A Turbulent Origin for Flocculent Spiral Structure in Galaxies:
II. Observations and Models of M33

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ABSTRACT

Fourier transform power spectra of azimuthal scans of the optical structure of M33 are evaluated for B, V, and R passbands and fit to fractal models of continuum emission with superposed star formation. Power spectra are also determined for Hα. The best models have intrinsic power spectra with 1D slopes of around \(-0.7\pm0.7\), significantly shallower than the Kolmogorov spectrum (slope = \(-1.7\)) but steeper than pure noise (slope = 0). A fit to the power spectrum of the flocculent galaxy NGC 5055 gives a steeper slope of around \(-1.5\pm0.2\), which could be from turbulence. Both cases model the optical light as a superposition of continuous and point-like stellar sources that follow an underlying fractal pattern. Foreground bright stars are clipped in the images, but they are so prominent in M33 that even their residual affects the power spectrum, making it shallower than what is intrinsic to the galaxy. A model consisting of random foreground stars added to the best model of NGC 5055 fits the observed power spectrum of M33 as well as the shallower intrinsic power spectrum that was made without foreground stars. Thus the optical structure in M33 could result from turbulence too.

Subject headings: turbulence — stars: formation — ISM: structure — galaxies: star clusters — galaxies: spiral
1. Introduction

The flocculent spiral structure in several nearby galaxies was recently shown to have a power spectrum for azimuthal scans that resembles the power spectrum of HI emission from the Large Magellanic Clouds (Elmegreen, Elmegreen, & Leitner 2003; hereafter Paper I). This power spectrum has a long-wavelength part that falls as $\sim 1/k$ for wavenumber $k$, an intermediate part that falls approximately as $k^{-5/3}$, and a short wave part that falls as $\sim k^{-1}$ again. Individual stars contribute most to the short waves, and the brightest stars can dominate this part of the power spectrum if they are not removed. The distance scale for the $k^{-5/3}$ part typically extends up to several hundred parsecs, depending on galaxy and galacto-centric radius. For NGC 5055, which is a large flocculent galaxy, the $\sim k^{-5/3}$ part extends up to 1 kpc.

Aside from the short-wave contamination from individual bright stars, these optical power spectra closely resemble the power spectrum of HI emission from the LMC (Elmegreen, Kim, & Staveley-Smith 2001) and dust absorption from the nuclear regions of two Sa-type galaxies (Elmegreen, Elmegreen & Eberwein 2002). This similarity led us to suggest that young stars follow the turbulent gas as they form, and that the outer scale for this turbulence is comparable to the disk thickness or the inverse Jeans length in the interstellar medium. The implication of this is not only that gravitational instabilities form flocculent arms, which was well-known before from numerical simulations (Sellwood & Carlberg 1984), but also that these instabilities generate much of the turbulence in the interstellar medium, which both structures the gas and causes the stars to form in fractal patterns (see also Huber & Pfenniger 2001; Wada, Meurer, & Norman 2002).

Paper I reviewed the observations of power spectra in local gas and dust (e.g., Crovisier & Dickey 1983; Gautier, et al. 1992; Green 1993; Armstrong et al. 1995; Stützki et al. 1998; Schlegel, Finkbeiner, & Davis 1998; Deshpande, Dwarakanath, & Goss 2000; Dickey et al. 2001) and in whole galaxies (Stanimirovic, et al. 1999; Stanimirovic et al. 2000; Elmegreen, Kim, & Staveley-Smith 2001). It also discussed the possible links between gaseous density structures and the structures that come from velocity in a turbulent medium (e.g., Lazarian & Pogosyan 2000; Goldman 2000; Stanimirovic & Lazarian 2001; Lazarian et al. 2001; Lithwick & Goldreich 2001; Cho & Lazarian 2003). Here we model the stellar light distribution in two galaxies to illustrate that star formation follows this turbulent gas and acquires a similar power spectrum. The nature of interstellar turbulence and star formation are not understood well enough to explain the results. Several processes are possible, including the formation of clouds and stellar complexes in moving sub-clouds that act like passive scalars in a large-

\footnote{Based on data obtained at the Canada-France-Hawaii Telescope}
scale turbulent flow (Goldman 2000; Boldyrev, Nordlund, & Padoan 2002; Paper I), the formation of clouds and stars in turbulence-compressed regions (Klessen, Heitsch, & Mac Low 2000; Ossenkopf, Klessen, & Heitsch 2001; Klessen 2001; Padoan et al. 2001a; Padoan & Nordlund 2002) and the formation of stars in shocks that are driven by other stars in a turbulent medium (Elmegreen 2002a). All of these processes give about the same power law structure for density and star formation.

The advent of large-scale digital CCD images measuring several thousand pixels on a side has made power spectrum analyses of galactic structure meaningful. Large images are necessary to get enough dynamic range to see the power law part of a power spectrum if there is one. In this respect, the optical survey of galaxies by the CFH12K camera of the Canada-France-Hawaii telescope (CFHT) is ideal. Here we analyze the CFHT image of M33 (Cuillandre, Lequeux, & Loinard 1999), which has an original pixel size of 0.206 arcsec and a binned size of 0.412 arcsec for the full galaxy mosaic. The maximum azimuthal circumference for the part we use is 10000 px, or 4120 arcsec, which gives scales covering 4 orders of magnitude. We take power spectra in both the azimuthal direction at equally spaced radii and along the spiral direction, following the arm and interarm regions.

We also model this power spectrum and the power spectrum of NGC 5055, a flocculent galaxy from Paper I, with fake-galaxy images of a fractal Brownian motion continuum plus discrete stars that follow this continuum.

### 2. Power Spectra of the Galaxy M33

Figure 1 shows two images of M33 with azimuthal and spiral lines at which scans of intensity were obtained and converted to 1D power spectra. The azimuthal scans are circles in the galaxy disk plane and the spiral scans go through both arm and interarm regions at a constant pitch angle (24.5 degrees). This pitch angle fits the strong arm in the south using deprojected coordinates (see Sandage and Humphreys 1980 for pitch angle fits to each arm). Power spectra were obtained for intensity scans along these directions, with each scan one pixel wide to maximize k-space resolution. For the azimuthal scans, 8 adjacent pixel-wide power spectra were averaged together to make the final power spectrum at each radius. These 8 adjacent scans for the 10 selected radii are shown in the figure as dark pixels. For the spiral scans, 21 adjacent pixel scans were used separately for each power spectrum, and then these 21 power spectra were averaged together to give the resultant power spectrum along each of the 8 spiral or interspiral cuts.

Figure 2 has the azimuthal intensity scan from the ellipse that is fourth out from the
center of the R-band image, and it shows the power spectrum of this scan multiplied by $k^{5/3}$. The original scan is on the left and a version with the brightest stars removed is on the right. The power spectrum is the sum of the squares of the sine and cosine Fourier transforms, plotted in log-log as a function of the wavenumber, $k$. The abscissa in the figure is the wave number normalized so that the right-most point, with a value of 1, has $k = 1/2$ pixels$^{-1}$. The distance corresponding to any normalized $k$ value is $2/k$ pixels. For our images one pixel is 1.68 parsecs, assuming a distance to M33 of 0.84 Mpc (Freedman, Wilson & Madore 1991).

The power spectra in Figure 2 and other figures here have been multiplied by $k^{5/3}$ in order to flatten what is normally a $k^{-5/3}$ spectrum for incompressible Kolmogorov turbulence and make this part easier to see. There are many possible explanations for a power spectrum that is approximately $k^{-5/3}$, as reviewed in detail in Paper I. It is probably not the result of motions in a perfectly incompressible fluid, as for the Kolmogorov problem, but some degree of incompressibility is still possible (Goldman 2000; Boldyrev, Nordlund, & Padoan 2002; Lithwick & Goldreich 2001). Even if it is highly compressible, the power spectrum of density structure is about the same (see Lazarian & Pogosyan 2000; Cho & Lazarian 2003).

Bright stars are clipped from the intensity scans in 3 steps. First we find the average intensity level of a clipped version of the scan, clipped below zero and at a high enough level to remove the brightest and saturated stars, all of which are foreground. Then we find the running boxcar average 31 pixels wide of the same scan, clipped a second time at some intermediate height above the average to remove the pedestals of the brightest stars, but not clipped low enough to remove the stars in M33. Finally, we clip the scan at a level above the running average that is 3 times the Gaussian $\sigma$ for the rms noise in this final clipped version; this last step is done with several iterations. The first step is necessary to account for the exponential disk; the second to include large-scale variations from star-formation regions and spiral waves, and the third to reduce the impact of stars in the power spectrum.

The power spectrum has a slightly steeper rise in Figure 2 when bright stars are included, and it has a deeper dip at high frequency. M33 has many foreground stars and bright point sources inside the galaxy so star removal is difficult. Thus there is a residual high-$k$ dip in all of the results shown here. The rising part could also be a bit too steep as a result of residual stars. This translates into a non-normalized power spectrum that is too shallow in its decline. The galaxies in Paper I had fewer foreground stars and shallower rising parts in their $k^{5/3}$—normalized power spectra. One of them, NGC 5055, will be modelled in the next section.

Power spectra of M33 for ten radii in three passbands are shown in Figure 3, and power spectra for the spiral scans in R band, along with the intensity profiles averaged
over 21 adjacent single-pixel wide scans (as discussed above), are shown in Figure 4. The power spectra are all similar regardless of color, radius, arm, or interarm position. After multiplication by \( k^{5/3} \) as in these figures, there is a slowly rising part on the left, a flat part in the middle-right, and a dip and second rising part on the far right. The density wave in M33 appears in the left-most few points in the azimuthal power spectra (Fig. 3), which turn up by a factor of \( \sim 10 \) compared to the extrapolated curves at low \( k \). The left-most point corresponds to a one-arm spiral, and the second-left most point corresponds to a two-arm spiral. There is no such systematic turn up in the spiral arm and interarm power spectrum (Fig 4).

The similarity of the power spectra for the different passbands may be surprising considering the galaxy looks smoother in red colors. However the power spectrum only gives a relative measure of power at each spatial frequency, whereas the visual impression of smoothness is based mostly on the ratio of high frequency emission to average brightness. Slight differences in the 3 passbands are visible at high \( k \), where the prominence of the stellar dip decreases toward the red.

The power spectra and intensity scans of a clipped H\( \alpha \) image of M33 taken with the same instrument are shown in Figure 5. The dashed line has a slope of 1, which is similar to the slope of the \( k^{5/3} \)-normalized power spectrum of the optical emission in Figure 3. There are many small H\( \alpha \) sources that act like stars in the intensity scans, giving dips in the power spectra at high frequency as for the optical images.

The top axes in Figures 3, 4, and 5 give the distance scale in parsecs that corresponds to \( 2/k \) on the abscissa. The outer scale for the horizontal part of the power spectrum is \( 2/0.02 \sim 100 \) pixels, which corresponds to 170 parsecs. This scale is about the same as what we found for galaxies in Paper I, although some aspects of the overall shape result from pixelation, as shown in the next section, and are therefore distance-dependent.

3. Model Power Spectra

Fractal Brownian motion models were made by filling half of a \( 640 \times 640 \times 640 \) cube in \( k \)-space with random complex numbers having real and imaginary values between 0 and 1, and then multiplying these values by \( k^{-\beta/2} \) for \( k = (k_x^2 + k_y^2 + k_z^2)^{1/2} \). The cube is only half full because of the symmetries in the Fourier transforms. The inverse Fourier transform of this cube, from complex to real numbers, gives a three-dimensional fractal with positive and negative numbers having a Gaussian distribution of values. This fractal is then exponentiated to give another fractal, now with a log-normal distribution of all-positive values. This last
step is done to mimic models of turbulence which produce a log-normal distribution of density (e.g., Wada & Norman 2001). An example of a fractal cube made this way is in Figure 2 of Elmegreen (2002b). Three dimensional power spectra of the model reproduce the input power spectrum slope $\beta$ (which is twice the input power of $k$ because the power spectrum is made from the square of the Fourier transform), and one-dimensional power spectra of the model have a slope that is shallower by 2. We denote the slope of the 1D model power spectrum by $\alpha$ and confirmed experimentally that $\alpha = \beta - 2$ for $\beta$ greater than 2. Input $\beta < 2$ give power spectra indistinguishable from noise ($\alpha = 0$). Other fractal Brownian motion models of interstellar structure and a more detailed discussion of $\beta$ are in Stützki et al. (1998).

We varied the intrinsic power $\beta/2$ from $-0.33$ to $+2.66$ and plotted the power spectrum for each model using density scans along a strip and along a circle in the midplane. We also integrated the cube through one dimension with a Gaussian weighting factor centered on the midplane, to simulate the average Gaussian profile of the interstellar medium viewed through a disk face-on. These power spectra should represent the observations of a fractal gas, and in fact models of the HI emission from the LMC were fitted in this way (Elmegreen, Kim & Staveley-Smith 2001).

The optical power spectra presented here are for starlight, and models of this require an extra step. To simulate stars, we use small points of “emission” with Gaussian profiles having a $\sigma$ of 2 pixels and an intensity equal to a random number between 0 and 30; these values were chosen to match the width and depth of the high-$k$ dip. We consider three cases: (1) an image made only of stars with a random distribution in space; (2) an image made only of stars with a fractal distribution in space, and (3) an image made of the fractal continuum in addition to stars with the same fractal distribution in space. The fractal distribution of stellar points is done by going to each cell in the fractal model cube and choosing a random number between 0 and 1. If the density in the cube at this point (differenced from the minimum density and divided by the maximum density for normalization) exceeds some factor times the random number, then we place a star there with the assumed Gaussian profile. If the density is less than this value, then we do not place a star there. If adjacent points have stars, then we add all the stellar profiles together. The multiplicative factor for the (0,1) random number determines how many stars there are. A value of 0.4 was chosen to give intensity profiles that looked reasonably dense in resolved stellar sources. In case (3), we chose stars in the same way but added this starlight to the fractal continuum in each cell.

The most reasonable case is (3). In a typical galaxy there will be both isolated bright stars and a haze of blended faint stars. Case (3) assumes that both follow the fractal distribution of the gas as a result of moderately recent star formation. The underlying
smooth distribution of old stars will not contribute much to the high spatial frequencies but will appear mostly in the lowest few $k$ values, like the spiral density wave. These can be ignored in both the modelled and the observed power spectra.

Figures 6, 7, and 8 show the results for three values of $\alpha$ which bracket our observations. For the best-fit $\alpha = 0.66$ case in Figure 7, the power spectrum and intensity scan from the 4th ellipse out in the R band image is shown at the top. The left panel shows power spectra multiplied by $k^{5/3}$, the middle panel shows conventional power spectra with no multiplier, and the right panel shows the corresponding intensity scans. The power spectrum below the R band observation is for a strip from left to right in the fractal cube. Below that is the power spectrum from a circle at the maximum radius of the model cube, giving a circumference 1700 pixels long. Next comes the power spectrum from continuum plus stars in a fractal pattern, and below that is the power spectrum from the pure stellar image with a fractal pattern. At the bottom is the spectrum for a pure-star image with a random distribution of stellar positions. This pure-star power spectrum is the same in Figures 6, 7, and 8 because it does not depend on $\alpha$.

The best match between the star+continuum power spectrum and the observation of M33 is for $\alpha \sim 0.66$. The $\alpha = 0$ case has a power spectrum that is slightly too steep in its rising part on the left (meaning that in a conventional plot, not normalized with multiplication by $k^{5/3}$, the power spectrum is too shallow in its decline here). The $\alpha = 1.33$ case has a power spectrum that is too slowly rising in this part (too steeply falling in a conventional plot). Generally, the star+continuum case is a better match than the fractal-star case, which has too large a dip at high frequency from the intrinsic stellar profiles. The random star case is a poor fit, having a power spectrum in the normalized plot with an increase as $k^{5/3}$, which corresponds to a flat power spectrum in a conventional plot (as seen at the bottom of the middle panel of Fig. 7).

The azimuthal power spectrum from the fifth ellipse out in the I-band image of NGC 5055 (Paper I; data from HST archives) is reproduced in Figure 9, along with several models. This galaxy has fewer foreground stars and so the dip at high frequency is very slight. The two models at the bottom are pure stars in a fractal pattern. The bottom one has a star brightness that is a random number multiplied by 5 and the next one up has a star brightness that is a random number multiplied by 20. Both star models have Gaussian $\sigma = 2$ px. The power spectra are identical although the intensity traces on the right show these differences clearly. Next up from the bottom are three models with a fractal distribution of stars and the associated continuum. They each have a star brightness equal to a random number times 5 and the stars have $\sigma = 2$ px again. This is only one-sixth the star brightness used for the M33 models, and the high-frequency dip is smaller too, in proportion. The three models
differ in the slope of the intrinsic power spectrum, decreasing from $\alpha = 1.70$ to 1.50 to 1.33 with increasing height in the figure. The power spectra show this difference at low frequency where the rising part gets slightly steeper as $\alpha$ decreases. The middle one, with $\alpha = 1.50$, is the best fit to NGC 5055. The power spectrum of the pure continuum with no stars in this case is shown next to the observation of NGC 5055. It differs from the star+continuum case only slightly at high frequency. We conclude from these models that NGC 5055 has relatively faint point sources and an intrinsic power spectrum for the optical emission with a slope of $-1.5 \pm 0.2$.

The $k^{-1.5\pm0.2}$ power spectrum fit to NGC 5055 is clearly steeper than the $k^{-0.66\pm0.66}$ fit to M33. The difference could be the result of residual stars and unresolved clusters in the M33 galaxy, although NGC 5055, which is 9 times more distant, has 4 times better resolution from HST so the pixel scale in parsecs (3.5 pc) is only a factor of 2 greater for NGC 5055 than for M33. Residual foreground stars could be more important. M33 has many more foreground stars than NGC 5055 because of its large angular size. Foreground stars are randomly positioned and their power spectrum varies like $k^0$, as in Figures 6, 7, and 8. Contamination from numerous random stars will flatten the spectrum.

Figure 10 shows a fit to M33 that starts with the underlying galaxy model for NGC 5055 and adds random foreground stars with an amplitude 6 times larger than in the NGC 5055 model itself and a stellar width of $\sigma = 2$ px. At the bottom is the power spectrum and intensity scan for the foreground stars. Next is the observation of NGC 5055, from Figure 9. The third curves up are the power spectrum and intensity scan for the sum of the best-fit model of NGC 5055 (the $\alpha = 1.5$ model in Fig. 9) and the foreground stars. The top curves are the observations of M33. The model of M33 with foreground stars has the same intrinsic galactic properties as NGC 5055, namely a superposition of galactic stars and a continuum that both follow a fractal pattern giving a 1D power spectrum slope of $-1.5$. The foreground stars make the slope appear shallower, $-0.66$, like the observed slope in M33.

4. Conclusions

The power spectra of optical emission in galaxies show a characteristic structure that can be modelled by a superposition of a fractal unresolved brightness distribution and a corresponding fractal distribution of point-like sources. The intrinsic power spectrum of these distributions has a slope of $-0.66 \pm 0.66$ for M33 and a slope of $-1.5 \pm 0.2$ for NGC 5055. The relatively flat slope for M33 is probably the result of foreground stars, which are difficult to remove from the observations. A model of M33 with foreground stars superposed on the NGC 5055 model demonstrates this point. The power spectrum slope for the foreground-
corrected version of M33, and the slopes for NGC 5055 and the other galaxies in Paper I are all comparable to the one-dimensional power spectrum from turbulence. We infer from this that young stars and clusters are distributed in a fractal pattern that is generated by a turbulent gas.

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Fig. 1.— R-band images of M33 with the azimuthal and spiral strips used for power spectrum analysis. The widths of the strips are 9 and 21 pixels, respectively. North is up. Power spectra were made from single-pixel wide strips and then averaged together.

Fig. 2.— Intensity scan for the fourth ellipse out from the center of the R-band image of M33 (top) and the power spectrum multiplied by $k^{5/3}$ of this scan (bottom). The original scan and power spectrum are on the left, and the version with the brightest stars removed is on the right. The dashed line on the right indicates the extent of the horizontal part in the power spectrum.

Fig. 3.— Power spectra multiplied by $k^{5/3}$ of ten radii in each of 3 passbands for M33. The radii are indicated in figure 1; here the radius increases toward the top of the plot.

Fig. 4.— Power spectra multiplied by $k^{5/3}$ and intensity scans for 8 spiral and interspiral traces in the R band image of M33. The exponential disk is evident in the intensity scans. The ends are tapered with a cosine function to prevent ringing in the power spectrum. Intensity peaks in this image can easily be correlated with bright patches along the spiral lines in Figure 1. The bottom intensity scan and corresponding power spectrum in this figure correspond to the southern spiral strip that goes through the main spiral arm at the bottom of Figure 1. The other scans going up here follow in clockwise order in Figure 1. The northern main spiral arm is the 5th up from the bottom here. The arm scans show the brightest emission in the intensity profiles and have the flattest power spectra at long wavelengths.

Fig. 5.— Power spectra multiplied by $k^{5/3}$ and intensity scans of a clipped Hα image of M33. The dashed line at the top of the left panel has a slope of 1.
Fig. 6.— Model results for a fractal cube made with an intrinsic 1D power spectrum having a power law slope of 0. The bottom power spectrum on the left and the bottom intensity scan on the right are for a random distribution of stars unrelated to the fractal. The dashed line near the power spectrum has a slope of $5/3$, meaning that the un-normalized power spectrum is flat. The second curves up from the bottom are for stars arranged in a fractal pattern. They are based on azimuthal profiles through the midplane of a 640-cubed fractal density distribution, placing a star everywhere the density of the continuum exceeds the peak density value multiplied by a random number that is uniformly distributed between 0 and 0.4. The third scan up from the bottom is this same fractal star distribution added to the fractal continuum distribution. The fourth scan is the continuum only, and the fifth scan is the continuum only in a linear scan through the midplane.

Fig. 7.— Model results for a fractal cube made with an intrinsic 1D power spectrum having a power law slope of $-0.66$. The bottom 5 curves are as in Fig. 5, and the top curves in each panel are the star-reduced R band result for the 4th ellipse out from the center of M33. The panel on the left contains the power spectra multiplied by $k^{5/3}$, as in the other figures here; the panel in the middle has the original power spectra without the normalization. This case with $\alpha = 0.66$ is the best fit to M33 for the model with a fractal continuum and stars (third curve up from the bottom in each panel). The top panel on the right is drawn separately to contain the longer scan length for M33.

Fig. 8.— Model results for a fractal cube made with an intrinsic 1D power spectrum having a power law slope of $-1.33$.

Fig. 9.— The power spectrum of the fifth intensity scan from the center of the I-band image of NGC 5055, from paper I, is shown at the top in this figure, along with the intensity scan itself on the right, and models that bracket the preferred fit are shown below.

Fig. 10.— Another model of M33 is shown here, third up from the bottom. This was made from the best-fit model of NGC 5055 by adding a Poisson distribution of foreground stars. The bottom scans here are the power spectrum and intensity scan for these foreground stars. The second and fourth scans up from the bottom are the observations of NGC 5055 and M33. Dashed lines show matching slopes. The top panel on the right is drawn separately to contain the longer scan length for M33.