abstract We propose a novel mathematical method to construct an exact polytropic sphere in self- 
gravitating hydrostatic equilibrium, improving the non-linear Poisson equation. The central boundary con-
dition for the present equation requires a ratio of gas pressure to total one at the centre, which is uniquely 
identified by the whole mass and molecular weight of the system. The special solution derived from the 
Lane-Emden equation can be reproduced. This scheme is now available for modelling the molecular cloud 
cores in interstellar media. The mass-radius relation of the first core is found to be consistent with the recent 
results of radiation hydrodynamic simulations.