The Fate of Non-Radiative Magnetized Accretion Flows: Magnetically-Frustrated Convection

Ue-Li Pen

Canadian Institute for Theoretical Astrophysics, University of Toronto, M5S 3H8, Canada; pen@cita.utoronto.ca

Christopher D. Matzner

Department of Astronomy and Astrophysics, University of Toronto, M5S 3H8, Canada; matzner@cita.utoronto.ca

Shingkwong Wong

Department of Physics, National Taiwan University; wshingkw@cita.utoronto.ca

ABSTRACT

We present a scenario for non-radiative accretion onto the supermassive black hole at the galactic center. Conducting MHD simulations with 1400^3 grid zones that break the axial and reflection symmetries of earlier investigations and extend inward from the Bondi radius, we find a quasi-hydrostatic radial density profile $\rho \propto r^{-0.72}$ with superadiabatic gradient corresponding to an $n \sim 0.72$ polytrope. Buoyancy generated by magnetic dissipation is resisted by the same fields so effectively that energy is advected inward: a state of magnetically-frustrated convection. This scenario is consistent with observational constraints on energetics and outer boundary conditions.

Subject headings: accretion – magnetohydrodynamics – black hole physics – outflows – galaxies: active – methods: numerical

1. Introduction

Stellar dynamical measurements indicate a black hole with mass $M_{\mathrm{BH}} = 2.4 \times 10^6 M_\odot$ (?) at the location of Sgr A* in the galactic center. Apart from its origin, a puzzling aspect of this object – and others like it in nearby galaxies – is its low X-ray luminosity, given the gaseous environment. High resolution X-ray imaging (?) shows hot gas with temperatures
\[ T = 2 \text{ keV} \text{ at densities near } n_e = 130 \text{cm}^{-3} \text{ within 1" of Sgr A*}. \text{ In } \eta')\text{’s theory, gas within the gravitational region of influence (Bondi radius)} \]

\[ r_B \equiv \frac{GM_{\text{BH}}}{c^2} \approx 0.03 \text{pc} \]  

(1)

falls inward, approaching free-fall. This scale is barely resolved by *Chandra* at the galactic center. The natural mass accretion rate is of order \( 4\pi\lambda r_B^2 \rho c_s \) if the background density is \( \rho; \lambda = 0.25 \) for a monatomic gas dominated by thermal (as opposed to magnetic) pressure. If matter is converted into radiation at an efficiency \( \eta \) by the black hole, the luminosity \( L \approx 2 \times 10^{40}(\eta/10\%) \text{ erg/s} \). Reported instead (\( ? \)) is a source, potentially the hole, with a luminosity of \( 2.4 \times 10^{33} \text{ erg/s} - 10^7 \) times fainter than this estimate. Even the advection of thermal energy across \( r_B \) at the Bondi rate would incur \( \sim 2 \times 10^{36} \text{ erg/s} \), exceeding the observations by three orders of magnitude. Because Bondi assumed spherical adiabatic flow of an ideal gas, many factors may be at work in this immense discrepancy.

First, the flow could be incredibly sporadic and we may have caught it in an off moment. However, the dynamical time \( r_B/c_s \) is only \( \sim 50 \) years, comparable to the history of radio observations and only a few times longer than the X-ray observations.

Second, it had been suggested (\( ?, \) as in the ADAF model of)\( ]1994\text{ApJ...}428\text{L..13N} \) that fluid can accrete at the Bondi rate without radiating \( (\eta \ll 10\%) \), which might be possible if electrons coupled to protons only by Coloumb collisions. However, observations of linear submillimeter polarization (? \( ) \) are interpreted (\( ? \)) to imply an accretion rate far below Bondi’s prediction. Moreover, Bondi-rate influx of electron thermal energy alone would exceed limits by \( \sim 10^{27} \).

Third, Bondi accretion passes through a sonic point only if the effective adiabatic index is smaller than \( 5/3 \). Gas with \( \gamma_{\text{eff}} = 5/3 \) accretes subsonically at all radii, and if \( \gamma_{\text{eff}} > 5/3 \) then quasi-hydrostatic settling flow is expected, becoming sonic just above the Schwarzschild radius \( r_{\text{Sch}} (?) \)\( 1978\text{AA}...70..583B \). Monatomic gas with \( \gamma = 5/3 \) has \( \gamma_{\text{eff}} > 5/3 \) if viscous or magnetic dissipation generates heat (which is not radiated) during inflow (?\( ]1978\text{ApJ...226.1041C,1981ApJ...246L..15S} \). Therefore, rapid inflow of the Bondi or ADAF type is unstable to motions that reverse the inflow of some fluid elements – potentially reducing \( \dot{M}_{\text{BH}} \) far below the Bondi estimate. The ADIOS model of (?) posits axial outflows cancelling the equatorial inflow. Similarly, the CDAF model of (?) invokes a rotating convecting atmosphere surrounding the hole. (? (2002, hereafter IN) and ?) discuss a similar, nonrotating, convecting flow (\( \S 3 \)).

Fourth and fifth, rotation and magnetic fields can both strongly affect the flow and interact with one another and with other physical effects in non-trivial ways.
If the angular frequency is $\Omega_B$ at the Bondi radius, conservation of angular momentum would bring it into orbit at a Kepler radius $r_K \simeq \Omega_B^2 r_B^3/c_s^2$. If cooling permits the formation of a thin disk, this signals the onset of efficient radiation ($\eta \simeq 10\%$ as used above). If angular momentum transport is weak, however, this can be a limiting step in accretion. \(^?\) has recently suggested a cold disk onto which the hot flow condenses, accreting sporadically with a long duty cycle. The disk of stars inferred by \(^?\) could have arisen this way. Recall, however, condensation onto this disk would exceed constraints if it emitted X-rays.

Similarly, magnetic fields can grow in strength as they are dragged inward by the flow. Accretion will shear embedded fields to become radial. An initial field, whether uniform or tangled \(^?\), will be stretched toward the split-monopole (hedgehog) configuration in which $B^2 \propto r^{-4}$ (e.g., IN), incurring a centrally divergent energy. Although radial fields exert no net force, this configuration is unstable and unattainable \(^?\)]. Instead, inflow can stall at a magnetic turnaround radius $r_{\text{mag}} \equiv GM/v_A^2(r_{\text{mag}}^2)$, where $v_A(r)$ is the Alfvén velocity. Mass inflow will be limited by the rate of magnetic energy dissipation via reconnection, by interchange instabilities, or by inflow along open field lines.

This contrasts with the widely-held picture of magnetic fields enhancing accretion through angular momentum transport, either via turbulent stresses \(^?\) or via magnetocentrifugal winds \(^?\). Also, if fields are generated locally through convection or magnetorotational instability, then they are limited in strength to partial equipartition with kinetic energy. Fields strengthened by inflow can reach equipartition with the gravitational potential, which can be a larger value (§2.1).

We pause here to note the critical importance of the density profile in the phenomenology and viability of models; see also \(^?\) and \(^?\). An atmosphere with $\rho \propto r^{-n}$ and, generically, $T \propto r^{-1}$, has a bolometric free-free luminosity $L \propto r^3 \rho^2 T^{1/2} \propto r^{(5-4n)/2}$ (or for a single frequency, $L_\nu \propto r^3 \rho^2 T^{-1/2} \propto r^{7/2-2n}$), so long as $T \gtrsim 3$ keV. Thus profiles with $n > 5/4$ (or $n > 7/4$) suffer a singularity in the bolometric (or single-frequency) luminosity. If $n < 5/4$, emission is dominated at outer radii; this is consistent with the small contrast in Chandra images.

Similarly, the thermal time varies as $T^{1/2}/\rho \propto r^{n-1/2}$ whereas the inflow time $r/v = 4\pi r^3 \rho/\dot{M}_{\text{BH}} \propto r^{3-n}$. Thus, flows with $n < 7/4$ are dynamically nonradiative. Finally, the local Bondi accretion rate varies as $r^2 \rho c_s \propto r^{3/2-n}$: flows with $n < 3/2$ accrete at $\dot{M} \simeq \dot{M}_B(R_B/R_{\text{Sch}})^{3/2-n}$.

Hydrostatic models are characterized by a polytropic index $n = 1/(\gamma_{\text{eff}} - 1)$, which describes the correlation between density and pressure $p \propto \rho^{\gamma_{\text{eff}}} = \rho^{1+1/n}$. In a Keplerian potential, $\rho \propto r^{-n}$ as used above. When $\gamma_{\text{eff}} = 5/3$, both hydrostatic and Bondi-like (e.g.,
ADAF) flows have $n = 3/2$, shallow enough to be advective but too steep to avoid the luminosity constraint. To force a shallower density profile requires additional pressure, either through rotation, magnetic fields, or an entropy inversion. The latter is convectively unstable.

CDAFs pass this test by achieving $n = 1/2$ in a combination of rotational, turbulent, and thermal support. However, the (?) model requires the inward angular momentum transport due to buoyant convection to exceed the outward transport due to magnetic fields. This may be possible for fields due solely to magnetorotational instability (?), but it is unlikely a property of field strengthened by shear in inflow.

More seriously, any model invoking rotational support is plagued by outer boundary conditions. Outside $r_B$ one expects solid-body rotation on radial shells, implying low specific angular momentum ($j$) near the axis. Sufficiently low-$j$ gas can fall directly within the innermost stable orbit; this comprises a fraction $\sim (r_S/r_K)^{1/2}$ of the total. If $j$ is somewhat higher, fluid falls inward to its own Kepler radius and shocks; the resulting pressure gradients drive a quadrupolar outflow of low-$j$ material along the equator. This has been observed in nonmagnetic simulations by (?) and by us. It could be avoided if the inner boundary supplied a strong jet that interfered with axial accretion, as occurs in protostars (?), but suggestions of such behavior in Chandra images are recent and tentative (?). Similarly, models that invoke rotational support to stem accretion require that rotational support remains important all the way out to $r_B$ – implying an asymmetry to the X-ray images which is not observed.

How can gas establish the flattened density profile required for low-luminosity accretion, while being fed low-angular-momentum, lightly magnetized ($\beta > 10$ for $B < 1$ mG) material at $r_B$? IN and (?) offer a solution to this conundrum (§3); our simulations suggest a different answer.

### 2. Simulations

In order to further explore the interaction between infall, rotation, magnetic fields, and buoyancy, we conduct MHD simulations with $1400^3$ zones arrayed in a uniform Cartesian grid, the largest MHD simulations to date. These were performed on the CITA McKenzie cluster: 512 Pentium-4 Xeon processors running at 2.4 GHz (?). At this resolution, each full dimensional sweep corresponding to two timesteps took 40 seconds. The code (?) is based on a 2nd order accurate (in space and time) high resolution Total-Variation-Diminishing (TVD) algorithm. It explicitly conserves the sum of kinetic, thermal and magnetic energy; hence magnetic dissipation (at the grid scale) heats gas directly. No explicit resistivity or viscosity is added, and reconnection and shocks occur through the solution of the flux conservation
laws and the TVD constraints. Magnetic flux is conserved to machine precision by storing fluxes perpendicular to each cell face.

Inner boundary conditions are imposed on a cube of width 24 grid cells. Interior to the largest inscribed sphere within this cube, gravitational forces are turned off. At the end of each time step magnetic fields in the cubical region are relaxed to the vacuum solution, permitting rapid reconnection in the interior zone. The Alfvén speed is matched to the circular speed at the surface of this region by the removal of matter, and the sound speed is matched to the same value by the adjustment of temperature. The pressure of this matter is always smaller than that of infalling material, and we never observe spurious outflows.

Whereas many prior simulations have started with a disklike, rotation dominated geometry and no low-$j$ material, we wished to investigate the potential role of axial infall. We attempt to separate generic physical effects from artefacts of boundary conditions. This is challenging, since the energy available at the center of the simulation is always larger than in any other region. Results may depend sensitively on the choice of inner boundary, and it is not feasible to simulate directly from $r_B$ to $r_{Sad}$. We search instead for scaling relations connecting these radii, bearing in mind the possibility of strong reactions such as outflows from an unresolved inner region.

For this reason, and since we are interested primarily in the effect of external parameters, most of our runs have worked inward from the Bondi radius. Simulations far inside $r_B$ elucidate local phenomena, but cannot easily be matched to their exterior environment. At each step, the outer 20 grid cells are replaced with the values from the initial conditions to enforce the continued inflow of new material. We avoid threading the outer boundary with any magnetic flux by adding flux lines to the fluid inside a region $1/2$–$3/4$ of the box size, with an admixture of random field loops and large loops that thread the whole box. (Most of the energy is in the coherent loops.) We maintain a mean value $\beta = 10$ in the flux generation region. To estimate $r_{mag}$, we assume $B \propto r^{-2}$ as in the split-monopole. This gives $r_{mag} \simeq 100$ grid cells. The estimate is not rigorous: fields are added in a cubical region; rotation changes the shearing rate, and $B \propto r^{-2}$ is not perfectly achieved.

To avoid immediate energetic feedback from the central regions due to the initial conditions, we start with an empty interior (an evacuated sphere of half the box radius), and let matter fall in. We then watch the evolution away from Bondi to draw conclusions about the fate of the flow. In grid units, box length is 1400, and $GM = 700$. We set $c_s(r = \infty) = 1$, and $\rho(r = \infty) = 1$. The Bondi radius was set at $r_B = 700$, touching the closest box edge. The simulations ran successfully for 6000 time steps to $t = 650$, or 1.5 free fall times from $r_B$. At that point, magnetic fields squeezed out along the midplane to the outer boundary, leading to numerical instabilities.
With a resolution of $10^{2.85}$ radial zones, of which $10^{1.51}$ is used for central and outer boundary conditions, we have 1.34 decades of scale in which to arrange the Kepler radius $r_K$ and the magnetic turnaround radius $r_{\text{mag}}$. We have performed runs that are initially rotationally supported ($r_{\text{mag}} < r_K$) and purely rotationally supported ones ($r_{\text{mag}} = 0$), as well as magnetized but nonrotating initial conditions ($r_K = 0$). The fluid is initialized with solid body rotation on shells, and constant specific angular momentum on cones of constant polar angle. The production simulation (fig. 1) chose $r_K = 336$, $r_{\text{mag}} = 100$. To further break any discrete symmetries, the velocity field was modulated at the 5% level at multipoles up to $l = 2$.

These simulations differ from previous ones in two important ways. First, magnetic and viscous dissipation occur only at the grid scale (or inner boundary); no enhanced diffusivity or viscosity is applied. Second, we break the alignment usually assumed between magnetic and rotational symmetry axes. This was accomplished by introducing large-scale flux loops, misaligned with respect to the rotational axis, that are dragged toward the central object.

2.1. Results

Simulations were run about one dynamical time at $r_B$. Although this does not give us any handle on the long-term evolution of the flow once its inner behavior begins to alter the conditions outside $r_B$, it does yield a picture of how the inner flow responds – for many inner dynamical times – to initial and boundary conditions. A snapshot of the resulting magnetic field structure is shown in 1, and various stresses balancing gravity are plotted versus radius in figure 2.
Fig. 1.— Magnetic field structure: midplane \((x - y)\) slice. Magnetic pressure is shown along with projected magnetic field vectors. The inner half of this picture corresponds to figure 2.
Fig. 2.— Run of supporting stresses (radial components of momentum fluxes), averaged on radial shells and compared to the local gravitational force per unit volume. Also plotted is the standard deviation of gravitational force per unit volume on radial shells.
At this point in the simulation, a central hydrostatic region supported by gas pressure – not rotation or magnetic fields – has grown outward. Within the region plotted in figure 2, $\rho \propto r^{-n}$ for $n \simeq 0.72$ also, $P \propto r^{-1.51}$ (so that $T \propto r^{-0.79}$, whereas $r^{-1}$ was expected), and magnetic pressure $P_{\text{mag}} \sim 10^{-1.5} P$. Gas pressure gradients dominate magnetic stresses by a factor $\sim 10$; Reynolds stresses, including rotation and inflow, are smaller still (and switch direction). Rotation is roughly one tenth the Kepler rate, despite that the entire region is a factor of two inside the initial $r_K$. The ratio of Alfvén to inflow velocities is similarly $\sim 10$, indicative of magnetic braking.

In contrast to Bondi and ADAF-type flows, inflow is very subsonic; correspondingly, the mass accretion rate is strongly suppressed relative to Bondi’s estimate. However, the accretion rate does agree with the Bondi rate derived from conditions at the inner boundary. This state is not rotationally-supported like the CDAF. It resembles in some ways the “CDBF” of IN and \(?\); but see §3 for differences.

To test the role of magnetic fields in the clogged inflow, we suddenly turned the magnetic fields off, and evolved the fluid for a dynamical time. The flow returned to Bondi’s solution.

3. Physical Interpretation

The fluid can be modelled as one in quasi-hydrostatic equilibrium with a polytropic index $\gamma_{\text{eff}} \simeq 2.25$. Since the adiabatic index is $\gamma = 5/3$, this represents a strongly superadiabatic state. In the usual description of entropy-driven convection, this can only occur when convective velocities approach the sound speed, which, in a power-law atmosphere, is roughly free-fall.

Saturation at a constant Mach number is a feature of \(?\)’s CDAF model and the CDBF model of IN, both of which have $n = 1/2$ and fall within class II of \(?\)’s classification of self-similar nonradiative flows. This slope is clearly flat enough to satisfy all of the observational constraints discussed in §1. The value of $n = 1/2$ derives from assuming a positive convective luminosity $L_{\text{conv}}$ – i.e., that the gravitational energy released bubbles out through the flow rather than being dragged inward (as it is in Bondi and ADAF flows). If $c_s$ scales with the Kepler speed $v_K$, then $L_{\text{conv}} \propto r^2 \rho v_K^3 M \propto r^{1/2-n} M$, where $M$ is the turbulent Mach number. Constancy of $L_{\text{conv}}$, required by steady state, implies $M \propto r^{n-1/2}$. If $n > 1/2$, $M$ rises outward until it saturates at $\sim 1$; thereafter, $n = 1/2$.

Our simulation shows $n = 0.72$ and mildly subsonic convective motions: $M \simeq 0.4$ – not increasing outward at $r^{0.22}$ as implied by the above argument. This has prompted us to calculate the total flow ($L_{\text{conv}}$) of magnetic, thermal, and gravitational energy. We find it to
be inward and small: \( L_{\text{conv}} \simeq -0.04 \times (4\pi r^2 \rho v_K^3) \).

How can subsonic convection persist in the presence of strong superadiabatic gradients? And, how can the net flow of energy be inward despite the existence of convective motions? Since the detailed treatment of magnetic field structures is the only physical effect distinguishing our final state from ordinary gaseous convection, the simulation results must represent a state of magnetically-frustrated convection in which magnetic shear stresses oppose buoyant motions. This is verified by the equipartition between magnetic stresses and buoyant stresses (variations of \( \rho g \) at fixed \( r \)) seen in figure 2. Correspondingly, kinetic energy is suppressed by a factor of three compared to the free energy available in density fluctuations (variations of \( \rho GM/r \) at fixed \( r \)). For further corroboration we note an anticorrelation between magnetic and buoyant stresses (cross-correlation coefficient -0.2) and also the relaxation to Bondi flow when fields were removed. An analogy to magnetic frustration is low-Reynolds-number convection, in which viscosity slows or halts buoyancy. (That said, energy inflow may be a feature of our inner boundary conditions – a question for further study.)

The coincidence in slope between gas and magnetic pressures most likely arises from the growth of magnetic fields to balance buoyancy: since buoyancy scales with gas pressure in the presence of a strong superadiabatic gradient, so must the magnetic stress.

One can discuss the plausible scenarios of the global gasdynamics of the Sgr A* region. In addition to the ambient gas, stellar winds may inject a larger mass flux into the region than would be lost by even ideal Bondi accretion \((?) \). If the natural accretion rate is sufficiently slow, the injected mass could conceivably push the ambient gas outwards. Similarly, the cooling time at the Bondi radius is approximately \( 10^5 \) years, and on this time scale matter has to either flow in, or be pushed out.

4. Conclusions

We have presented a new picture of the hot plasma in the vicinity of the galactic center black hole based on the results of new very large three dimensional MHD simulations. We find that the code produces results consistent with observational constraints, including boundary conditions and total luminosity. Fluid remains in quasi-static equilibrium supported primarily by thermal pressure, with a radial density profile \( \rho \propto r^{-0.72} \) in reasonably spherical symmetry. This implies a significant entropy inversion; we believe buoyancy is impeded by subdominant magnetic fields.

A distinguishing feature of the flow is that energy is advected inward, despite convection,
albeit quite slowly compared to the local estimate $4\pi r^2 \rho(r) v_K(r)^3$. The same is true of central regions of Bondi flow, where $\dot{E} = -\dot{M} c_s(r > t_B)^2/(\gamma - 1)$. Unlike Bondi, the $\dot{M}$ is also much slower than the dynamical accretion rate.

This feature provides a point of contrast with related suggestions for the Sgr A* flow: CDAF (\textsuperscript{?}) and CDBF (IN), both of which obey $\rho \propto r^{-1/2}$ and contain different levels of rotational support. The 1/2-law in these models is a consequence of a positive outward convective luminosity, plus the saturation of convective motions at constant Mach number (\textsuperscript{?}, see § 3 and)\textsuperscript{2001astro.ph..4113G}. However, the density profile is essentially unconstrained when $\dot{E}$ and $\dot{M}$ (as well as the angular momentum flux $\dot{J}$) are all effectively zero (\textsuperscript{?})his class-IV flow\textsuperscript{2001astro.ph..4113G}.

The influx of energy also provides a method of distinguishing magnetically-frustrated flows from the other models even if the central density profile cannot be probed. In CDAFs and CDBFs, energy must be convected outward and will heat gas outside the Bondi radius at a rate comparable to $\dot{M} c_s^2 \sim 10^3 L_\odot$. If present, effects of this heating on the hot gas surrounding the hole may be visible. On the other hand, winds from massive stars in the same volume may overwhelm inward advection.

(\textsuperscript{?}) criticize ADAFs on the basis that they possess a positive Bernoulli function ($B > 0$), and that this predisposes them to outflow (realized in their ADIOS model). Note however that $B$ is positive (though small) in Bondi flow, where it is set by the external pressure; however, no outflow develops. We find $B > 0$ in the magnetically-frustrated flow as well. We do observe upwelling, but it takes the form of an overall quadrupolar circulation with inflow along the axis and outflow along the equator. There is no indication that this upwelling becomes supersonic at any radius.

We speculate that non-radiative MHD flows are in general very inefficient at accreting. The corollary is that accretion only occurs when the cooling time is short.

This research was funded by NSERC, by the Canadian Foundation for Innovation, and by the Canada Research Chairs program (for CDM). We thank Phil Arras, David Ballantyne, Robert Fisher, Chris McKee, and an anonymous referee for comments.