Mirror dark matter and large scale structure.

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Mirror matter is a dark matter candidate. In this paper, we re-examine the linear regime of density perturbation growth in a universe containing mirror dark matter. Taking adiabatic scale-invariant perturbations as the input, we confirm that the resulting processed power spectrum is richer than for the more familiar cases of cold, warm and hot dark matter. The new features include a maximum at a certain scale \( \lambda_{\text{max}} \), collisional damping below a smaller characteristic scale \( \lambda_{5} \), with oscillatory perturbations between the two. These scales are functions of the fundamental parameters of the theory. In particular, they decrease for decreasing \( x \), the ratio of the mirror plasma temperature to that of the ordinary. For \( x \sim 0.2 \), the scale \( \lambda_{\text{max}} \) becomes galactic. Mirror dark matter therefore leads to bottom-up large scale structure formation, similar to conventional cold dark matter, for \( x \lesssim 0.2 \). Indeed, the smaller the value of \( x \), the closer mirror dark matter resembles standard cold dark matter during the linear regime. The differences pertain to scales smaller than \( \lambda_{5} \) in the linear regime, and generally in the non-linear regime because mirror dark matter is chemically complex and to some extent dissipative. Lyman-\( \alpha \) forest data and the early reionisation epoch established by WMAP may hold the key to distinguishing mirror dark matter from WIMP-style cold dark matter.

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I. INTRODUCTION

The dark matter problem provides one of the strongest reasons to suspect the existence of physics beyond the standard model. We will explore the possibility that dark matter is mirror matter in this paper. Our objective is to understand the growth of density perturbations in the linear regime in such a universe, taking adiabatic scale-invariant perturbations as the input. In doing so, we both confirm and extend the results of Ref. [1]. By comparing mirror dark matter (MDM) with conventional cold, warm and hot dark matter (CDM, WDM and HDM, respectively), we hope to explain the physics of mirror dark matter in as clear a way as possible, and to pinpoint the data that are most sensitive to its characteristic features.

Before launching into the analysis, we should set the stage by briefly reviewing the evidence for non-baryonic dark matter, and explaining why we think mirror matter is an interesting candidate.

It is very well established that the dynamics of objects ranging in size from galaxies up to clusters of galaxies cannot be understood using standard gravity unless one postulates that invisible matter dominates over the visible by a factor of \( 10 - 30 \). (It is also logically possible that our understanding of gravity at large scales is incomplete [2], though we will not pursue that possibility here.) It has been known for some time that the conservative option

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of the dark material being simply ordinary matter in the form of non-luminous objects such as “Jupiters”, neutron stars, and so on, is ruled out if one accepts that the light elements H, D, $^3$He and $^7$Li were created through big bang nucleosynthesis (BBN). The baryon-to-photon ratio is the one free parameter in standard BBN, and the value required to produce fair agreement with the primordial abundance data is a factor of five or so too small to account for all the dark matter required to successfully model the gravitational dynamics of clusters. Ordinary baryonic dark matter is also inconsistent with successful large-scale structure formation, principally because perturbation growth begins too late. In addition, acoustic peak data from cosmic microwave background anisotropy measurements, including those very recently reported by the Wilkinson Microwave Anisotropy Probe (WMAP) collaboration, have independently pointed to a matter-to-baryon density ratio of about six $^6$ $^7$. So, if one accepts standard hot big bang cosmology, then one must perforce accept the existence of non-baryonic dark matter.

Acknowledging the reality of non-baryonic dark matter, one can maintain conservatism by supposing that massive ordinary neutrinos provide the additional matter density. While this is a natural and obvious possibility, it runs foul of large-scale-structure data. A successful account of large-scale structure, the concern of this paper and one of the most important problems in physics $^8$, must be part of a successful cosmology. Neutrino dark matter is the archetype of HDM, inducing “top-down” structure formation whereby very large structures form first, with smaller ones arising from subsequent fragmentation. Hot dark matter driven “top-down” scenarios are now ruled out by the data: more structure is observed at small scales than possible with HDM (see, for instance, Refs. $^7$ and $^9$).

To sum up: the confluence of galactic/cluster dynamics, big bang nucleosynthesis, acoustic peak, gravitational lensing and large-scale structure data strongly point to a universe whose material or positive-pressure component is roughly 3% luminous baryonic, 15% dark baryonic, and 82% exotic. This is a remarkable conclusion.$^3$

It is very interesting that the exotic dark component must consist of stable forms of matter, or at least extremely long-lived. While exotic unstable particles abound in extensions of the standard model, completely new stable degrees of freedom pose a more profound model-building challenge. The stability challenge is fully met by mirror matter.

The mirror matter model arose from the aesthetic desire to retain the improper Lorentz transformations as exact invariances of nature despite the $V - A$ character of weak interactions $^1$. $^2$ $^3$. $^4$ $^5$ It does so by postulating that, first, the gauge group of the world is a product of two isomorphic factors, $G \otimes G$, and, second, that an exact discrete $Z_2$ parity symmetry, unbroken by the vacuum, interchanges the two sectors.$^5$ The minimal mirror matter model takes $G$ to be simply the standard model gauge group $SU(3) \otimes SU(2) \otimes U(1)$. All ordinary particles except the graviton receive a mirror partner. A mirror particle has the same mass as the corresponding ordinary particle, and mirror particles interact amongst themselves in the same way that ordinary particles do, except that mirror weak interactions are right-handed rather than left-handed.

The two sectors must interact with each other gravitationally, with certain other interaction channels also generally open, though controlled by free parameters that can be arbitrarily small. The possible non-gravitational interactions include photon–mirror-photon kinetic mixing $^12$ $^16$, neutrino–mirror-neutrino mass mixing $^17$, and Higgs-boson–mirror-Higgs-boson mixing $^{12}$ $^{13}$. We will assume, for simplicity, that all of these parameters are small enough to be neglected. (One should bear in mind, however, that photon–mirror-photon kinetic mixing can cause remarkable phenomena even with a controlling parameter as small as $10^{-6} - 10^{-9}$ $^{17}$ $^{19}$.)

Mirror protons and mirror electrons are stable for exactly the same reason that ordinary matter is stable. One would expect that mirror matter would have existed in the cosmological plasma of the early universe. If so, mirror

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$^1$ We prefer to add the electron-neutrino chemical potential to the baryon-to-photon ratio in the parameter count of BBN, with “standard BBN” then defined as the zero chemical potential line in a two-dimensional parameter space. Some quite persistent discrepancies in the data actually hint that a nonzero chemical potential may be necessary $^3$ $^4$ $^5$.

$^2$ Large scale structure may be observationally probed via galaxy surveys $^4$ and gravitational weak lensing $^8$.

$^3$ Acoustic peak and other data also require the total matter density to be about 30% of the critical value giving a spatially flat universe $^2$ $^4$. With the strong evidence for flatness from the acoustic peak features, one is also obliged to add a 70% non-material, negative-pressure component called “dark energy”, as also suggested by SN1a observations $^{19}$. Note also that much of the dark baryonic matter, for high red shifts, has now been detected through Lyman-$a$ studies.

$^4$ Alternative motivations include $E_6 \otimes E_8$ string theory, and brane-world constructions such as the “manyfold” universe $^{14}$.

$^5$ For the alternative of spontaneously broken mirror symmetry see Refs. $^{16}$. 

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matter relics in the form of gas clouds, planets, stars, galaxies and so on might well be common in the universe today, manifesting observationally as dark matter. Most prior work on MDM, with the notable exception of Ref. [1], has focused on the astrophysical phenomenology of compact mirror matter objects, hybrid ordinary-mirror systems, and diffuse mirror matter gas/dust in our own solar system [13, 20]. The discovery of mirror matter through such means would, obviously, be a major breakthrough. In this paper, however, we turn to the other generic purpose of dark matter: to assist the growth of density perturbations in the early universe, thus initiating large scale structure formation. We want, ultimately, to know if mirror dark matter is consistent with large scale structure data, and, if it is, to develop observables that can discriminate between MDM and the current paradigm of collisionless CDM (and whatever other candidates might be dreamed up).

From the macroscopic perspective, mirror matter is a much more complicated dark matter candidate than standard CDM particles such as axions and WIMPs. Rather than just one species of particle, mirror dark matter is chemically complicated, consisting of all the mirror analogues of ordinary matter: protons, neutrons and electrons. Further, MDM is self-interacting, and a background of mirror photons and mirror neutrinos interacts with the mirror-baryonic matter. However, since (i) the self-interactions of the mirror particles are by construction identical to those of ordinary particles except for the chirality flip, and (ii) the interaction between ordinary and mirror matter is by assumption dominated by standard gravity, the MDM universe can be analysed through well-defined physics despite the complex nature of the dark sector. It is not necessarily a virtue for DM to consist of a single exotic species such as a WIMP or axion. Indeed, in their recent review Peebles and Ratra [21] emphasised that standard CDM can be viewed as the calculationally simplest DM scenario that, in broad terms, is phenomenologically acceptable, but which may be subject to revision or replacement when more detailed large scale structure data are collected. They then point out that certain data already challenge standard CDM on points of detail, though they caution that these discrepancies might in the end be due to calculational problems only. We take the view that all well-motivated standard model extensions supplying stable exotic species should be investigated for their DM potential.

The rest of this paper is structured as follows. In Sec. II, we review the elements of cosmology in a universe with mirror matter, and explain why there need not be a 50/50 mixture of ordinary and mirror matter. Sections III and IV then discuss two key scales in the perturbation evolution problem: the Jeans length and Silk scale for mirror baryons. The former determines the scale at which sub-horizon sized modes can begin to grow in the dark matter (mirror) sector, while the latter determines the scale below which growth is damped. Section V discusses the outcomes of the linear growth regime through final processed spectra for mirror dark matter perturbations. It also discusses Lyman-α forest data, CMBR anisotropy and early reionisation. We conclude in Sec. VI.

II. COSMOLOGY WITH MIRROR DARK MATTER

We begin by dispelling a common misconception regarding mirror matter. One might naively expect that the exact discrete symmetry between the ordinary and mirror sectors in the Lagrangian would require the universe to contain, and to have always contained, a precisely 50/50 mixture of ordinary and mirror particles. Such a universe would be inconsistent with the standard cosmological framework. First, the doubling of the universal expansion rate due to mirror photons, neutrinos and antineutrinos would completely spoil big bang nucleosynthesis. Even if some way could be found to counteract the additional relativistic species, there would be a second objection. As we reviewed above, observations favour a DM to baryon density ratio of about five, comfortably larger than two.

6 We emphasise that the microscopic theory is by contrast very simple.
7 Initial conditions must also be supplied (see the next section).
8 A coincident epoch with a temporarily negative cosmological constant of the right magnitude perhaps?
temperature $T'$ of the mirror plasma in the early universe, and the background mirror photons today $T'_0$, is different from that of the ordinary plasma, $T$, and the ordinary cosmic microwave background photons today, $T_0$. One of the fundamental parameters in our cosmology will therefore be

$$x \equiv \frac{T'_0}{T_0}. \quad (2.1)$$

Since the energy density of relativistic species goes as the fourth power of temperature, the contribution of the light mirror degrees of freedom to the cosmological density is strongly suppressed by $x^4$. Even a small difference between the temperatures, such as a factor of $1/2$, is enough to comply with the BBN upper bound on extra relativistic energy density. This removes the first objection to mirror matter cosmology.

One might be uncomfortable with ascribing the macroscopic asymmetry of the universe to asymmetric initial conditions. However, standard Friedmann-Robertson-Walker cosmology already has a raft of problems associated with initial conditions: homogeneity, spatial flatness, etc. One approach to those problems is, of course, inflationary cosmology. Interestingly, prior work has shown that inflation can also effectively initialise a mirror matter universe to have $T' \neq T$ \[23, 24\]. One way is to introduce a mirror inflaton to partner the ordinary inflaton within the chaotic inflation paradigm. Since the stochastic processes germinating inflation will not comply with the discrete symmetry on an event-by-event basis, the main result follows.

What about the second objection to mirror matter cosmology? Given that $T' < T$, one might conclude that the mirror baryons would have a correspondingly lower density than their ordinary counterparts, thus exacerbating the problem of not enough MDM. This conclusion would be true if the magnitudes of the baryon asymmetries in the two sectors were equal.

However, we have to take into account that the inequality of temperatures of ordinary and mirror matter will in general change the outcome of baryogenesis in the two sectors, even though the microphysics is the same \[24\]. One expects in fact that

$$\eta' \equiv \frac{n'_{B}}{n'_{\gamma}} \neq \frac{n_{B}}{n_{\gamma}} \equiv \eta, \quad (2.2)$$

where $n_X$ is the number density of ordinary species $X$ and $n'_X$ is the number density of mirror species $X'$. (We denote the mirror partner to a given particle by a prime.) The ratio of mirror baryon and ordinary baryon number densities can be written as

$$\frac{n'_{B}}{n_{B}} = \frac{\eta'}{\eta} x^3. \quad (2.3)$$

Because the baryons and mirror baryons have equal rest masses and are highly non-relativistic for the epochs we consider, this quantity is also approximately the energy density ratio,

$$\frac{\Omega'_{B}}{\Omega_{B}} \simeq \frac{\eta'}{\eta} x^3, \quad (2.4)$$

where, as usual, $\Omega_X$ denotes the energy density of $X$ in units of the critical density. In Refs. \[1, 24\], it was shown that the mirror baryon asymmetry can be greater than the ordinary baryon asymmetry, and can in fact overwhelm the $x^3$ factor in Eq. \[2.4\]:

$$\frac{\eta'}{\eta} > \frac{1}{x^3}. \quad (2.5)$$

Two quite different baryogenesis scenarios were analysed in Ref. \[1\]: the out-of-equilibrium baryon-number violating decays of massive bosons and electroweak baryogenesis. Interestingly, in both cases $\Omega'_{B} > \Omega_{B}$ with an acceptable $\Omega_{B}$ could be obtained provided that

$$x \gtrsim 0.01. \quad (2.6)$$
We will use this value as an indicative lower limit to cosmologically interesting values of $x$. It obviously should not be taken as definitive, because we do not yet know what baryogenesis mechanism actually operates in nature. (A new mechanism involving mirror matter has been very recently proposed in Ref. [25].)

Motivated by the above results, we take $\Omega'_{B}/\Omega_B$ as the second free parameter in our cosmology, fixed only by observational data.

Before moving on, we should deal with a third possible objection to mirror dark matter, this one based on the results of recent works constraining self-interacting CDM. Recall that an extension of standard CDM through self-interactions was proposed to circumvent the problem of over dense cores for some types of galaxies [26]. The required properties were that the elastic scattering cross-section of DM particle on DM particle should lie in the interval \( \sigma/M \simeq 10^{-23} - 10^{-24} \text{ cm}^2/\text{GeV} \), and the DM should remain dissipationless. These constraints are violated by MDM because it is dissipative and, if we take an atomic hydrogen cross-section as a guide, then the self-interaction strength is too high. However, the two cases are not directly comparable. The evolution of MDM is much more complicated than that of the self-interacting CDM considered in Ref. [26]. For instance, MDM would form more intrinsic structure (mirror stars and other compact objects) than self-interacting CDM, so it is not just a question of scattering cross-sections. Exactly what sorts of compact mirror matter objects would form, and how they would be distributed, is a very complicated question beyond the scope of this work. This development will not parallel that of the ordinary sector. For instance, one of the key parameters affecting the galaxy formation process – the rate of star creation – will be different because primordial nucleosynthesis in the mirror sector will produce much more mirror helium relative to mirror hydrogen than is the case for their ordinary analogues [1].

Let us see how the main cosmological equations are changed by the presence of mirror matter [1]. The Friedmann equation for a flat universe becomes

\[
H^2 = \frac{8\pi G}{3} \rho_{\text{tot}},
\]

where $\rho_{\text{tot}}$ is the total energy density, ordinary plus mirror (plus vacuum)

\[
\rho_{\text{tot}} = \rho + \rho' + \rho_\Lambda,
\]

and the Hubble parameter \( H = \dot{a}/a \) where \( a \) is the scale factor. In terms of the present-day energy densities, $\Omega_{r,m,\Lambda}$ for radiation, matter and vacuum, respectively, and the present-day Hubble parameter $H_0$ (the so-called Hubble constant), the Friedmann equation can be rewritten as

\[
H(z)^2 = H_0^2 \left[ \Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_\Lambda \right]
\]

where \( z \) is red-shift. (We include the vacuum energy contribution for completeness and self-consistency only. Its effects are negligible for the early universe epoch we will consider.)

The present energy density of relativistic particles, $\rho_r$, is the sum of contributions from ordinary photons, mirror photons, and presently relativistic neutrinos and mirror neutrinos. It is given by

\[
\rho_r = \frac{\pi^2}{30} [g_*(T_0) T_0^3 + g_*'(T'_0) T'_0^4] = \frac{\pi^2}{30} g_*(T_0) T_0^4 (1 + x^4),
\]

where \( g_*(T_0) \) and \( g_*'(T_0) \) are the effective numbers of relativistic degrees of freedom in the ordinary and mirror sectors respectively:

\[
g_*(T_0) = g_*'(T'_0) \simeq 2(1 + 0.23 N_{0}^{rel\nu}).
\]

The number of presently non-relativistic neutrino flavours, $N_{0}^{rel\nu}$, is either zero or one, depending on whether the neutrino masses are degenerate or hierarchical, respectively. From Eq. (2.11) we see that the contribution of mirror particles to the relativistic energy density can be neglected at all times because \( x^4 \ll 1 \) due to the BBN constraint \( x \lesssim 0.5 \).

Observations require us to take the mirror baryons to dominate the total present-day matter density, viz.

\[
\Omega_m = \Omega_B + \Omega'_B \simeq \Omega'_B.
\]
After WMAP, the favoured range at 68% C.L. is

$$\Omega_m h^2 \simeq 0.14 \pm 0.02,$$

(2.13)

where $h \simeq 0.72$ is the Hubble constant in units of $H \equiv 100$ km/s/Mpc. For future convenience, we will use

$$y \equiv \frac{0.14}{\Omega_m h^2}$$

(2.14)

instead of $\Omega_B/\Omega_B$ as the second a priori free parameter in our mirror matter cosmology. The 3σ preferred range for $y$ is $0.7 - 1.75$.

There are a number of the critical moments in the process of perturbation growth. One of them occurs when the universe is equally dominated by radiation and matter. The corresponding redshift, $z_{eq}$, is found from

$$(\Omega_{\gamma} + \Omega_{\gamma}')(1 + z_{eq})^4 + \frac{\rho_{rel \nu}(z_{eq})}{\rho_c} = \Omega_m (1 + z_{eq})^3,$$

(2.15)

where $\rho_{rel \nu}(z)$ is the energy density in relativistic neutrinos at matter-radiation equality. Using Eq. (2.10), the observed present-day background photon temperature and $x \ll 1$, this evaluates to

$$1 + z_{eq} \simeq \frac{5500}{y \xi},$$

(2.16)

where

$$\xi \equiv 1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N^{rel \nu} \simeq 1 + 0.23 N^{rel \nu},$$

(2.17)

with $N^{rel \nu}$ denoting the number of relativistic neutrino flavours at the designated moment. For the epoch prior to ordinary photon decoupling, it is most likely that all three neutrino mass eigenstates are always relativistic, so we will set $\xi \simeq 1.69$ in all of our numerical estimates. Adopting this, Eq. (2.16) becomes

$$1 + z_{eq} \simeq \frac{3300}{y}$$

(2.18)

yielding $z_{eq}$ in the range 1900 – 4600 from the 3σ allowed interval for $y$.

Two other critical moments are matter-radiation decoupling in the ordinary and mirror sectors. The exponential factor in the Saha equation describing decoupling implies that these events occur at about the same temperature, $T_{dec} \simeq T'_{dec}$, so that

$$1 + z'_{dec} \simeq \frac{1 + z_{dec}}{x} \simeq \frac{1100}{x}.$$  

(2.19)

Matter-radiation decoupling in the mirror sector precedes that in the ordinary sector because of the temperature hierarchy $x < 1$. In the following, it turns out that we will have to consider two cases defined by $x > x_{eq}$ and $x < x_{eq}$, where

$$x_{eq} \simeq 0.34 y.$$  

(2.20)

The distinction follows from Eqs. (2.16) and (2.19): for $x > x_{eq}$, mirror radiation-matter decoupling occurs during the matter-dominated epoch, while for $x < x_{eq}$ it occurs during the radiation dominated epoch. Numerically, $x_{eq}$ takes values in the approximate interval 0.24 – 0.6 (the upper end of this range is disfavoured by BBN).

### III. THE JEANS LENGTH

The Jeans length for mirror matter determines the minimum scale at which sub-horizon sized perturbations in the mirror matter will start to grow through the gravitational instability in the matter-dominated epoch. The mirror
baryon perturbations begin growing first, with the perturbations in the ordinary matter catching up subsequently. This process is similar to the standard CDM scenario, with the points of difference to be discussed later.

Physically, the Jeans length sets the scale at which the gravitational force starts to dominate the pressure force. It is defined as the scale at which the sound travel time across a lump is equal to the gravitational free-fall time inside the lump. The Jeans length for mirror matter is given by

\[ \lambda_J(z) = \frac{\sqrt{\pi} v_s'(z)(1 + z)}{\sqrt{G \rho_{tot}(z)}}, \]  

(3.1)

where \( v_s'(z) \) is the sound speed in the mirror matter, and the \((1 + z)\) factor translates the physical scale at the time of redshift \((1 + z)\) to the present time.

Let us examine the mirror matter Jeans length for the period between mirror-neutrino decoupling and mirror-photon decoupling. The sound speed is calculated from

\[ (v_s')^2 = \frac{dp'}{d\rho'} \]  

(3.2)

where the pressure is dominated by the contribution from mirror photons, \( p' \sim \rho_B'/3 \), and the relevant density is given by

\[ \rho' = \rho_B' + \rho_N'. \]  

(3.3)

Using \( \rho_B' \sim T^4 \) and \( \rho_N' \sim T^3 \), we see that

\[ (v_s')^2 = \frac{1}{3} \frac{1}{1 + \frac{3}{4} \rho_B'}. \]  

(3.4)

To transform the sound speed expression into a more useful form, we first note that

\[ \frac{\rho_B'(z)}{\rho_N'(z)} = \frac{\Omega_B'}{\Omega_N'} \frac{1}{1 + z}. \]  

(3.5)

Then, from the definition of \( z_{eq} \) as given by Eq. (2.15), we obtain that

\[ \frac{\Omega_m}{\Omega_\gamma} \simeq (1 + z_{eq})(1 + x^4)\xi. \]  

(3.6)

Using \( \Omega_B' = \Omega_m - \Omega_B \), we therefore deduce that

\[ \frac{\rho_B'(z)}{\rho_N'(z)} = \frac{1 + z_{eq}}{1 + z} \left( \frac{1}{x^4} + 1 \right) - \frac{\Omega_B'}{\Omega_\gamma} \frac{1}{(1 + z)x^4}. \]  

(3.7)

For our purposes it will be good enough to make the approximations \( \Omega_B' \simeq \Omega_m \) and \( 1/x^4 \gg 1 \), so that \( \rho_B'(z)/\rho_N'(z) \simeq \xi(1 + z_{eq})/[x^4(1 + z)] \). It is easy to check that \( 3\rho_B'/4\rho_N' \gg 1 \) for the epoch of interest. Using these approximations, Eq. (3.4) yields

\[ v_s' \simeq \frac{1}{\sqrt{3}} \frac{2x^2\xi}{\sqrt{3}} \sqrt{\frac{1 + z}{1 + z_{eq}}}. \]  

(3.8)

Observe that the sound speed in the mirror photon-baryon plasma is approximately proportional to \( x^2 \): the low mirror sector temperature suppresses the sound speed by diluting the relativistic component of the mirror plasma [1].

Substituting Eq. (3.8) in Eq. (3.1) and evaluating the resulting expression we obtain

\[ \lambda_J(z) \simeq \frac{2.1 \times 10^4}{\sqrt{2 + z + z_{eq}}} x^2 y^{1/2} \text{ Mpc}, \]  

(3.9)
which implies that

\[ \lambda'_{J}(z_{eq}) \sim 260x^2 y \text{ Mpc}, \]

(3.10)

and

\[ \lambda'_{J}(z'_{dec}) \sim \sqrt{\frac{2x}{x + x_{eq}}} \lambda'_{J}(z_{eq}) \text{ Mpc}. \]

(3.11)

Beware that if mirror photon decoupling occurs before matter-radiation equality \((z'_{dec} > z_{eq}, x < x_{eq})\), then Eq. (3.10) is inapplicable. The mirror baryon Jeans length plummets to very low values after mirror photon decoupling, because the pressure supplied by the relativistic component of the mirror plasma disappears, and the sound speed greatly decreases.

IV. THE SILK SCALE

For the case of mirror dark matter, perturbations on scales smaller than a characteristic length \(\lambda'_S\) will be washed out by the collisional or Silk damping that arises while mirror photons decouple from the mirror baryons [28]. The microphysics of this process is identical to that of Silk damping in a universe containing only ordinary baryons. We use the latter (imaginary) universe as a familiar reference.

Elementary considerations involving photon diffusion may be used to estimate that in our baryonic reference universe, the Silk scale is given by

\[ (\lambda'_S)^2 = \frac{3t^0_{dec} \lambda_{\gamma}(t^0_{dec})}{5(a^0_{dec})^2}, \]

(4.1)

where \(t^0_{dec}\) is the photon decoupling time, \(\lambda_{\gamma}\) the photon mean free path, and \(a^0_{dec}\) the scale factor at decoupling. The mean free path is given by

\[ \lambda_{\gamma} = \frac{1}{X_e n_e \sigma_T}, \]

(4.2)

where \(X_e\) is the electron ionisation fraction at decoupling (so that \(X_e n_e\) is the total number density of free electrons) and \(\sigma_T = 8\pi \alpha^2 / (3m_e^2)\) is the Thomson scattering cross-section. Using \(n_e \sim n_B\), this expression yields

\[ \lambda_{\gamma} \sim \frac{Gm_B m_e^2}{H^2 \alpha^2 X_e (1 + z_{dec})^2} \left( \frac{1}{\Omega_B h^2} \right). \]

(4.3)

Using \(X_e \sim 0.1, a^0_{dec} \sim z_{dec} \sim 1100\), and incorporating some refinements, one obtains the estimate

\[ \lambda^0_S \sim 2.5(\Omega_B h^2)^{-3/4} \text{ Mpc} \]

(4.4)

for our reference baryonic universe [29].

The mirror matter Silk length at mirror photon decoupling can be obtained by deducing how the various quantities in Eq. (4.1) scale with \(x\). The results depend on whether mirror photon decoupling occurs before or after matter-radiation equality, i.e. whether \(x < x_{eq}\) or \(x > x_{eq}\), respectively [see Eq. (2.20)].

A. Case I: \(x > x_{eq}\)

The sequence of events in this case is:

1. Matter-radiation equality occurs at redshift \(z_{eq}\).

2. At a smaller redshift \(z'_{dec}\) mirror photons decouple from mirror baryons.
3. Later still, at \( z = z_{\text{dec}} \), ordinary photon decoupling occurs. After \( z = z_{\text{eq}} \), perturbation growth is no longer damped by the expansion rate, but it is still retarded by mirror-photon induced pressure. The latter disappears at \( z = z'_{\text{dec}} \), and perturbations above the scale \( \lambda_S' \) (determined below) begin to grow in the mirror sector. After \( z = z_{\text{dec}} \), ordinary photon pressure stops preventing the ordinary baryons from falling into the potential wells created by the now growing perturbations in the mirror baryons.

To evaluate the Silk scale, we note that

\[
\frac{t'_\text{dec}}{t_{\text{dec}}} = \left( \frac{a'_\text{dec}}{a_{\text{dec}}} \right)^{3/2} = \left( \frac{1 + z_{\text{dec}}}{1 + z'_\text{dec}} \right)^{3/2} = x^{3/2},
\]

where \( t_{\text{dec}} \) is the actual photon decoupling time. The expression for \( \lambda'_\gamma \) tells us that

\[
\frac{\lambda'_S}{\lambda'_{\gamma}} = x^3.
\]

Hence,

\[
\lambda'_S = x^{5/4} \lambda^0_S,
\]

where it is understood that \( \Omega_B \) is replaced by \( \Omega_m \) in the expression for \( \lambda^0_S \). Observe that the mirror Silk scale is suppressed relative to the reference universe analogue by the temperature ratio to the stated power.

Numerically, we find that

\[
\lambda'_S(x > x_{\text{eq}}) \sim 11x^{5/4}y^{3/4} \text{ Mpc}.
\]

\textbf{V. PROCESSED POWER SPECTRA FOR MIRROR DARK MATTER}

Having obtained the values of the characteristic scales for our problem, we are now in a position to qualitatively discuss the shapes of the MDM processed power spectra in the linear regime, treating cases I and II defined above separately.

**A. Processed power spectrum for case I: \( x > x_{\text{eq}} \)**

Figures 1 and 2 schematically depict the processed power spectrum in MDM when \( x > x_{\text{eq}} \) at the two important moments of matter-radiation equality \( (z = z_{\text{eq}}) \) and mirror photon decoupling \( (z = z'_{\text{dec}}) \), respectively. The vertical axes are \( \log(\delta \rho_B/\rho'_B) \), the logarithm of the mirror baryon density perturbation at scale \( \lambda \), at these two moments, plotted as functions of \( \log(\lambda/\text{arbitrary scale}) \). We will now explain how these “cartoons” arise.
A number of scales are highlighted along the horizontal axis. The largest of these is $\lambda_{eq}$, the horizon scale at $z = z_{eq}$, which is easily found to be given by

$$\lambda_{eq} \sim 130y \text{ Mpc.}$$

(5.1)

Scales smaller than $\lambda_{eq}$ have already entered the horizon and have thus been processed by gravitationally-driven growth and microphysical effects such as Silk damping. Perturbations on super-horizon sized scales obey $(\delta \rho / \rho)_{\lambda} \sim \lambda^{-2}$ for the usual reason, the prior period having been radiation dominated.

The next scale is the mirror baryon Jeans mass, as given by Eq. (3.10). Notice that the BBN bound $x < \sim 0.5$ guarantees that the mirror baryon Jeans scale is always within the horizon at matter-radiation equality. The (sub-horizon sized) perturbations between $\lambda_{eq}$ and $\lambda'_{J}(z_{eq})$ have grown only logarithmically because of radiation dominance.

The spectrum peaks at $\lambda'_{J}(z_{eq})$. Below that scale, the perturbations are oscillatory and suppressed, being washed out completely below the Silk scale $\lambda'_{S}(z_{eq})$. To understand the detailed behaviour in this interval, we need to look more closely at the evolution of the perturbations.

Before doing so, however, we should pause to note the similarities and differences between Case I MDM and CDM. For scales $\lambda > \lambda'_{J}(z_{eq})$, the processed spectrum is identical to that of CDM. Below $\lambda'_{J}(z_{eq})$, however, the story is different. Recall that the CDM spectrum maintains its slow logarithmic rise as $\lambda$ decreases below $\lambda_{eq}$ until extremely small scales. This is because both the Jeans and free-streaming lengths for WIMP CDM are extremely small, the former because there is no pressure support to fight, and the latter because the WIMPs are very massive and thus slow-moving. For all practical purposes, CDM just has two regimes ($\lambda$ larger or smaller than $\lambda_{eq}$), while the physically richer MDM has four.

We now turn to the relatively complicated behaviour in the interval $\lambda'_{S}(z_{eq}) < \lambda < \lambda'_{J}(z_{eq})$. Consider the time evolution of a perturbation at scale $\lambda$ within this interval. A relevant consideration is whether or not $\lambda$ was smaller
or larger than the Jeans length at the $\lambda$ horizon-crossing time. One can compute that

$$z_{\text{ent}}(\lambda) \sim 4.6 \times 10^5 \frac{\lambda}{\text{Mpc}},$$

with Eq. (3.9) then yielding

$$\lambda'_{J}(z_{\text{ent}}(\lambda)) \sim \frac{360 x^2 y}{\sqrt{1 + \frac{130 y}{\lambda/\text{Mpc}}}} \text{ Mpc.}$$

The special scale $\lambda_C$ which is equal to the Jeans length at $z_{\text{ent}}(\lambda_C)$ is then

$$\lambda_C \sim 140 y \left( -1 + \sqrt{1 + 6 x^2} \right) \text{ Mpc.}$$

For $\lambda > \lambda_C$, the scale is larger than $\lambda'_{J}(z_{\text{ent}}(\lambda))$, otherwise it is smaller. Perturbations on scales $\lambda_S < \lambda < \lambda_C$ therefore start to oscillate about their horizon-entry values upon entering inside the horizon; the averaged power spectrum is flat within this interval.

Perturbations on scales $\lambda_C < \lambda < \lambda'_{J}(z_{eq})$ exhibit more complicated behaviour. Upon entering the horizon, the perturbation begins to grow (logarithmically) slowly. But the Jeans length increases as $z$ decreases, and at some moment $z = z_{\text{end}}$ it overtakes $\lambda$. One may estimate that

$$\frac{z_{\text{ent}}(\lambda)}{z_{\text{end}}(\lambda)} \sim \frac{(\lambda/\text{Mpc}) y}{920 x^4 y^2 - 0.008(\lambda/\text{Mpc})^2}.$$  

The larger this ratio, the larger the perturbation growth. It is easy to see that the ratio in fact increases monotonically from $\lambda = \lambda_C$ until $\lambda = \lambda'_{J}(z_{eq})$, which means that the growth factor for the perturbation also grows in this interval. This completes the explanation of the qualitative features of Fig. 1.

Figure 2 depicts the processed spectrum at the other critical moment: mirror photon decoupling at $z = z'_{\text{dec}}$. After this moment, the mirror Jeans and Silk lengths fall to very small values, and of course the universe is now matter...
dominated. Perturbations at all scales therefore begin to grow in proportion to the cosmological scale factor $a$, the processed spectrum retaining its $z = z_{\text{dec}}'$ shape (until linearity breaks down).

Many of the qualitative features of Fig. 2 have the same explanation as their counterparts in Fig. 1. There are some differences, though. Perturbations on scales larger than $\lambda_J'(z_{\text{dec}}')$ [see Eq. (3.11)] grow linearly with $a$ because the universe is matter-dominated. A scale between $\lambda_J'(z_{\text{eq}})$ and $\lambda_J'(z_{\text{dec}}')$ is larger than the Jeans length at its horizon entry time, so the associated perturbation grows until that scale is overtaken by $\lambda_J'$ before $z = z_{\text{dec}}'$.

The spectrum therefore has a maximum at the scale

$$\lambda_{\text{max}} = \lambda_J'(z_{\text{dec}}').$$  \hfill (5.6)

Using Eq. (3.11), this evaluates to

$$\lambda_{\text{max}} \sim 370 \frac{x^2 y}{\sqrt{1 + x_{\text{eq}} x}} \text{ Mpc}. \hfill (5.7)$$

B. Processed power spectrum for case II: $x < x_{\text{eq}}$

For this case, mirror photon decoupling precedes matter-radiation equality ($z_{\text{dec}}' > z_{\text{eq}}$). Figures 3 and 4 depict the processed power spectra at these two moments. Their qualitative features can be explained in a very similar way to the preceding case. There are some points of difference, however.

Examine Fig. 3 first. It is similar to Fig. 1 but $\lambda_{\text{dec}}'$ (the horizon scale at $z = z_{\text{dec}}'$) plays the role previously held by $\lambda_{\text{eq}}$. Between $\lambda_{\text{dec}}'$ and $\lambda_J'(z_{\text{dec}}')$, the spectrum grows logarithmically slowly. Between $\lambda_J'(z_{\text{dec}}')$ and $\lambda_S'(z_{\text{dec}}')$, the spectral curve looks similar to the analogous region in Fig. 1 except that there is no flat part because the scale $\lambda_C'$ is always smaller than the Silk length: the curve is growing, on average in this region, with the perturbations also oscillating about the mean for a given $\lambda$. The spectrum has a maximum at the same scale as computed for case I [see Eq. (5.7)]. Once again, the relatively quick fall off as the scale decreases from $\lambda_{\text{max}}$ to the Silk scale represents qualitatively different behaviour compared to standard CDM.
FIG. 4: As for Fig. 3 except that $z = z_{eq}$.

Turning to Fig. 4, the case II processed power spectrum at matter-radiation equality, we see perturbations falling off as $\lambda^{-2}$ for $\lambda > \lambda_{eq}$ and growing logarithmically as $\lambda$ falls from $\lambda_{eq}$ to $\lambda'(z_{dec}')$. For smaller scales, the form of the power spectrum remains unchanged from its character at $z = z_{dec}'$, displaying slow logarithmic growth in the direction of increasing $\lambda$. Recall that after mirror photon decoupling, the mirror baryon Jeans length plummets to a very small value, so all physically interesting scales are now greater than the Jeans length and thus can grow. The growth imprinted on the processed spectrum at $z = z_{eq}$ is, however, only logarithmic simply because the preceding period was radiation dominated. The oscillations in this regime created prior to $z = z_{dec}'$ remain as a feature of the spectrum because there is no process that can damp them out. The position of $\lambda_{max}$ thus remains unchanged.

For $z < z_{eq}$, the universe is matter-dominated and perturbations on all scales grow in proportion to the cosmological scale factor $a$. The spectrum thus retains its shape at $z = z_{eq}$.

C. Discussion

1. Linear regime

We have seen that for both cases I and II, the processed power spectrum at the relevant moment, $z_{dec}'$ and $z_{eq}$ respectively, displays a peak at $\lambda_{max}$ as given by Eq. (5.7). For scales above $\lambda_{max}$ the MDM perturbation spectrum is identical to that of standard CDM. At scales below the peak, the perturbations oscillate about either a constant mean or one that is slowly rising towards the peak. Such oscillations are not a feature of standard CDM.

The extent of the difference between MDM and CDM therefore hinges on the value of $\lambda_{max}$ (and $\lambda_B'$), and hence on the temperature ratio $x$. Furthermore, the first structures to form will do so at the scale $\lambda_{max}$. It is therefore
interesting to compare $\lambda_{\text{max}}$ with the typical galactic scale,\(^9\)

$$\lambda_{\text{gal}} \sim 3.7 y^{1/3} \text{ Mpc.}$$  (5.8)

Putting $y = 1$ we see that $\lambda_{\text{max}}$ is equal to the galactic scale at

$$x \sim 0.2.$$  (5.9)

For $x > 0.2$, the first structures to form would be larger than galaxies, while for $x < 0.2$ the scale falls rapidly into the sub-galactic regime.

If one classifies dark matter as CDM-like, WDM-like and HDM-like according to whether the first structures are sub-galactic, galactic or super-galactic, respectively, then we conclude that MDM is

1. CDM-like for $x \lesssim 0.2$,
2. WDM-like for $x \sim 0.2$, and
3. HDM-like for $x \gtrsim 0.2$.

Given that top-down structure formation appears to be disfavoured, we conclude that

$$x \lesssim 0.2$$  (5.10)

is the favoured temperature range for large scale structure formation.

The smaller $x$ is, the more closely the MDM processed power spectrum in the linear regime resembles its analogue for standard CDM. It is interesting that MDM becomes CDM-like for $x$’s that are not too small relative to our indicative lower limit of about 0.01 [recall the discussion leading to Eq. (2.4)].

A stringent test of the perturbation spectrum at small scales in the linear regime arises from Lyman-$\alpha$ forest data \(^{30}\). A detailed study of the implications of these data for MDM is certainly well motivated. In this paper we will have to settle for the rough guidance provided by existing analyses constraining warm dark matter. MDM and WDM are similar in that each exhibits small scale wash-out, though the mechanisms are different (Silk damping and free-streaming, respectively). In Ref. \(^{31}\), Narayanan et al use Lyman-$\alpha$ data to constrain the mass $m_{WDM}$ of the WDM particle through its free-streaming scale $R$, given by

$$R \simeq 0.2 (\Omega_{WDM} h^2)^{1/3} \left( \frac{m_{WDM}}{\text{keV}} \right)^{-4/3} \text{ Mpc}$$  (5.11)

in terms of the cosmological WDM density $\Omega_{WDM}$. They obtain

$$m_{WDM} > 0.75 \text{ keV}$$  (5.12)

for an $\Omega_{WDM} h^2 = 0.2$ universe. Adopting the resulting upper bound on $R$ as a rough bound on $\chi_S$, we get from Eqs. (4.10), (5.11) and (5.12) that

$$x \lesssim 0.06,$$  (5.13)

where $y = 0.14/0.2$ was used for consistency. This bound appears to be more stringent than the $x < 0.2$ range obtained by requiring that $\lambda_{\text{max}}$ be sub-galactic.

\(^9\) As usual in this context, what we mean by this is the size the material forming a galaxy would have today had non-linearity not set in.
2. CMBR acoustic peaks

Over the past decade the measurements of the Cosmic Microwave Background anisotropy provided us with a powerful tool for finding the cosmological parameters and studying the structure formation in the Universe. In particular, recent WMAP data have established that the adiabatic perturbation scenario is favoured over isocurvature perturbations. A natural question is then: could the CMB data help us in discriminating between MDM and other types of dark matter?

From our previous discussion it follows that the new features of the MDM power spectrum might leave an imprint on the CMB anisotropy at small angular scales, dictated by Eq. (5.7). With decreasing these angular scales become smaller which corresponds to the larger values of . Thus we could expect a non-negligible effect only at larger which is comparatively less interesting in view of the arguments presented above. However, it could be worthwhile to give further precision to this qualitative argument before a final conclusion can be made. In any case, this follow-up study should be done in combination with the analysis of the large-scale structure surveys such as 2dF and the Sloan Digital Sky Survey.

3. Non-linear regime

No matter what the value of , there is no doubt that the non-linear evolution of MDM must be very different from that of standard CDM. We certainly expect MDM to eventually form mirror stars, mirror galaxies, and so on, by analogy with ordinary matter. However, the details of this evolution cannot be an exact parallel to that of ordinary matter. One very important reason for this is that the mirror-helium to mirror-hydrogen ratio from mirror primordial nucleosynthesis will be significantly higher than its ordinary counterpart, thus affecting star formation and evolution, which in turn will influence mirror galaxy formation. No one has yet attempted a detailed study of this interesting topic, apart from pointing out relatively obvious consequences such as the faster evolution of mirror stars, and we have no progress to report ourselves. Suffice it to say that purely observational searches for compact mirror objects within and in the halo of our galaxy should be pursued irrespective of the status of theoretical investigations.

The detection of early reionisation by WMAP is potentially highly relevant for MDM. Early reionisation implies early (ordinary) star formation, which in turn requires sufficient power on small scales to encourage gravitationally collapsed objects to form. Indeed, since WMAP claims to have ruled out WDM through this means, one might be concerned that MDM is similarly ruled out. This is not obviously so, however. The earliest stars must arise from some rare large amplitude fluctuations going non-linear. But since WDM is non-dissipative, whereas MDM is chemically complex and dissipative, the analogy between the two breaks down in the non-linear regime. It is a priori possible that the collisional damping of small scale perturbations is compensated by the greater capacity of MDM to clump compared to regular WDM. This is a very interesting topic for future studies of MDM.

VI. CONCLUSION

Mirror matter is a natural dark matter candidate because it is stable. Furthermore, the mirror matter model has aesthetic appeal through its invariance under the full Poincaré group, including improper Lorentz transformations. Because the microphysics of mirror matter is basically identical to that of ordinary matter, the study of its cosmological implications is well-defined, depending on a small number of a priori free parameters (the ratio of the relic mirror photon temperature to that of ordinary relic photons and the mirror baryon mass density ).

In this paper, we have looked at the linear regime of density perturbations for mirror dark matter in more detail than the previous major study of Ref. 1. We deduced the semi-quantitative features of the processed power spectrum for MDM, compared it to standard CDM and HDM and explained the origin of the differences. A MDM power spectrum is characterised by a peak at the scale which is itself a function of and . The first structures to form will have size . Requiring this scale to be sub-galactic implies that should be less than about 0.2. Because
bottom-up structure formation is favoured over top-down, we conclude that this is in fact the favoured range for the temperature ratio.

It is encouraging that MDM can behave very similarly to standard CDM for reasonable values of $x$ and $\Omega' B$. However, observational differences hopefully also exist and should be used to discriminate between the two candidates. Future Lyman-\(\alpha\) forest data probing structure at small scales will further challenge the standard CDM paradigm, with the prospect of uncovering a subtle discrepancy, or of constraining models featuring small scale damping even further. For MDM, a constraint on small scale damping translates into an upper bound on $x$. By using an analogy with warm DM, we estimated an upper bound of about 0.06. At some point, the need for a smaller $x$ could clash with plausible mechanisms for baryogenesis, though our ignorance of the correct baryogenesis mechanism will prevent a rigorous ruling out of MDM by this means. More optimistically, the hypothetical discovery of small scale damping would specify the value of $x$.

The non-linear regime obviously will differ strongly, with the generic expectation of compact mirror objects (stars, planets, meteorites) and structures (galaxies and so on). The discovery of early reionisation by WMAP potentially provides important input into the comparison of the MDM scenario with the real universe. Exploitation of this opportunity is not straightforward, however, because the dissipative and chemically complex nature of MDM acts as a barrier to confident theoretical analysis of the formation of the earliest stars in the MDM universe.

Finally, if mirror matter supplies all of the non-baryonic dark matter, then existing dark matter searches for WIMPs and axions should fail. (It is of course possible that the universe contains both MDM and standard CDM, even though we did not focus on that hypothesis here.)

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[30] See, for example, D. H. Weinberg et al., astro-ph/0301186 for a review and additional references.