We discuss the relations between Generalized Parton Distributions (GPDs) and nucleon resonances. We first briefly introduce the concept of “transition” GPDs. Then we discuss a straightforward application to the modelization of the $N$-$\Delta$ magnetic transition form factor. Finally, we discuss the experimental aspects of the subject and present first preliminary experimental investigations in this field.

1 Brief introduction to “transition” GPDs

The Generalized Parton Distributions provide one of the most complete information on the structure of the nucleon: they allow to access the longitudinal momentum distributions of the partons in the nucleon as well as the nucleon’s transverse profile, the parton correlations, their orbital momentum contribution to the total nucleon’s spin, quark-antiquark components in the nucleon, etc...

The GPDs can be accessed through the hard (i.e. large photon virtuality $Q^2$) exclusive electroproduction of photons ("Deep Virtual Compton Scattering" -DVCS-) and mesons $\pi^0, \rho^0, \rho^\pm, \omega, \phi$, etc...- on the nucleon.

The formalism introducing the Generalized Parton Distributions can be found in Refs. 1, 2, 3, 4 and is summarized in the contribution of A. Belitsky to these proceedings. We refer the reader to these articles for the standard definitions and notations used in the following.

The GPD formalism was originally developed for exclusive reactions on the nucleon, both in the initial and final states, but was recently extended to more general baryonic final states 5, in particular for the simplest non-elastic transition the $N$-$\Delta$ case. Like “transition” $N$-$\Delta$ form factors, one can introduce about “transition” $N$-$\Delta$ GPDs and these are in principle different from the nucleon GPDs.

$N$-$\Delta$ GPDs contain the same type of information as the nucleon GPDs, i.e. the quarks longitudinal momentum distributions and their transverse positions in the $\Delta$, their spin distribution, etc... and, in the case of $\Delta$-$\Delta$ GPDs, similarly to Ji’s sum rule, the orbital contribution of the quarks to the spin of the $\Delta$.

At leading twist in QCD, the $N$-$\Delta$ transition can be parametrized in terms of 3 vector $N$-$\Delta$ GPDs and 4 axial-vector $N$-$\Delta$ GPDs. Among them, one expects 3 $N$-$\Delta$ GPDs to dominate at small $|t|$: the magnetic vector $N$-$\Delta$ GPD $H_M$, whose first moment corresponds to the $N$-$\Delta$ magnetic transition form factor $G_M^M(t)$, and the axial vector $N$-$\Delta$ GPDs $C_1$ and $C_2$ whose first moments correspond to the axial and pseudoscalar $N$-$\Delta$ form factors respectively.

In order to provide numerical estimates for the $N$-$\Delta$ amplitudes, these three GPDs have been estimated in Ref. 5 in the large $N_c$ limit. In this limit (valid to about 30%), they can be related to three nucleon GPDs: $C_1 \propto \tilde{H}^{(3)}$, $C_2 \propto \tilde{E}^{(3)}$.
and \( H_M \propto E^{(3)} \), where the superscript \(^{(3)}\) refers to the isovector nucleon GPDs combination. As various modelizations of the nucleon GPDs are available in the literature, these relations allow to make numerical estimates and calculations for observables (cross sections, asymmetries,...) related to exclusive reactions involving the N-\( \Delta \) transition, such as \( ep \rightarrow e'\Delta\gamma \) (usually referred to as \( \Delta VCS \) by analogy with DVCS -see Fig. 1-), \( ep \rightarrow e'\Delta^{++}\pi^- \), etc...

Figure 1. The exclusive process \( ep \rightarrow e'\Delta\gamma \) expressed in terms of the 3 \( N - \Delta \) transition GPDs, which are expected to dominate at small \( |t| \): \( C_1, C_2 \) and \( H_M \). The longitudinal momentum fractions of the quarks \( (x, \xi) \) are indicated on the figure as well as the definition of the momentum transfer \( t \).

In the case of the \( \Delta VCS \) process, similarly to the DVCS process, there is an interference with the associated Bethe-Heitler mechanism (where the outgoing photon is emitted by the incoming or scattered electron), and this produces a beam helicity asymmetry, which is shown on Fig. 2.

The large \( N_c \) relations, although not extremely precise, allow also to interpret the \( \Delta VCS \) reaction in terms of nucleon GPDs and therefore permits to access a different \textit{flavor} combination of the nucleon GPDs : for instance, in DVCS, one accesses in general the combination \( \frac{1}{2}\tilde{H}^u + \frac{1}{2}\tilde{H}^d \) whereas, in \( \Delta VCS \), one accesses the isovector part \( \tilde{H}^u - \tilde{H}^d \). In this way, the \( \Delta VCS \) process is therefore also useful to carry out a \textit{flavor} decomposition of the nucleon GPDs.

A new way to study baryon spectroscopy is therefore opening up through these transition \( N \rightarrow N^*, \Delta^* \) GPDs, where not only the \( t \)-dependence, which can currently be determined through transition form factors and which reflects the transverse spatial distribution of the partons in the nucleon, can be accessed but also their \( x \)-dependence, reflecting the longitudinal momentum distributions of the quarks in the \( N^* \)'s.

2 Modelization of the N-\( \Delta \) transition form factor

In this part, as a basic illustration of the information contained in the GPDs, we show how the N-\( \Delta \) transition form factor can be estimated from the (nucleon)
GPDs, relying on very simple assumptions. We start from the model independent sum rules which relate the GPDs to the form factors:

\[ F_2^q(t) = \int_{-1}^{+1} dx \ E^q(x, \xi, t), \quad G^*_M(t) = \int_{-1}^{+1} dx \ H_M(x, \xi, t) \quad (1) \]

where \( F_2 \) is the nucleon Pauli form factor and \( G^*_M \) the N-\( \Delta \) magnetic transition form factor.

In the large \( N_c \) limit, the N-\( \Delta \) GPDs can be expressed in terms of the isovector combination of nucleon GPDs and one arrives at the following sum rule \( ^5 \):

\[ G^*_M(t) = \frac{G^*_M(0)}{\kappa_V} \int_{-1}^{+1} dx \ \left( E^u(x, \xi, t) - E^d(x, \xi, t) \right) = \frac{G^*_M(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\} \quad (2) \]

where \( \kappa_V = \kappa_p - \kappa_n = 3.70 \). In the large \( N_c \) limit, the value \( G^*_M(0) \) is given by \( ^6 \): \( G^*_M(0) = \kappa_V/\sqrt{3} \), some 30 \% smaller than the experimental number. In calculations, we therefore use the phenomenological value \( G^*_M(0) \approx 3.02 \) \( ^{10} \).

One can choose \( \xi = 0 \) in the previous equations, and by modeling the nucleon GPD \( E(x, 0, t) \), one can therefore estimate the N-\( \Delta \) transition form factor. A plausible ansatz at low \( |t| \), inspired from a Regge form, for the GPD \( E(x, 0, t) \) could be \( ^6 \):

\[ E^q(x, 0, t) = \kappa_q q_v(x) \frac{1}{x^{\alpha_t} t} \quad (3) \]
where \( q_v(x) \) is the \((u,d)\) valence quark distribution normalized to \((2,1)\) respectively and where \( \kappa_u \) and \( \kappa_d \) are given by

\[
\kappa_u = 2\kappa_p + \kappa_n, \tag{4}
\]
\[
\kappa_d = \kappa_p + 2\kappa_n. \tag{5}
\]

In equation (3), \( \alpha'(t) \) is the slope of standard Regge trajectory (to be fitted) around \( 1 \text{ GeV}^2 \). It has been shown in Ref. 7 that such kind of ansatz was able to reproduce rather well the electric and magnetic radii of the nucleon and the Dirac and Pauli form factors at low \(| t |\) \((< 1 \text{ GeV}^2)\). We show on figure 3 the decent agreement for the N-\( \Delta \) transition form factor.

\[
N - \Delta \text{ MAGNETIC FORM FACTOR}
\]

**Figure 3.** The \( N \rightarrow \Delta \) magnetic transition form factor, relative to the dipole. The dotted curve corresponds to \( \alpha' = 1.102 \text{ GeV}^{-2} \). The other curves correspond to extensions of the present model by allowing for non-linear Regge trajectories (see Ref. 8 for a more detailed discussion). The data for \( G^*_M \) are from the compilation of Ref. 10. For the JLab data points at 2.8 and 4 \( \text{GeV}^2 \), both the analyses of 9 (upper points) and 10 (lower points) are shown.

### 3 Experimental aspects

Finally, on the experimental side, first hints of the observation of the \( ep \rightarrow e'\Delta \gamma \) reaction in the CLAS detector at JLab are showing up. The analysis, led by S. Bouchigny from IPN Orsay, has consisted in selecting \( e'n\pi^+ \) final states in data taken with a 4.2 \( \text{GeV} \) incident electron beam. Selecting events whose missing mass \( MM(e'n\pi^+X) \) is around zero \(^a\), in order to be compatible with a missing photon

\(^a\)It should be kept in mind that there is certainly a contamination of the \( \Delta VCS \) signal by \( \Delta n^0 \) final states at this preliminary stage of analysis, the current resolution of CLAS not permitting...
(see figure 4-a), and imposing a $W > 2$ GeV cut so that one is above the resonance region, one was able to observe the $n\pi^+$ invariant mass spectra of figure 4-b where one clearly distinguishes, not only $\Delta\gamma$ final states, but also higher $N^*\gamma$ final states, providing evidence for $N\rightarrow N^*$ DVCS processes.

Figure 4. a (Left) : Squared missing mass spectrum $e'n\pi^+X$ in GeV$^2$, corresponding to the missing mass of a $\gamma$ or a $\pi^0$. b (Right) : Invariant mass spectrum of the $n\pi^+$ system in GeV for $W > 2$ GeV, therefore corresponding to $N^*(\gamma, \pi^0)$ final states.

The statistics in this figure (effectively equivalent to a few days of beam time) are not enough yet to extract a significant beam asymmetry to be compared to the theoretical calculations of figure 2 but the observed signals are certainly very encouraging.

Data have been taken last year in CLAS with a 5.75 GeV beam and are in the process of being analysed (with 5 times more statistics than with the 4.2 GeV run of figure 4) and should allow to have a first glance at a $\Delta$VCS beam asymmetry very soon.

Finally, the forthcoming equipment of CLAS with a fine (angular and energy) resolution and high counting rate capability calorimeter, to be installed and operational mid-2004 (see for instance Ref. 11), will also boost tremendously the potential of analysis of this channel. The full detection of the photon(s) of the final state will be possible and will unambiguously allow to distinguish $e'\Delta\gamma$ from $e'\Delta\pi^0$ final states.

the separation of the two by the missing mass technique.
4 Conclusion

We have briefly discussed here a couple of aspects of the relation between (transition) Generalized Parton Distributions and nucleon resonances. The exclusive electroproduction of a photon or of a meson with baryonic resonances opens a brand new way to understand the nucleon excitation spectrum, allowing to access a wealth of information not contained in direct (i.e. $s$-channel) resonance excitation. This opens up the perspective to arrive at a 3-dimensional picture of the nucleon and its resonances through the formalism of the GPDs.

References

11. JLab experiment E01-113,
   “http://www.jlab.org/exp_prog/generated/apphallb.html”