Effective R-parity violation from supersymmetry breaking

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(Dated: June 4, 2005)

We present a scenario in which Yukawa-like R-parity violating (RPV) couplings are naturally suppressed. In our model, RPV is assumed to originate from the SUSY breaking mechanism and then transmitted into the SUSY Lagrangian only through soft SUSY breaking operators in the scalar potential. The RPV Yukawa-like operators of the superpotential, conventionally parametrized by the couplings \( \lambda, \lambda' \) and \( \lambda'' \), are then generated through loops containing the SUSY scalars, the gauginos and the soft RPV interactions and are, therefore, manifest as effective operators with a typical strength of \( \mathcal{O}(10^{-3}) \).

PACS numbers: 12.60.Jv, 11.30.Er, 14.80.Ly

One of the unresolved puzzles associated with supersymmetric (SUSY) theories is whether or not R-parity \((R_P)\) is conserved in the SUSY Lagrangian. On the one hand, a general SUSY theory allows the existence of RPV trilinear operators whose strength is naturally of \( \mathcal{O}(1) \). On the other hand, experimental searches for RPV interactions yield null results, which seem to indicate that such RPV couplings, if exist, are much smaller than \( \mathcal{O}(1) \) as long as the typical SUSY mass scale is not much larger than the electroweak scale. The question then arises: why are the RPV Yukawa-like interactions so suppressed wherever they can emerge?

In this paper we propose a way for generating RPV Yukawa-like effective operators, with a structure similar to the ones which may appear in the superpotential (i.e., the usual \( \lambda, \lambda' \) and \( \lambda'' \) type RPV operators) \( \textsuperscript{†} \), however, with a typical strength \( \ll \mathcal{O}(1) \). We assume that the superpotential conserves \( R_P \) and that RPV “leaks” into the SUSY Lagrangian through the mechanism that breaks supersymmetry. In particular, in this scenario, the only place where RPV interactions show up is in the scalar potential - in the form of three-scalar “soft” operators. Being “soft”, such scalar RPV operators cannot renormalize the Yukawa-like RPV operators and, so, \textit{the latter remain zero at all scales}. Such a non-vanishing low-energy RPV soft operator will, however, generate an effective RPV Yukawa-like interaction through loops involving the soft interactions (for a related work see \( \textsuperscript{2} \)). In general, one expects that these effective RPV interactions will show additional patterns (compared to the “regular” \( \lambda, \lambda' \) and \( \lambda'' \) ones) due to their explicit dependence on the particles that run in the loops.

To illustrate how the new effective RPV couplings are generated, assume that \( R_P \) is violated in the Lagrangian \textit{only} through the pure leptonic (lepton number violating) soft SUSY breaking operators:

\[
\Delta V_{\text{soft}}^{R_P, \Phi} = \epsilon_{ab} \frac{1}{2} a_{ijk} \hat{L}_i^a \hat{L}_j^b \hat{E}_k^c + \text{h.c.} , \quad (1)
\]

where \( \hat{L}(\hat{E}^c) \) are the scalar components of the leptonic SU(2) doublet(charged singlet) supermultiplets \( \hat{L}(\hat{E}^c) \), respectively, \( \hat{L} = (\hat{\nu}_L, \hat{\ell}_L) \) and \( \hat{E}^c = \hat{\nu}_R \). Also, \( a_{ijk} = -\bar{a}_{ijk} \), due to the SUSY(2) indices \( a, b \).

This obviously means that in our framework the superpotential conserves \( R_P \), i.e., \( \lambda, \lambda' \) and \( \lambda'' \) as well as the bilinear RPV terms \((\mu_L \hat{L} \hat{H}_u)\) are absent. For example, \( \lambda_{ijk} \rightarrow 0 \) in

\[
\Delta W_{R_P, \Phi} = \epsilon_{ab} \frac{1}{2} a_{ijk} \hat{L}_i^a \hat{L}_j^b \hat{E}_k^c + \text{h.c.} . \quad (2)
\]

In order to realize this scenario, let us suppose that SUSY breaking occurs spontaneously in a hidden sector at the intermediate scale \( \Lambda \sim 10^{10} - 10^{11} \text{ GeV} \), described by the \( R_P \)-conserving Fayet-I?Raiffaartha superpotential:

\[
W = m_{12} \hat{\Phi}_1 \hat{\Phi}_2 + g \hat{\Phi}_3 \left( \hat{\Phi}_2^2 - M^2 \right) , \quad (3)
\]

where \( m_{12} \sim M \sim \Lambda \) and under \( R_P \) the chiral superfields in \( \textsuperscript{3} \) transform as: \( \hat{\Phi}_1, \hat{\Phi}_2, \hat{\Phi}_3 \rightarrow -\hat{\Phi}_1, -\hat{\Phi}_2, \hat{\Phi}_3 \). SUSY breaking can then be triggered by the vacuum expectation values (VEV’s) of the auxiliary F-term \((F_{\Phi_i})\) of \( \hat{\Phi}_1, \hat{\Phi}_2, \hat{\Phi}_3 \). We choose the minimum of the potential in this model to lie at \( \Phi_{\Phi_1} = A_{\Phi_1} = 0, A_{\Phi_2} = M \sqrt{1 - m_{12}^2/(2g^2M^2)} \), which satisfies \( F_{\Phi_1} = m_{12} A_{\Phi_2}, F_{\Phi_2} = 0 \) and \( F_{\Phi_3} = g(A_{\Phi_2}^2 - M^2) \).

Supergravity mediation of SUSY breaking can then be parametrized by the following \( R_P \)-conserving superpotential:

\[
\frac{1}{M_{Pl}} \int d^2 \theta \left[ \hat{\Phi}_1 \hat{L} \hat{L} \hat{E}^c + \hat{\Phi}_2 \hat{L} \hat{L} \hat{E}^c \right] + \text{h.c.} , \quad (4)
\]

which will spontaneously break \( R_P \) and induce the soft operator in \( \textsuperscript{†} \) with \( a \sim \frac{F_{\Phi_2}}{M_{Pl}} \sim \frac{A_{\Phi_2}^2}{M_{Pl}} \sim m_W, \) for
\[ m_{12} \sim M \sim \Lambda. \] The superpotential in (1) will also generate the operators \( \propto \lambda \) in (2) with an extremely suppressed coupling: \( \lambda \propto \frac{A_{\Phi i}}{M_{P}} \sim \frac{\Lambda}{M_{P}} \sim 10^{-9} - 10^{-8} \) at the high scale, essentially causing the soft operator in (1) to be the only source for RPV in this model. Alternative mechanisms for the generation of RPV were suggested in (3).

In general, other operators which couple the hidden sector superfields \( \Phi_{1}, \Phi_{2}, \Phi_{3} \) to the observable sector can be constructed, which will generate an RPV \( \mu \)-like term as well as a soft bilinear term \( B_{L} \overline{L} H_{u} \) (for a somewhat similar example see e.g., (4)). One can always rotate away the \( \mu \) term at the high scale by a field redefinition. In the absence of the Yukawa-like trilinear terms, \( \mu \) will remain zero at all scales at the one loop order (see (4)). The \( B_{i} \)'s can also be rotated away at the high scale, however, even if they are zero at the high scale they will be radiatively (one-loop) generated by the non-zero soft trilinear \( \lambda \) term in (1) and evolved down to the electroweak (EW) scale through the RGE. Since, for \( \lambda \rightarrow 0 \) and \( \mu_{i} \rightarrow 0 \) the RGE for \( B_{i} \) takes a relatively simple form (4), \( B_{i}(M_{Z}) \) can be easily estimated. In the leading log we get:

\[ B_{i}(M_{Z}) \sim -\frac{1}{16\pi^{2}}h_{\tau}(M_{Z})\mu(M_{Z})a_{i33}(M_{Z}) \ln \left( \frac{M^{2}_{Z}}{M^{2}_{\chi_{n}}} \right), \]

where \( h_{\tau} \) is the \( \tau \) Yukawa coupling and \( \mu \) is the usual \( \mu \) term \( (\mu H_{u}H_{d}) \).

Let us now define the effective RPV terms which are generated at one-loop through the soft operator in (1) via diagrams of the type shown in Fig. 1 as follows:

\[ \mathcal{L}^{eff}_{RP} \equiv \frac{1}{2} \left( \frac{a_{ijk}}{16\pi^{2}} \right) \left[ \overline{L}_{i} \hat{c}_{k} (A_{L,ijk} L + A_{R,ijk} R) e_{j} + B_{L,ijk} \overline{\nu}_{i} \nu_{j} \right] + h.c., \] (6)

where \( i, j, k \) are generation indices and \( L(R) \equiv (1 - (+)\gamma_{5})/2. \) Note that the customary RPV operators in (2) are obtained by setting: \( A_{R,ijk} = B_{L,ijk} = 0, A_{L,ijk} = C_{L,ijk} = D_{L,ijk} = 1 \) GeV\(^{-1} \) and \( a_{ijk} = 16\pi^{2}\lambda_{ijk} \) GeV.

The particles running in the loop diagrams for each of the form factors in (2) are: (I) \( \hat{e}_{L,\hat{c}} R X_{n}^{0} \) for the \( \nu_{L,\hat{c}} e j \) term, where \( \chi_{n}^{0} \) are the four neutralinos \( (n = 1 - 4), \) (II) \( \hat{e}_{L,\hat{c}} R X_{m} \) for the \( \nu_{L,\hat{c}} \nu_{j} \) term, where \( \chi_{m} \) are the two charginos \( (m = 1, 2), \) (III) \( \hat{e}_{R,\hat{c}} L X_{n}^{0} \) for the \( \hat{e}_{R,\hat{c}} \nu_{j} \) term and (IV) both \( \hat{e}_{L,\hat{c}} \nu_{L} X_{n}^{0} \) and \( \hat{e}_{R,\hat{c}} \nu_{j} \) for the \( \hat{e}_{R,\hat{c}} \nu_{j} \) term.

Calculating these loop diagrams we find that \( B_{L,ijk} \propto m_{\nu}, \) i.e., no \( \nu \nu \nu \) term in the limit of zero neutrino masses. Also, \( A_{R,ijk} \ll A_{L,ijk} \) since \( A_{R,ijk} \) is proportional to the leptonic Yukawa couplings. Thus, neglecting terms which are proportional to the external lepton masses and to the leptonic Yukawa couplings we find

\[ A_{L,ijk} = \frac{e^{2}}{s_{W}c_{W}} \sum_{n=1}^{4} m_{\chi_{n}^{0}} c_{0}^{n(ijk)} Z_{N}^{1n} \left( Z_{N}^{1n} s_{W} + Z_{N}^{2n} c_{W} \right), \]

\[ C_{L,ijk} = -\frac{e^{2}}{s_{W}c_{W}} \sum_{n=1}^{4} m_{\chi_{n}^{0}} c_{0}^{n(ijk)} Z_{N}^{1n} \left( Z_{N}^{1n} s_{W} - Z_{N}^{2n} c_{W} \right), \]

\[ D_{L,ijk} = \frac{e^{2}}{2s_{W}^{2}c_{W}^{2}} \sum_{n=1}^{4} m_{\chi_{n}^{0}} c_{0}^{n(ijk)} \left( (Z_{N}^{1n})^{2} s_{W}^{2} - (Z_{N}^{2n})^{2} c_{W}^{2} \right) + \sum_{m=1}^{2} \frac{2c_{W}^{2}m_{\chi_{m}} c_{0}^{m(ijk)} Z_{1m}^{1m} Z_{1m}^{1m}}{Z_{N}^{1n} s_{W}^{2} + Z_{N}^{2n} c_{W}^{2}} \right), \] (7)

where \( Z_{N}^{ij}, Z_{ij}^{+}, Z_{ij}^{-} \) are the matrices that diagonal-ize the neutralino and chargino mass matrices, respect-
tively as defined in $\Box$, $s_W(c_W)$ is the sine(cosine) of the weak mixing angle $\theta_W$, and $c_0^{a(n,ijk)}$, $c_0^{c(n,ijk)}$, $c_0^{d(n,ijk)}$, $c_0^{d2(n,ijk)}$ are the three-point loop integrals defined via:

$$C_0 \left( m_1^2, m_2^2, m_3^2, p_1, p_2, p_3^2 \right) = \int \frac{d^4q}{i\pi^2} \frac{1}{(q^2 - m_1^2)(q^2 - m_2^2)(q - p_3)^2 - m_3^2}$$

and

$$C_0^{n(n,ijk)} = C_0 \left( m_0^2, m_{e_{kR}}^2, m_{e_{jL}}^2, m_{e_{kL}}^2, m_{e_{jL}}^2, m_{e_{jL}}^2 \right),$$

$$C_0^{c(n,ijk)} = C_0 \left( m_0^2, m_{e_{kR}}^2, m_{e_{jL}}^2, m_{e_{kL}}^2, m_{e_{jL}}^2, m_{e_{jL}}^2 \right),$$

$$C_0^{d(n,ijk)} = C_0 \left( m_0^2, m_{e_{kR}}^2, m_{e_{jL}}^2, m_{e_{kL}}^2, m_{e_{jL}}^2, m_{e_{jL}}^2 \right),$$

$$C_0^{d2(n,ijk)} = C_0 \left( m_{\nu_t}^2, m_{\nu_t}^2, m_{\nu_t}^2, m_{\nu_t}^2, m_{\nu_t}^2, m_{\nu_t}^2 \right).$$

Since the soft RPV trilinear $a$-term in $\Box$ generates the soft RPV bilinear term $B_i$ through the RGE, an effective $lll$ interaction ($l$ for lepton and $\tilde{l}$ for slepton) of the type $\Box$ can also arise from slepton-Higgs mixing ($\propto B_i/B_0$, where $B_0$ is the $R_P$-conserving soft bilinear term) followed by the Higgs couplings to leptons ($\propto h_{ij}^i$, where $h_{ij}^i$ is the Higgs-$l_j l_k$ Yukawa coupling). The contribution of this diagram to the $lll$ coupling is therefore $\propto h_{ij}^i \times B_i/B_0$, and can be easily estimated when all tree-level low energy SUSY mass parameters are assumed to be of the same size, i.e., $\sqrt{B_0(M_Z)} \sim \mu(M_Z) \sim a_{ijkl}(M_Z) \sim M_{SUSY}$. Thus, from $\Box$ it is clear that $h_{ij}^i \propto (B_i/B_0) \sim (h_{ij}^i h_{ij}^k/16\pi^2) \times \ln(M_{Pl}/M_2^2)$. For $j \neq k$ and for $j = k = 2$, this is negligible, while for $j = k = 3$ (couplings involving the $\tau$) we have $\tilde{u}_{ij3} \sim (h_{ij}^i/16\pi^2) \times \ln(M_{Pl}^2/M_2^2)$, which gives $\tilde{u}_{ij3} \sim 10^{-4}$ for $\tan^2\beta \sim O(10)$ and $\tilde{u}_{ij3} \sim 10^{-2}$ if $\tan^2\beta \sim O(1000)$. This effect will be studied in further detail in $\Box$.

In Table I we give a sample of our numerical results for the three effective RPV couplings in $\Box$, corresponding to the “Snowmass 2001” benchmark points SPS1, SPS2, SPS4 and SPS5 of the minimal SuperGRAVity (mSUGRA) scenario $\Box$. The ranges of values in Table I for each effective operator are due to slight differences in the slepton masses for different generations $i, j, k$. We see that the SPS1 and SPS5 scenarios give the largest effective couplings, of the order of $10^{-4} - 10^{-3}$ if $a_{ijkl} \sim 16\pi^2$ GeV $\sim 150$ GeV.

Let us also evaluate the effective RPV form factors $A_{L,ijkl}$, $C_{L,ijkl}$ and $D_{L,ijkl}$ in a low energy SUSY framework where the SUSY parameter space is defined at the electroweak scale. In particular, in the most general case $A_{L,ijkl}$, $C_{L,ijkl}$ and $D_{L,ijkl}$ depend on: $\tan\beta$, the $R_P$ conserving bilinear Higgs mass term $\mu$, the three gaugino mass parameters $M_1$, $M_2$ and $M_3 = m_{\tilde{g}}$ ($m_{\tilde{g}}$ is the gluino mass) and the slepton masses. For simplicity, we assume the GUT relationship between the SU(3), SU(2) and U(1) gaugino mass parameters, namely, $M_2 = (\alpha_2/\alpha_3)m_{\tilde{g}}$ and $M_3 = (5/3)\tan^2\theta_W M_2$, where $m_{\tilde{g}}$ is taken to be a free parameter. We further assume that all the charged sleptons and the sneutrinos get a universal contribution, $m_0$, to their masses at the electroweak scale. Thus, the masses of the charged sleptons and the sneutrinos are given by (including the usual D-term contributions)

$$m_{\tilde{e}_{R}}^2 = m_0^2 - \sin^2\theta_W m_Z^2 \cos 2\beta$$

$$m_{\tilde{e}_{L}}^2 = m_0^2 - (1/2 - \sin^2\theta_W) m_Z^2 \cos 2\beta$$

$$m_{\tilde{\nu}_{L}}^2 = m_0^2 + \frac{1}{2} m_{\tilde{\nu}^c}^2 \cos 2\beta$$

Under these conditions, our relevant low-energy SUSY parameter space is reduced to four parameters: $\tan\beta$, $\mu$, $m_0$ and $m_{\tilde{g}}$.

Using the above SUSY parameter space, in Fig.2 we show contour plots of our effective RPV couplings $A_{L,ijkl}$, $C_{L,ijkl}$ and $D_{L,ijkl}$ (in GeV$^{-1}$) in the $\mu - M_2$ plane ($M_2$ is fixed by $m_{\tilde{g}}$ due to the GUT relation mentioned above), for two discrete choices of $\tan \beta$, namely, $\tan \beta = 3$ and 35. It should be noted that the numbers corresponding to a particular effective operator are flavor blind since we have assumed a universal contribution to the soft scalar masses. We can see from Fig.2 that for slepton masses of 100 GeV (the values of $m_0$ in each of the plots are chosen to give slepton masses of 100 GeV) and $|\mu|$ as well as $M_2$ are of the order of several hundreds GeV, then the $\tilde{e}\bar{e}$, $\tilde{e}l_e l\nu$ and $\tilde{e}_R l_e^{c\nu}$ effective couplings in $\Box$ range between $4 \times 10^{-3} - 2 \times 10^{-3}$, where the $\tilde{e}_R l_e^{c\nu}$ effective coupling is the largest of the three.

The existing upper bounds on $\lambda_{ijkl}$ (see e.g., Allanach et al. in $\Box$) are larger than the expected values of our effective couplings if the trilinear soft breaking RPV coupling, $a_{ijkl}$, is of the order of the electroweak mass scale. We note that in the general case of soft lepton number violation, the scalar potential may also contain the RPV soft operator $c_{ijkl} e'_{ijkl} l'_i l'_j Q_j D_k^c$, in which case both $\lambda$-like and $\chi'$-like RPV interactions may be effectively loop-generated. Then, the more stringent limits on the $\lambda'\chi'$ coupling products can be applied to constrain our scenario. This will be investigated in a future work $\Box$. 

<table>
<thead>
<tr>
<th>Effective coupling (GeV$^{-1}$)</th>
<th>SPS1</th>
<th>SPS2</th>
<th>SPS4</th>
<th>SPS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>A_{L,ijkl}</td>
<td>\times 10^4$</td>
<td>3.5</td>
<td>3.6</td>
</tr>
<tr>
<td>$</td>
<td>C_{L,ijkl}</td>
<td>\times 10^4$</td>
<td>3.4</td>
<td>3.5</td>
</tr>
<tr>
<td>$</td>
<td>D_{L,ijkl}</td>
<td>\times 10^4$</td>
<td>4.8</td>
<td>4.9</td>
</tr>
</tbody>
</table>
Finally, in our model the soft RPV $a$-term in $\lambda$ gives rise to a neutrino mass only at the two-loop level (see [2]), or equivalently, as an effective “tree-level” neutrino mass which is generated by the sneutrino VEVs ($v_i$) and is $\propto v_i^2$. Since $v_i \propto B_i/M_{\text{SUSY}}$ and since in our model $B_i$ is a one-loop quantity, clearly this also is essentially a two-loop effect. Similarly, ”one-loop” neutrino masses involving two vertices of our effective $\lambda$s are essentially a three-loop effect.

To summarize, we have postulated that RPV can emerge in the hidden sector through the same mechanism that breaks SUSY and then transmitted to the SUSY dynamics at some high energy scale, only in the form of soft operators. A viable hidden sector model that gives rise to softly broken RPV was presented.

In this scenario Yukawa-like RPV operators are generated only through loops involving the RPV soft interactions. Such effective Yukawa-like RPV interactions are similar (albeit richer) in structure to the conventional $\lambda$, $\lambda'$ and $\lambda''$ ones which, in most RPV scenarios, are added ad-hoc in the superpotential. We have shown that this framework naturally explains the smallness and, therefore, the present non-observability of RPV interactions in low and high energy processes. The implications of our effective RPV scenario to the $\lambda'$ and $\lambda''$-type couplings, as well as to bilinear RPV will be detailed in [9].

We thank J. Wudka, Y. Shadmi, Y. Grossman and S.K. Vempati for very helpful discussions. S.R. thanks the Lady Davis Fellowship Trust for financial support.
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