Tadpoles and Closed String Backgrounds in Open String Field Theory

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ABSTRACT: We investigate the quantum structure of Witten’s cubic open bosonic string field theory by computing the one-loop contribution to the open string tadpole using both oscillator and conformal field theory methods. We find divergences and a breakdown of BRST invariance in the tadpole diagram arising from tachyonic and massless closed string states, and we discuss ways of treating these problems. For a Dp-brane with sufficiently many transverse dimensions, the tadpole can be rendered finite by analytically continuing the closed string tachyon by hand; this diagram then naturally incorporates the (linearized) shift of the closed string background due to the presence of the brane. We observe that divergences at higher loops will doom any straightforward attempt at analyzing general quantum effects in bosonic open string field theory on a Dp-brane of any dimension, but our analysis does not uncover any potential obstacles to the existence of a sensible quantum open string field theory in the supersymmetric case.

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1. Introduction

String field theory is a space-time formulation of string theory which may have the capacity to describe all string backgrounds in terms of a common set of degrees of freedom. Much recent interest in Witten’s open string field theory (OSFT) has been centered around the discovery that this theory can describe D-branes as classical solitons, so that distinct open string backgrounds not related through marginal deformations can appear as solutions of a single set of equations of motion (for reviews see ). Most of the recent work in this area has focused on classical aspects of OSFT (although, for some recent papers which address quantum features of the theory, see ).

In order for string field theory to have a real chance at addressing any of the deep unsolved problems in string theory/quantum gravity, it is clearly necessary that the theory should be well defined quantum mechanically. In an earlier phase of work, some progress was made in understanding the quantum structure of OSFT. This work is summarized and described in the language of BV quantization by Thorn in his review. In this paper we extend this earlier work by carrying out a systematic analysis of the one-loop open string tadpole diagram in Witten’s bosonic OSFT. We analyze the divergence structure of this diagram and the role which closed strings play in the structure of the tadpole, and we describe the implications of this analysis for the quantum theory.

An important aspect of quantum open string field theory is the role which closed strings play in the theory. As has been known since their first discovery, closed strings appear as poles in nonplanar one-loop amplitudes of open strings. An analysis of these poles in the one-loop nonplanar two-point function of OSFT was given in . Because of the existence of these intermediate closed string states, any unitary quantum open string field theory must include some class of composite asymptotic states which can be identified with closed strings. These asymptotic states have not yet been explicitly identified in OSFT, although related open string states which can be used to compute amplitudes including closed strings in OSFT are described in ; other approaches to understanding how closed strings appear in OSFT were pursued in . In this paper, we consider the appearance of closed strings in OSFT from a different point of view than has been taken in previous work on the subject. We show that an important part of the structure of the open string
tadpole comes from the closed string tadpole, which in the presence of a D-brane describes the linearized gravitational fields of that D-brane. This demonstrates that not only do the closed strings appear as poles in the open string theory, but that they also take expectation values in response to D-brane sources within the context of OSFT; this provides a new perspective on the role of closed strings in OSFT.

The relationship between open and closed strings is central to the concept of holography and the AdS/CFT correspondence [?, ?, ?]. In the AdS/CFT correspondence, a decoupling limit is taken where open strings on a brane are described by a conformal field theory; this theory has a dual description as a near-horizon limit of the closed string (gravity) theory around the D-brane. A complete quantum open string field theory would generalize this picture; if OSFT can be shown to be unitary without explicitly including the closed strings as additional dynamical degrees of freedom, we would have a more general holographic theory in which the open string field theory on a D-brane would encode the gravitational physics in the full D-brane geometry in a precise fashion. While we do not directly address these ideas in this paper, some further discussion in this direction is included at the end of the paper.

Until recently, only a few diagrams had been explicitly evaluated in OSFT: the Veneziano amplitude [?, ?, ?], and the non-planar two-point function [?]. These diagrams were computed by explicitly mapping the Witten parameterization of string field theory to a parameterization more natural for conformal field theory, and then computing the diagram explicitly in CFT. There has recently been some renewed interest in studying perturbative aspects of OSFT by developing new techniques for calculating diagrams in the theory [?, ?]. Using these methods it is possible to compute any OSFT diagram to a high degree of accuracy using the level truncation method on oscillators. This method provides an alternative to the CFT method, and gives some information about a wider range of diagrams while lacking the analytic control of the CFT method. In this paper we use both methods, finding that each gives useful information.

The one-loop tadpole diagram we consider in this paper is perhaps the simplest of the one-loop diagrams in OSFT. While a preliminary study of this diagram was done in [?, ?], we expand on the analysis presented in [?] and augment it by using the conformal field theory method to give an alternate expression for the diagram. We also generalize the discussion by computing the diagram for OSFT defined on a Dp-brane background for any p.

The one-loop tadpole diagram has divergences of several kinds. As the modular parameter $T$ describing the length of the internal open string propagator becomes large, there is a divergence from the open string tachyon. This divergence is easy to understand, and can be removed by analytic continuation in the oscillator approach to OSFT. In addition to the large $T$ divergence of the diagram, there are divergences as $T \to 0$. In the conformal frame natural to OSFT this limit corresponds to a pinching off of the world-sheet. In an alternate conformal frame, however, the small open string loop gives rise to a long closed string tube. These two conformal frames are displayed in figure 1. Since the propagation
Figure 1: Two conformally equivalent pictures of the one-loop open string tadpole. a) The open string tadpole is represented as a purely open string process in which a single open string splits into two open strings which then collide. b) The open string tadpole is represented as a transition between an open string and a closed string. The closed string is absorbed into the brane.

over long distances of massive fields is suppressed, only the tachyon and the massless sector of the closed strings contribute to the $T \rightarrow 0$ divergences of the tadpole. Using both the conformal field theory and oscillator approaches, we isolate the $T \rightarrow 0$ divergences in the one-loop tadpole diagram. By extracting the leading divergences of the tadpole, we can separate the divergence arising from the tachyon from any divergences associated with the graviton/dilaton in the massless sector.

The divergence from the closed string tachyon arises because of the usual problem that the Euclidean theory has a real exponent in the Schwinger parameterization of the propagator, and diverges for tachyonic modes. This problem is usually dealt with by a simple analytic continuation. In this case, however, the analytic continuation is rather subtle, as the closed string degrees of freedom which are causing the divergence are not fundamental degrees of freedom in the theory. In the one-loop diagram we study in this paper, the analytic continuation can be done by hand in the CFT approach by explicitly using our understanding of the closed string physics underlying the divergence. Even for this relatively simple diagram, however, there are a number of subtleties in this analytic continuation, and to ensure that we completely remove all the tachyonic divergences we are forced to consider lower-dimensional brane backgrounds. In a more general context, such as for higher-loop diagrams, it would be difficult to systematically treat this type of divergence using open string field theory.

Assuming that the divergence from the closed string tachyon is dealt with by a form of analytic continuation, we are left with possible divergences from the massless closed string states. Such divergences appear only when considering the open string theory on a Dp-brane with $p \geq 23$. This is essentially because the open string tadpole is generated directly from the closed string tadpole, and the closed string tadpole arises from the solution of the linearized gravitational equations with a Dp-brane source [?]. Since a brane of codimension 2 has a long-range potential which goes as $\ln r$, while a brane of codimension 3 has a potential going as $1/r$, we need at least three codimensions to remove the divergences from the massless sector. In general, whether the tadpole is finite or divergent, we find that the structure of the linearized closed string fields in the D-brane background is encoded in the open string tadpole.
After analyzing the divergence structure of the one-loop tadpole, we also consider briefly what one might expect of OSFT at two or more loops. We consider in particular the two loop non-planar diagram which represents a a torus with a hole in it. This diagram contains as a subdiagram the closed string one point function which suffers from a BRST anomaly [?, ?, ?, ?] in world-sheet perturbation theory. We conjecture that in OSFT this diagram will lead to a divergence and BRST anomaly. Since this divergence is purely closed-string in nature, it should occur for the theory on a Dp-brane for any p, and poses a serious problem for any attempt to make sense of the bosonic open string field theory as a complete quantum theory.

In Section 3 we compute the one-loop tadpole using conformal field theory methods. In section 4 we compute the same diagram using oscillator methods. Section 5 contains a discussion of the one-loop open string tadpole in Zwiebach’s open-closed string field theory, where the structure of the diagram is somewhat more transparent. Section 6 synthesizes the results of the preceding sections, and contains a general discussion of the divergences of the tadpole and the role of closed strings in the tadpole. Section 7 contains some concluding remarks. In two appendices we include some technical points. Appendix A contains a discussion of the BRST anomaly in the D25-brane theory. Appendix B contains some comments on the infinite-level limit of the level truncation method used in Section 4.

2. Perturbation theory in open string field theory

We begin with a summary of Witten’s formulation of open bosonic string field theory (OSFT) [?] with an emphasis on perturbation theory. For general reviews of OSFT see [?, ?, ?, ?]. The classical field for OSFT is a ghost number one state, Ψ, in the free open string Fock space. The string field, Ψ, has a natural expansion in terms of the open string fields

\[ \Psi = \int d^{26}p \left[ \phi(p)c_1|0;p\rangle + A_\mu(p)\alpha_\mu c_1|0;p\rangle + \psi(p)c_0|0;p\rangle + \ldots \right], \tag{2.1} \]

where \( \phi \) is the open string tachyon, \( A_\mu \) is the gauge field and \( \psi \) is an auxiliary field. The vacuum \( |0\rangle \) denotes the \( SL(2,\mathbb{R}) \) vacuum. The classical action is given by

\[ S(\Psi) = \frac{1}{2} \int \Psi \star Q_B \Psi + \frac{g}{3} \int \Psi \star \Psi \star \Psi. \tag{2.2} \]

The definitions for the \( \star \)-product and string integration are given in [?, ?, ?, ?, ?, ?] in terms of both oscillator expressions and conformal field theory correlators. We will also make use of two-string and three-string vertices, \( \langle V_2 \rangle \) and \( \langle V_3 \rangle \), which are defined by

\[ \langle V_2||\Psi_1\rangle\Psi_2 \rangle = \int \Psi_1 \star \Psi_2, \tag{2.3} \]
\[ \langle V_3||\Psi_1\rangle\Psi_2\rangle\Psi_3 \rangle = \int \Psi_1 \star \Psi_2 \star \Psi_3, \tag{2.4} \]
and are elements of the two-string and three-string Fock spaces respectively.

The theory has a large gauge group. Infinitesimally the gauge transformations are given by

$$\Psi \to \Psi + Q_B \Lambda + g(\Psi \star \Lambda - \Lambda \star \Psi),$$

(2.5)

where $\Lambda$ is any ghost number 0 field.

To quantize the theory we must fix a gauge. The standard choice for gauge fixing is Feynman-Siegel (FS) gauge fixing which imposes the condition $b_0 \Psi = 0$. It is straightforward to perform tree-level calculations in this gauge, but some care is required when trying to impose this condition on path integrals. Roughly speaking, it turns out that if one tries to introduce Fadeev-Popov ghosts to fix $b_0 \Psi = 0$, the ghosts themselves suffer from a gauge redundancy similar to the gauge redundancy of the original action. To fix this new gauge redundancy one must introduce ghosts for ghosts. As the new ghosts have their own redundancy, this process proceeds forever, creating an infinite tower of ghost fields. Happily, at the end of the day this entire procedure can be summarized as follows [?]:

1. The field $\Psi$ is fixed by $b_0 \Psi = 0$.

2. The ghost number of $\Psi$ is allowed to range over all ghost numbers, not just ghost number 1. The fields of ghost numbers other than one are all ghost fields.

3. $\Psi$ is a grassmann odd field. To define what this means, suppose the states $\{|s\rangle\}$ form a basis for the open string Fock space such that each $|s\rangle$ has definite ghost number. Then if we write $\Psi$ in a Fock space expansion as $\Psi = \sum_s |s\rangle \psi_s$, then $\psi_s$ has the opposite grassmannality of $|s\rangle$.

The form of the action remains the same as in equation (2.2). Using the FS gauge condition of $\Psi$ we can simplify the kinetic term:

$$S_{FS}(\Psi) = \frac{1}{2} \int \Psi \star c_0 L_0 \Psi + \frac{g}{3} \int \Psi \star \Psi \star \Psi.$$  

(2.6)

Given the gauge fixed action we can now develop the Feynman rules for perturbation theory. We can do this in two ways, which we will refer to as the conformal field theory method and the oscillator method.

In the conformal field theory method, the Feynman rules are given in terms of rules for sewing strips of world-sheet together. Amplitudes may be evaluated by conformally mapping the resulting diagrams to the upper half plane for genus 0 or the cylinder for genus 1.

In the oscillator method the Feynman rules are calculated directly from the action using the usual methods from field theory but summing over the infinite number of fields. For any amplitude this gives rise to correlators which can be evaluated using squeezed state methods.

In the next two sections we consider each of these two methods in turn.
In this section we calculate the one-point function using conformal field theory methods. We begin the calculation with the assumption that we are working with the theory on a D25-brane. In section 3.1 we describe how the diagram can be computed by constructing a map from the original Witten diagram to the cylinder. In section 3.2, we specialize to the limit where the internal loop of the diagram is small and study the divergences in this limit. In section ?? we discuss the origin of these divergences from the propagation of closed string modes over long distances (these divergences are discussed further in Section 6). Finally, in section ?? we discuss how the calculation differs for the theory on a Dp-brane with $p \neq 25$.

### 3.1 Mapping the Witten diagram to the cylinder

We begin with a brief discussion of the world-sheet interpretation of FS gauge fixed OSFT. The derivation of this interpretation is given in [? , ? , ? , ?]. The Feynman rules consist of one propagator and one vertex.

The propagator is given by an integral over world-sheet strips of fixed width. By convention the strips are of width $\pi$, and length $T$, where $T$ is integrated from 0 to $\infty$. To ensure the right measure on moduli space $b(\sigma)$ is integrated across the strip [?].

The only vertex in the theory is a prescription for gluing three strips together. The right half of the first strip is glued to the left half of the second and similarly for the second and third strips and the third and first strips.

Using these rules we can construct the one-point amplitude. We start with an external state $|A\rangle$ which propagates along a strip of length $T_A$. We then take a second strip of length $T$ and glue both ends of it and the end of the first propagator together using the vertex. The resulting diagram is pictured in figure 2.

We wish to study this diagram using conformal field theory methods. To do this we use the methods of [?] to map the diagram to a cylinder. For another approach to this conformal mapping problem, see [?]. Taking the limit $T_A \to \infty$, we can map the external state, $|A\rangle$, to a puncture at the boundary of the cylinder.

It is convenient to flatten the diagram by cutting along the folded edge of the external propagator in figure 2 and cutting the internal propagator in half. The resulting diagram is displayed in figure 3 a. We will let $\rho$ be the coordinate on the Witten diagram and $u$ be the coordinate on the cylinder. To enforce Neumann boundary conditions along the boundaries of the diagram, we use the doubling trick. Since the double of the cylinder is a torus, we may use the theory of elliptic functions to determine $\rho(u)$.

![Figure 2: The world-sheet diagram for the one-point function.](image-url)
Consider the image of the Witten diagram under \( \rho \to u \) shown in figure 3 b. The top and bottom of the diagram are identified as well as the left and right edges. The external state, \( |A\rangle \), is mapped to a puncture at point \( A \) which we choose to be at \( u = 0 \). The midpoint is mapped to point \( \beta \). By the symmetry in the diagram, we can set the real part of \( \beta \) to zero. Since the vertex in the original Witten diagram had an angle of \( 3\pi \), the function \( \rho(u) \) must behave as \( \rho(u) - \rho(\beta) \sim (u - \beta)^{3/2} \) near \( \beta \). This implies that \( d\rho/du \) has a branch cut. 

The integral around the puncture \( A \) of \( d\rho/du \) corresponds, in the original Witten diagram, to the total width of the external propagator plus its double, which is fixed to be \( 2\pi i \). This implies that \( d\rho/du \) has a simple pole at \( A \) of residue one.

We now define the quadratic differential \( \phi(u) = (d\rho/du)^2(u) \). From the form of \( d\rho/du \) near \( \beta \) we see that \( \phi(u) \) has no branch cut, just a simple zero at \( \beta \). In order to preserve Neumann boundary conditions we must also include a zero at the image of \( \beta \) under the doubling. Since we have put the top of the diagram along the real axis, we just get a second zero at \( \beta^* \). The only other piece of analytic structure we need is that since \( d\rho/du \) had a simple pole at \( A \), \( \phi(u) \) has a double pole at \( A \).

Now since \( \phi(u) \) is a meromorphic function on a torus we may determine it using \( \vartheta \)-functions:

\[
\phi(u) = C \frac{\vartheta_1(u - \beta, q)\vartheta_1(u - \beta^*, q)}{\vartheta_1^2(u, q)},
\]

(3.1)

\footnote{One has to be careful that the contour does not cross the branch cut so that, in the \( \rho \) coordinates, the contour is continuous.}

\( Figure 3: \) a) The tadpole diagram laid flat by cutting the external propagator along its middle and cutting the internal propagator in half. The edges to be identified are indicated by dashes. b) The doubled image of the tadpole under the conformal map \( u(\rho) \). The left and right sides of the image are identified as well as the top and bottom. The image of the midpoint of the vertex is denoted by \( \beta \).
where \( q = e^{-\pi \tau} \). The constant \( C \) is determined from the condition that \( \sqrt{\phi(u)} \) has a pole of residue one at \( A \):

\[
C = \frac{(\vartheta_1'(0))^2}{\vartheta_1(-\beta) \vartheta_1(-\beta^*)}.
\]

The two constants \( \beta \) and \( \tau \) can be determined from the height and width of the diagram by integrating \( d\rho/du \) along the curves \( \gamma_1 \) and \( \gamma_2 \). We have

\[
\int_{\gamma_1} du \sqrt{\phi(u)} = 2\pi i,
\]

\[
\int_{\gamma_2} du \sqrt{\phi(u)} = T.
\]

In general these relations cannot be solved analytically, but it is straightforward to solve them numerically and thus to determine \( \tau \) and \( \beta \) as functions of \( T \).

At this point one could, in principle, evaluate the diagram for any given \( A \). If we suppose that the state \( A \) is defined by a vertex operator inserted on a half-disk with coordinates \( v \), we can easily compute the map \( \rho(v) \) from the half-disk to the tadpole diagram. The diagram at a fixed modular parameter is then computed by evaluating

\[
\langle (u(\rho \circ \rho(v) \circ A)(u(\rho) \circ \frac{1}{2\pi i} \int d\rho b(\rho))) \rangle_{\text{torus}}
\]

where the contour of integration runs across the internal propagator. This correlator implicitly defines a fock space state, \( |T(T)\rangle \), given by

\[
\langle A|T(T)\rangle \equiv \langle (u(\rho \circ \rho(v) \circ A)(u(\rho) \circ \frac{1}{2\pi i} \int d\rho b(\rho))) \rangle_{\text{torus}}.
\]

Note that the state \( |T(T)\rangle \) is a function of the modular parameter \( T \). The full tadpole diagram is given by integrating over this modular parameter. We thus define the full tadpole state \( |T\rangle \) by

\[
|T\rangle = \int_0^\infty dT \; |T(T)\rangle.
\]

The expression (3.4) can only be evaluated numerically, since we do not know \( \rho(u) \) explicitly (only its derivative) and we cannot analytically solve the constraints (3.3). Furthermore, there are several types of divergences in the integral over \( T \) in (3.6). The integrand (3.5) diverges as \( T \to \infty \) due to the open string tachyon. While this divergence is difficult to treat in the CFT approach, its physical origin is clear and is quite transparent in the oscillator approach, where this divergence can be treated by a suitable analytic continuation. We discuss the open string tachyon divergence further in Sections 4 and 6. In addition to the divergence as \( T \to \infty \), there are further divergences as \( T \to 0 \) arising from the closed string. These divergences are much more subtle, as closed strings are not explicitly included among the degrees of freedom in OSFT, but arise as composite states of highly excited open strings. We thus seek to evaluate the tadpole diagram in an expansion around \( T = 0 \), where much of the interesting physics in the diagram is hidden.
We now focus on the region of moduli space near $T = 0$. Unfortunately the map $\sqrt{\phi(u)}$ cannot easily be expanded around this limit. To get around this we use a trick. It turns out that the conformal map greatly simplifies if, instead of fixing the integral along $\gamma_1$ to be $2\pi i$, we set it equal to some parameter $H$ and take $H \to i\infty$ holding $T$ fixed. This is equivalent to gluing a semi-infinite cylinder to the bottom of the tadpole. Later we will see how to replace this long cylinder with a boundary state to reduce back to the finite length cylinder case, but for the moment we just consider the conformal map in this limit.

By solving the constraints (3.3) numerically, one can verify that as $H \to \infty$ with $T$ fixed, $\beta$ limits to a constant $\beta_0$, while $\tau \to i\infty$. Recalling that $q = e^{i\pi \tau}$, we see that since $\tau$ is pure imaginary, $q \to 0$ as $H \to \infty$. Thus we may set $q = 0$ in our map to get

$$\left.\frac{d\rho}{du}\right|_{q=0} = \sqrt{\phi(u)} \left|_{q=0} = \sqrt{\cot^2(u) - \cot^2(\beta_0)}. \right. \tag{3.7}$$

We can now solve for $T$ in terms of $\beta_0$ by performing the integral along $\gamma_2$:

$$T = \oint_{\gamma_2} du \sqrt{\cot^2(u) - \cot^2(\beta_0)} = -\frac{i\pi}{\sin(\beta_0)}. \tag{3.8}$$

Using this relation, we can eliminate $\beta_0$ from the definition of our conformal map:

$$\lim_{H \to \infty} \left(\frac{d\rho}{du}\right) = \sqrt{1 + \left(\frac{T}{\pi}\right)^2 + \cot^2(u)}. \tag{3.9}$$

This function may even be integrated analytically although the resulting expression is cumbersome. For notational simplicity we now consider $d\rho/du$ only in the limit of $H \to \infty$ and we will assume that the tadpole diagram has an infinitely long tube extending from the bottom.

Before we consider the effect of replacing the long tube at the bottom of the diagram with a boundary state, we consider the effect of the map $\rho(u)$ on the external state $A$ as $T \to 0$. Note that if we take the limit that $T \to 0$, $d\rho/du$ simplifies even further.

$$\lim_{T \to 0} \left(\frac{d\rho}{du}\right) = \sqrt{1 + \cot^2(u)} = \frac{1}{\sin(u)}, \tag{3.10}$$

where one must be careful about the interpretation of the branch cuts. Integrating this function yields

$$\int du \frac{1}{\sin(u)} = \log \left(-\tan \left(\frac{u}{2}\right)\right). \tag{3.11}$$

While this may not seem like a familiar map, it is actually a representation of the identity state. Putting

$$h(z) = \frac{1 + iz}{1 - iz}, \tag{3.12}$$
Figure 4: A circle of conformal maps showing the equivalence of the two prescriptions for the identity state. In the limit that $T \to 0$ traversing the diagram clockwise from the surface in the upper left to the tadpole in the bottom left is equivalent to the trivial map $f(z) = z$.

We consider the circle of conformal maps pictured in figure 4. One can verify that traversing the diagram counterclockwise (starting from the vertical strip at the bottom and proceeding to the representation of the identity in the upper left), gives the same map as $\rho(u)$ when $T \to 0$. This implies that the tadpole diagram with a long tube attached to the bottom is conformally equivalent to the identity state with an operator inserted in the corner in the limit $T \to 0$. Such states are known as Shapiro-Thorn states [?, ?].

We now must deal with the fact that the original diagram had a tube of finite length extending from the bottom. We thus consider the effect of replacing the infinite tube at the bottom of the tadpole with the closed string boundary state with Neumann boundary conditions. The boundary state in disk coordinates for Neumann boundary conditions can be written

$$\langle B| = \langle 0| (\tilde{c}_{-1} + c_1)(\tilde{c}_0 + c_0)(\tilde{c}_1 + c_{-1}) \exp \left( \sum_{m>1} b_m \tilde{c}_m + \tilde{b}_m c_m \right) \exp \left( \sum_{n\geq 1} \frac{1}{n} \alpha_n \cdot \tilde{\alpha}_n \right) \right) \tag{3.13}$$

where the oscillators are the usual closed string oscillators and $\langle 0|$ is the closed string $SL(2, \mathbb{R})$ vacuum. When we map to cylinder coordinates, the disk becomes a semi-infinite tube with the boundary state being propagated in from infinity. By rescaling the size of the cylinder we can make it the same circumference as the long tube at the bottom of the tadpole.