New constraints on space-time Planck scale fluctuations from established high energy astronomy observations

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Abstract

The space-time metric is widely believed to be subject to stochastic fluctuations induced by quantum gravity at the Planck scale. This work is based on two different phenomenological approaches being currently made to this topic, and theoretical models which describe this phenomenon are not dealt with here. By using the idea developed in one of these two approaches in the framework of the other one, it is shown that the constraints on the nature of Planck scale space-time fluctuations already set by the observation of electrons and gamma-rays with energies above 15 TeV are much stronger than have been shown so far. It is concluded that for the kind of Planck scale fluctuations implied by several models, including the most naive one, to be consistent with the observations, the transformation laws between different reference frames must be modified in order to let the Planck scale be observer-independent.

Key words: Quantum Gravity, Planck scale, space-time: quantum fluctuations, TeV, special relativity
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1 Introduction

Merging relativity and quantum mechanics is one of the greatest challenges of modern physics. It is a very difficult task because the models involved in it are hardly testable experimentally, if at all. Though, recently, various attempts to do so have appeared in the literature. This paper focuses on two particular

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approaches made in this respect, which both search for modifications of kine-
matics and photon propagation at high energy by adding an extra term to the
dispersion relation. These two approaches differ in the nature of this term, and
hence obtain different kinds of results. The first approach [1–3,24] searches for
effects induced by Planck scale space-time fluctuations, therefore introducing
a stochastic term in the dispersion relation, whereas the second one [8–11,16]
introduces a constant term in the dispersion relation. Both approaches have
succeeded in constraining the parameter they add to the dispersion relation
using experimental data from high energy astronomy. Other kinds of observa-
tions also constrain these parameters even more strongly. In the framework of
the first approach such constraints come from optical interferometry [3] but
they are controversial [18,19]. In the case of the second approach they come
from atomic and nuclear physics experiments [23].

The aim of this paper is to show that the application to the first approach
of one of the developments made in the framework of the second approach
allows us to derive new constraints on the nature of space-time at Planck
scale. In this respect, the parametrization of the dispersion relation modifying
term will be generalized as being the sum of a constant and a stochastic term.
In the following paragraphs the first approach will be introduced, and the
modification of the dispersion relation due to Planck scale fluctuations will
be explained. The implications of the latter concerning the nature of the laws
of coordinate transformation between different inertial reference frames will
be discussed. Then, one of the developments made in the framework of the
second approach will be introduced, applied to the first approach and some
details considered. The conclusion and a short discussion will then follow.

2 First approach: the effects of Planck scale space-time fluctuations
on kinematics

Let us first introduce space-time Planck scale fluctuations naively. Using Heisen-
berg’s uncertainty principle one can find that virtual particles having the
Planck energy can pop in and out of the vacuum within the Planck scale
of time and space. The spatial extension of such particles would match the
Schwarzschild radius associated to their mass, therefore their presence would
curve the space-time continuum into something looking like a foam. This is
where the concept of “space-time foam” comes from. This description is very
naive, and quantum gravity models, of course, introduce the phenomenon dif-
ferently. An interesting concept related to this topic is that of quanta of length
and area [22]. Meanwhile, space-time Planck scale fluctuations have not yet
been observed. Following the work of [1,2], we will introduce stochastic fluc-
tuations into the space-time metric.
If space-time is subject to stochastic fluctuations at the Planck scale, then each measurement of lengths and time intervals must be affected by fluctuating terms and one could write:

\[
\begin{align*}
  l &\to l \pm \sigma_l; \quad \sigma_l \simeq l_P \\
  t &\to t \pm \sigma_t; \quad \sigma_t \simeq t_P
\end{align*}
\]  

(1)

where \( l_P \) and \( t_P \) are the Planck length and time intervals respectively. As developed in [1,2], assuming that the de Broglie wavelengths of particles follow the fluctuations of space time (assumption 1) leads to, using \( \hbar = c = 1 \),

\[
\begin{align*}
  \delta T &\simeq t_P \Rightarrow \delta E = \delta \left( \frac{1}{T} \right) \simeq E^2 t_P = \frac{E^2}{E_P} \\
  \delta \lambda &\simeq l_P \Rightarrow \delta p = \delta \left( \frac{1}{\lambda} \right) \simeq p^2 l_P = \frac{p^2}{E_P}
\end{align*}
\]  

(2)

\( E_P \) being the Planck energy. Assuming that the fluctuations on \( t \) and each component of \( l \) are uncorrelated (assumption 2: rotational invariance), one can write [1,2]:

\[
\begin{align*}
  E &\simeq \bar{E} + \zeta \frac{E^2}{E_P} \\
  p &\simeq \bar{p} + \xi \frac{p^2}{E_P}
\end{align*}
\]  

(3)

with \( \zeta \) and \( \xi \) being distributed as a gaussian of mean value \( \mu = 0 \) and variance \( \sigma = 1 \).

A more general way of describing the space-time fluctuations at the Planck scale is to write \( \sigma_x/x = f(x_P/x) \), where \( x \) stands for \( l \) or \( t \), with \( f \ll 1 \) for \( x \gg x_P \) and \( f \gg 1 \) for \( x \lesssim x_P \). In this case, \( f(x) \) can be approximated with the lower order term of its expansion in the range \( x \gg x_P \) in the following way [3]:

\[
\frac{\sigma_x}{x} \simeq a_0 \left( \frac{x_P}{x} \right)^\alpha
\]  

(4)

where both \( \alpha \) and \( a_0 \) are positive constants of order 1. The naive choice for \( \alpha \) is 1, which is equivalent to eq.1 and is indeed the first order term given by quantum loop gravity (see [3]), but it is worth keeping in mind that some models of quantum space-time give other values like \( \alpha = 1/2 \) (random-walk scenario) or \( \alpha = 2/3 \) (holographic principle of Wheeler and Hawking). A discussion about this and citations of these models are given in [3,24] and references therein.
The choice of the reference frame in which to apply the above equations raises an important issue concerning special relativity: in which reference frames do the fluctuations have the Planck scale, if they exist? Consider the three following cases:

- **case A:** Planck scale space-time fluctuations do not exist.
- **case B:** If the fluctuations have the Planck scale in all reference frames, then the laws of coordinate transformations between different inertial reference frames would have to depart from pure Lorentz transformations to let this scale be invariant. This is the milestone of Doubly Special Relativity (DSR) theories, in which both the velocity of light and the Planck scale of length and mass are observer-independent scales [4–7,17].
- **case C:** Space-time fluctuations may have the Planck scale in one preferred reference frame only, and boosted values of this scale in other reference frames. Indeed, there is a preferred reference frame in the Universe: the one where the Cosmic Microwave Background (CMB) appears isotropic. This case has been considered in many phenomenological studies of the effects that a fluctuating space-time would have on kinematics [8], although it implies the abandonment of the relativity principle which stipulates that laws of physics should be the same for all inertial observers.

Expanding eq.3 at the first order in $E/E_P$ in the usual dispersion relation, in the energy range $m^2 \ll E^2 \simeq p^2 \ll E_P^2$ and using assumption 2, one obtains:

$$m^2 = E^2 - p^2 + \frac{\eta E^3}{E_P}$$  \hspace{1cm} (5)

where $\eta$ is distributed as a gaussian with $\mu = 0$ and $\sigma = 2\sqrt{2}$. This expression is obtained in [1,2] as well, although the value of $\eta$’s distribution width is only assumed to be of order unity in the latter. However the $2\sqrt{2}$ value used here can be retrieved in [3]. One can notice that this expression is not invariant under ordinary Lorentz transformations: it is either valid in one reference frame only, or valid in all possible reference frames if the laws of coordinates transformation between different inertial reference frames were to depart from ordinary Lorentz transformations.

Using equation 4 instead of 3 as a starting point, one can derive a more general expression for the dispersion relation:

$$m^2 = E^2 - p^2 + \frac{\eta a_0 E^\alpha E^2}{E_P^2}$$  \hspace{1cm} (6)

equation 5 being the case $a_0 = 1$ and $\alpha = 1$ of this equation.

Equations 3, 5 and 6 express the fact that at each measurement, the measured
values of $E$ and $p$ are different from their mean value. Consider interactions where the energy exchanged by the particles involved is $\ll E_P$. The typical scales of length and time of these interactions are much larger than the Planck ones. Hence, one should stipulate independent fluctuations for each initial and final particle [2]. It is worth noticing that the conservation of energy-momentum then still applies on the macroscopic scale for the mean values of $E$ and $p$, but does not apply any more in individual interactions within the amplitude defined by the fluctuating term of equations 5 and 6.

3 Introduction of the second approach and demonstration

In the analysis performed in [1,2] and [3] $\eta$ is a stochastic variable which accounts for the space-time fluctuations specified in eq.1. On the other hand, it is introduced as a constant in the analysis performed in [8,9], [10,11] and [16]. In these approaches the modification of the dispersion relation does not come explicitly from space-time fluctuations. Instead, it comes from the introduction of a minimum length in [10,11] (see [5,6]) and [16], or from quantum field theory or theoretical approaches to quantum gravity in [8,9]. In the latter approach, $\eta$ depends on the nature of the particle involved and does not have to be of order 1. The general form of the modified dispersion relation is then:

$$m_k^2 = E_k^2 - p_k^2 + \frac{\eta_k E_k^{\alpha+2}}{M_0^\alpha}$$

where $M_0$ is of the order of $E_P$, and the subscript $k$ stands for the nature of the particle involved.

A priori the dispersion relation could also be altered by both a constant term and a stochastic term due to Planck scale space-time fluctuations. Let’s write $\eta = \eta_a + \eta_s$, where $\eta_a$ is the constant term and $\eta_s$ the stochastic one. From now on, the constant and the stochastic $\eta$ approaches will be referred to as the $\eta = \eta_a$ and $\eta = \eta_s$ approaches respectively.

Many phenomenological studies of Lorentz violating kinematics have been made following either the $\eta = \eta_a$ or $\eta = \eta_s$ approach. Consider here the case where $\eta = \eta_a + \eta_s$. The key point of this paper is to show that conclusions can be obtained by using a study already done in the $\eta = \eta_a$ case and bringing it into the case of $\eta = \eta_a + \eta_s$. This study concerns the 1 vertex electromagnetic interactions involving an electron and a photon, which are forbidden in usual kinematics by the conservation laws of $E$ and $p$ but can be allowed if Planck scale space-time fluctuations are taken into account.
Equation 5 shows that in electromagnetic interactions involving electrons the fluctuating term becomes significant when \( E \geq E_C = (E_pm^2)^{1/3} \approx 15 \text{ TeV} \), where \( m \) is the mass of the electron. If we consider case C, this would hold in the CMB reference frame only and the TeV photons above 15 TeV such as those seen by CANGAROO [20] and HEGRA [13] would obey modified kinematics, as would the very high energy (100 TeV) electrons inferred from ASCA X-ray observations of the Crab nebula [14]. On the other hand, if the Universe is in case B then space-time fluctuations have the same scale in all reference frames, hence one can choose the centre of mass of the interaction to determine \( E_C \). Modified kinematics would then occur in interactions where particles have \( E \geq E_C \) in the centre of mass. In this case the Planck scale fluctuation term of eq.5 would be negligible in the above processes since the energy they involve in their centre of mass is \( \ll E_C \).

In [8,9] and [12], it has been shown that the above observations of very high energy photons and electrons rule out the possibility that \( \eta \) takes negative values\(^1\) in the case \( \eta = \eta_a \) and \( \alpha = 1 \), in the framework of case C. The demonstration is based on the following argument: if \( \eta < 0 \) in case C, photons and electrons of energy above \( E_C \) can undergo 1 vertex interactions. As a result the photons and electrons of energies above \( E_C \) would respectively decay into \( e^+e^- \) pairs and radiate spontaneously, and hence would not be observed. On the other hand, positive values of \( \eta \) are still allowed in the case \( \eta = \eta_a \), because it does not allow the 1 vertex interactions to happen. These conclusions are the same for any value of \( \alpha \leq 1 \).

Now, what happens to these conclusions if \( \eta = \eta_a + \eta_s \), if we assume \( |\eta_a| \lesssim 1 \) (assumption 3)\(^2\)? In this case \( \eta \) could take any sign, hence 1 vertex interactions would be made possible for any value of \( \alpha \leq 1 \). Since such interactions have not been observed, it means that case C would be ruled out if the assumptions made so far are correct. This would imply that Planck scale space-time transformations don’t exist (case A) or that Lorentz transformations would have to be rewritten as it is aimed at in DSR theories (case B). If on the other hand one or more of the assumptions made so far was wrong, it would be a very important clue for the development of quantum gravity theories. Experimental results in atomic and nuclear physics have already set the constraint \( |\eta_a| \ll 1 \) in the case \( \alpha = 1 \) [23], which confirms assumption 3. (High energy astronomy also provides constraints on \( |\eta_a| \) [15], but they are not as stringent as those from [23].)

Let us consider in more detail how these assertions can be demonstrated by calculating the cosine of the angle \( \theta \) between the two outgoing particles of the 1

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\(^1\) The reader who wishes to read [8,9] should note that \( \eta \) is defined here with the opposite sign as in these references.
vertex interactions in the case $\alpha = 1$ [12]. Let us assume here that the standard conservation of $E$ and $p$ still applies in the DSR framework. This is not the case according to the developments made in [6]. However, the modification of this conservation law would not be a fluctuating term, but would come from the effects of the constant component of the modified dispersion relation which has been severely constrained by [23]. Hence this assumption should not affect the result qualitatively.

Using the unmodified dispersion relations, one finds respectively, in the processes $\gamma \rightarrow e^+e^-$ and $e^\pm \rightarrow e^\pm \gamma$:

\[
\begin{cases}
\cos \theta = 1 + \frac{m^2}{2E^2} \frac{1}{u(1-u)^2} > 1 \\
\cos \theta = 1 + \frac{m^2}{2E^2} \frac{1}{(1-u)^2} > 1
\end{cases}
\]  

where $E$ is the energy of the initial particle and $u$ is the fraction of it taken by the positron in the first line and by the photon in the second line. In both cases the cosine is $> 1$ and thus unphysical: the interactions are forbidden.

Now, if one uses the modified dispersion relation and a first order expansion in $(m/E)^2 \sim E/E_P \sim 10^{-15}$, one obtains respectively:

\[
\begin{cases}
\cos \theta = 1 + \frac{m^2}{2E^2} \frac{1}{u(1-u)^2} + \frac{E}{2E_P} \left[ \eta_p u \left( 1 - \frac{1}{1-u} \right) + \eta_e (1-u) \left( 1 - \frac{1-u}{u} \right) + \eta_{\gamma\gamma} \frac{1}{u(1-u)} \right] > 1 \\
\cos \theta = 1 + \frac{m^2}{2E^2} \frac{1}{(1-u)^2} + \frac{E}{2E_P} \left[ \eta_i \left( 1 - u + \frac{(1-u)^2}{u} \right) - \eta_f \left( 1 - u + \frac{(1-u)^2}{u} \right) - \eta_{\gamma\gamma} \left( u + \frac{u^2}{1-u} \right) \right] > 1
\end{cases}
\]

where in the first line $\eta_{\gamma\gamma}, \eta_e, \eta_p$ apply respectively to the photon, the electron and the positron, and in the second line $\eta_i, \eta_f, \eta_{\gamma\gamma}$ apply respectively to the initial electron, the final electron and the photon.

Both cosines can be physical if, respectively:

\[
\begin{cases}
\eta_p u^3 (1-u)(1-2u) + \eta_e u(1-u)^3(2u-1) + \eta_{\gamma\gamma} u(1-u) < \left( \frac{E_C}{E} \right)^3 \\
\eta_{\gamma\gamma} \frac{1-u}{u} - \eta_f \frac{(1-u)^2}{u} - \eta_{\gamma\gamma} u(1-u) < \left( \frac{E_C}{E} \right)^3
\end{cases}
\]

Since the various $\eta$ terms can take stochastic values, these conditions are allowed. In both relations the leading $\eta$ term is the one multiplied by the lowest power of $u$ or $(1-u)$, which is the one associated with the initial particle. It is interesting to note that the second relation has an IR divergence, similar to its 2 vertex equivalent: if $u$ is allowed to be $\ll 1$, this relation can be satisfied even when $E \ll E_C$. This divergence doesn’t have to be treated here, since the study of the phenomenon is conclusive at high energy anyway.
4 Conclusion

The conclusion of this paper is that Planck scale space-time fluctuations described by an exponent $\alpha \leq 1$ are consistent with the observations only if the Planck scale is observer-independent, in the framework of the assumptions made here. Concerned models are: the naive description of space-time, as described by eq.1 [1,2] and implied by quantum loop gravity; the random walk scenario; and the holographic principle of Wheeler and Hawking (see [3]). As a result, there are two possibilities. The first one is that Planck scale space-time fluctuations don’t exist. The second one is that if they exist according to one of the above models the Planck scale has to be observer-independent, which implies that the laws of coordinate transformations between different inertial reference frames have to be changed. If both possibilities were ruled out by other means, one could show that at least one of the assumptions made here is wrong, or that $\alpha > 1$. This would also be a significant clue concerning the development of quantum gravity theories.

Remark: Reference [3] is a recent study of Planck scale space-time fluctuations based on stellar interferometry. It is cited several times in this paper, regarding the developments made in its beginning. It concludes that space-time does not fluctuate at the Planck scale; however, the demonstration it uses is controversial [18,19]. Laser interferometers which will be used in gravitational wave detectors like VIRGO and LIGO will also be able to detect Planck scale space-time fluctuations if they exist, or rule them out [21], giving one more independent insight on the nature of space-time at the Planck scale.

A paper dealing with the same problem and reaching very similar conclusions [25] has been submitted shortly before the submission of the present paper.

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