that they contain essentially no adjustable parameters, since the entropy threshold is set by equating the cooling time of gas to a Hubble time of 15 Gyr. However, the predicted \(L/T\) relation fails to steepen sufficiently at low temperatures to pass through the bulk of the galaxy group points, and the \(M/T\) relation, whilst steeper than the self-similar relation \((M \propto T^{1.5})\), is less steep than that derived from the present sample in Paper I.

One of the most recent numerical studies of cluster structure to incorporate the effects of both cooling and feedback is the work of 7). These simulations explore the effects of setting an entropy floor through instantaneous preheating at high redshift, and also through heating scaled to the expected supernova energy input in overdense regions through-out the evolutionary history of the Universe. The effects of radiative cooling are also included in one of the supernova heating runs. The main conclusions from this work, are that \(\geq 1\text{ keV per particle of energy input is required to give a reasonable match to the observed steepening of the }L/T\) relation. The authors point out that such a large energy boost appears to exceed what is expected from supernova input alone, even if a high heating efficiency of the IGM is assumed.

7) find that a somewhat larger suppression of \(L_X\) in groups is achieved by injecting the same amount of energy progressively, compared to global preheating of the IGM. Entropy profiles generally agree well with the 7) slope of 1.1, but are flattened in low mass systems by the effects of energy injection, as are gas density profiles. The gas temperature profile becomes more centrally peaked in low mass haloes subject to additional heating. The \(M/T\) relation is found to be little affected by heating, either in slope or normalization. However, cooling is found to decrease the normalization and steepen the slope of the relation, bringing it into better agreement with observations. In comparison to these results, we certainly observe flatter gas density profiles in cool systems (Paper I), and Fig. 2 provides some tentative evidence that the entropy slope may be rather shallower in lower temperature \((T < 3\text{ keV})\) systems. In the case of the \(M/T\) relation, 7) and 7) show that systematic variations in the concentration parameter of dark matter haloes with mass, may play a key role in steepening its slope, as discussed in Paper I.

6 CONCLUSIONS AND SUGGESTIONS

6.1 Existing models

Drawing the above results together, it seems that cooling-only models cannot provide a viable explanation for the similarity-breaking observed in clusters, unless one is willing to admit the presence of very large quantities of baryonic dark matter, which would have to dominate the baryon budget in groups.

Preheating models appear to have the generic property of generating large isentropic cores in low mass systems, in conflict with our observations. This seems to be an inescapable feature of simple preheating models in which the IGM is raised uniformly and ‘instantaneously’ to a high adiabat, since such gas can only be shocked when falling into potential wells deep enough for its motion to be supersonic. More complex preheating models may enable this problem to be circumvented, as we discuss below.

Models involving a mixture of cooling and star formation (or possibly AGN heating) appear more natural and promising. However, a number of features of our data appear to conflict with all models proposed to date:

(i) The very large entropy excesses seen at large radii (Figs. 6 and 7) are a surprise. Models tend to show entropy enhancement in the inner regions of low mass systems, with a normal shock-generated entropy profile re-establishing itself at larger radii.

(ii) Closely related to this, is the fact that the entropy profiles appear to be approximately self-similar apart from a normalization constant, and in particular, that larger isen-
tropic cores are not seen in galaxy groups. In fact, Fig. 1 shows that the lowest temperature systems actually have entropy profiles which appear to drop all the way into the centre, unlike hotter systems. The fact that the scaling suggested by (?) brings our profiles into good agreement (Fig. 2) is a puzzle, given that this scaling is really not justified by their model as a scale factor for entropy profiles.

(iii) The temperature profiles shown in Fig. 3, are also not quite what is expected from the models. Any mechanism which gives an entropy boost should produce the strongest results in low mass systems. Since a rise in entropy has to be coupled with the maintenance of hydrostatic equilibrium, the natural consequence is a rise in central temperature, coupled with a decrease in density. The density drop is certainly observed, and there are indications that the temperature has been raised in cool systems outside the core, but the temperature profiles in groups seem to lack the expected central cusp. This is related to the lack of an entropy core in groups referred to in point (ii), above.

(iv) The behaviour of the gas core radius, discussed in Paper II, is similarly unexpected. As a fraction of $R_{200}$, the gas core radius is approximately constant at $\sim 10\%$ over most of our temperature range, but falls dramatically, by an order of magnitude, in groups. The only study to predict this sort of behaviour is that of (?), who show that fitting a convex profile which steepens progressively, with a beta model, can lead to smaller fitted core radii as the outer radius of detection shrinks. In Paper I we reported tests with fits to real data for two clusters, truncated at different radii, which suggested that this effect is not dominating our fits. However, X-ray surface brightness profiles extending to larger fractions of $R_{200}$ for groups are required to definitively settle this issue.

6.2 New possibilities

What might these disagreements with the models be pointing towards? (?) argued that the large excess entropy at $R_{200}$ in groups indicates that the entropy profiles in cool systems are dominated throughout by the effects of preheating. Two features of our results make us uneasy about this conclusion. Firstly, these excesses are now seen (Fig. 7) to extend up to moderately rich clusters, and to involve entropies several times those expected in the IGM at the present epoch (?). Secondly, the slope of all our entropy profiles (Fig. 1) seem to scatter about the value predicted from shock heating. This is an uncomfortable coincidence if shock heating plays no role in establishing them. It is certainly possible to devise preheating scenarios which could reproduce any of our profiles, by varying the history of energy injection, and the density gradient of the gas into which this energy was injected, in the vicinity of systems of a given mass. However, such a solution would require a large amount of fine tuning.

If, on the contrary, we wish to retain shock heating as the basic mechanism responsible for generating the rising entropy profiles seen outside the core in both clusters and groups, how can this be achieved? The post-shock temperature is essentially determined by the infall velocity of gas into a halo, which could be reduced by pressure forces, but not increased above the free-fall value. Hence, the only way to raise the whole entropy profile in low mass systems, in the way implicit in Fig. 1, seems to be to reduce the density of the pre-shock gas accreting into lower mass systems, relative to that falling into rich clusters. In simple spherical collapse models, the matter turning around and accreting into haloes at a given epoch is expected to have a given overdensity. However, in reality, cosmological simulations show us that accretion is far from spherical, and that much of it takes place along filaments.

This change in topology does not in itself change the relationship between the mass of virialized systems and the density of the gas accreting into them, since structure formation is (almost) self-similar. Bigger systems have bigger filaments, and the mean density of the accretion flow at a given epoch is still set by the overdensity at turnaround, which is independent of system mass. However, this situation changes if preheating of gas in the filaments introduces an additional scale into the problem. In general, the scale height of gas in filaments will be set by hydrostatic equilibrium of the gas in the local gravitational potential of the dark matter (?, e.g.,) ?val02. The temperature to which gas is shock heated during the collapse of filaments will scale with their mass, and hence the effect of any thermal input into the IGM from galaxy formation or the growth of supermassive black holes will be more prominent in the smaller filaments associated with lower mass systems. Preheating will act to increase the scale height, and hence to reduce the density of gas in filaments. Even in filamentary structures which are still collapsing, preheating of the gas will increase its pressure, and retard its collapse relative to that of the dark matter.

The calculations of (?) show (albeit in the spherically symmetric case) that even in the preheated case, the accretion shock rapidly becomes strong once it is established, so that the post-shock entropy of gas accreting into a cluster will scale as $T_{200}/n_1^{2/3}$, where $n_1$ is the pre-shock density of the gas, and $T_{200}$ is the virial temperature of the cluster. Hence, if preheating reduces $n_1$ in lower mass systems, relative to rich clusters, then the entropy will scale sub-linearly with $T_{200}$, which is what we observe.

It is worth noting some features of this model. Firstly, the effect we invoke is intimately linked to the fact that the accreting gas is largely confined to filaments, and hence will not be seen in analytical and semi-analytical models, such as those of (?) and (?), which assume spherically symmetric accretion. In the spherical case, the gas has nowhere to expand to in response to the initial preheating. Consequently any entropy rise due to preheating serves to raise the temperature of the gas, rather than to lower its density. In the lowest mass systems, this temperature rise can delay shock formation, and hence result in an isentropic core, but in high mass systems (and at large radii in low mass ones) it has essentially no effect, since the post-shock temperature is just $T = \frac{\mu m_p v_i^2}{2}$ (?), where $v_i$ is the velocity at which gas, with mean particle mass $\mu m_p$, flows into the shock. In contrast, if preheating serves primarily to reduce the density of the pre-shock gas, then this will lead to a rise in post-shock entropy whatever the shock strength, since (for a given shock Mach number) the post-shock density simply scales with $n_1$, so that the post-shock entropy $S_2 \propto n_1^{-2/3}$.

Secondly, this mechanism is a very efficient way of raising the entropy of intracluster gas for two reasons. As has been noted previously (?) a given injection of energy produces a larger rise in entropy when the gas density is lower.

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Hence it is more effective to deposit energy into low overdensity structures such as filaments, than into fully collapsed systems like clusters. Moreover, if this injection places the gas onto a higher adiabat by reducing its density, then the effects of the accretion shock serve to raise it further. For example, if effects of the accretion shock serve to raise it further. For example, ?) show that the ratio between the post- and preshock adiabats is well approximated by

\[ S_2/S_1 = A(1 + \frac{8 \mu m_p v_r^2}{5kT_1}), \]  

(10)

where A is a constant of order unity. So, a rise in \( S_1 \) by some factor, due to a density change (and hence with no change in \( T_1 \)) driven by preheating, will boost \( S_2 \) by the same factor. Hence the shock has a multiplier effect, and if \( S_2/S_1 \) is large, a modest rise (in absolute terms) in \( S_1 \), may result in a much larger rise in \( S_2 \). Since the entropy will typically jump by an order of magnitude in the accretion shock, as its temperature is raised from \( \sim 10^6 \) K to \( 10^7-10^8 \) K, entropy excesses of the magnitude reported here within clusters (i.e. 100-1000 keV cm\(^2\)), might be generated from a rise in entropy of only \( \sim 10-100 \) keV cm\(^2\) within the filaments which feed them.

Thirdly, if preheating acts primarily to reduce the density of the pre-shock gas, rather than to increase its temperature, then the Mach number of accretion shocks is not much reduced, compared to the unheated case. The post-shock entropy profiles are therefore almost entirely due to the evolution of the accretion shock, and \( S(r) \propto r^{1.1} \) profiles are expected, outside a small unshocked central core, in good agreement with observations.

Whilst the viability of this model clearly cannot be investigated by spherically symmetric analytical treatments, one might hope to find evidence to test it from 3-dimensional simulations of cluster formation. Rather few studies have been published which incorporate the effects of feedback into the IGM, together with a study of its effects on the entropy profiles of clusters. The results from the work of ?) and ?), are not very encouraging. In both these studies, the effects of heating of the IGM serve to flatten the entropy profiles in clusters, with the effects being more pronounced in lower mass systems. This accords better with observations than do most 1-dimensional analytical models, insofar as distinct isentropic cores are not generally present. However, the progressive flattening of \( S(r) \) towards lower mass systems does not agree with our observed properties (Fig. 1).

It is premature to conclude at this stage that the preheated filament model can be rejected. Feedback is very poorly understood, and its incorporation into numerical codes is notoriously difficult. In the study of ?), feedback was included by simply raising the temperature of all gas particles by 1.5 keV at a redshift of 4. ?) adopted the approach of imposing an entropy floor on the gas in overdense (\( \delta > 5 \)) regions at a variety of redshifts from 1 to 5, whilst ?) attempted to model the effects of energy injection from galaxies in a more realistic way by using a semi-analytic scheme to predict the supernova rate, and sharing the resulting energy amongst gas particles in regions with \( \delta > 50 \). Unfortunately, the interplay of feedback and cooling is extremely complex, and hard to model. Results tend to depend strongly on spatial resolution, and current simulations (including those of Borgani et al. and Muanwong et al.) generally fail to reproduce the observed low cooled fraction of baryons. A wider exploration of the effects of different feedback schemes, and a study of the density of gas flowing into accretion shocks in simulations, should clarify whether the mechanism proposed here is actually the dominant effect in modifying the entropy of intracluster gas. At the same time, higher quality observations from \textit{XMM-Newton} and \textit{Chandra} will provide more robust determinations of observed entropy profiles in a wide range of virialized systems. Initial results from \textit{XMM-Newton} for a small number of groups (??) confirm that no central isentropic core is present in these systems.

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