Gravity and Matter in Extra Dimensions

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Abstract

In this paper, we derive from the viewpoint of the effective 4D theory the interaction terms between linearized gravity propagating in $N \geq 2$ large extra dimensions and matter propagating into one extra dimension. This generalizes known results for the interactions between gravity and 4D matter in ADD-type models. Although we assume that matter is described by an Universal Extra Dimensions (UED) scenario (with all fields propagating into the fifth dimension), we present our results in a general form that can be easily adapted to various other scenarios of matter distribution. We then apply our results to the UED model on a fat brane and consider some phenomenological applications. Among these are the computation of the gravitational decay widths of the matter KK excitations and the effect the width of the brane has on the interactions of gravity with Standard Model particles. We also estimate the cross-section for producing single KK excitations at colliders through KK number-violating gravitational interaction.

1 Introduction

At present, various theoretical constructions (most notably string theory) imply that there may exist more space-time dimensions than the four directly accessible to our senses. For a long time, it was assumed that such extra dimensions are compactified on a scale of order $1/M_{Pl}$, and thus inaccessible to the present day colliders, or those of the foreseeable future. In light of recent ideas [1], however, it is conceivable that the compactification radius of these extra dimensions is close to the inverse TeV scale or larger, which may have implications for the phenomenology of current day colliders.

There are several types of models with extra-dimensions that can lead to interesting low-energy physics. Relevant for our paper are the Arkani-Hamed–Dimopoulos–Dvali (ADD)
type of models [2], which exhibit factorizable geometry, and where the extra dimensions form a compact manifold. Each field that can propagate beyond the four non-compact dimensions (the bulk) can then be expanded in Kaluza-Klein (KK) modes, and manifests itself in the 4D effective theory as a tower of fields with increasing mass, the interval between the masses of two consecutive excitations being of order $1/r$.

The ADD scenario was initially conceived as a way to explain the hierarchy between the electro-weak scale and the Planck scale. The reason for gravity being so weak is that its strength is diluted by propagating in extra dimensions. The ADD relation between the 4D Planck scale $M_{Pl}$ and the fundamental Planck scale $M_D$ is given by

$$M_{Pl}^2 = M_D^{N-2} r^N,$$

which implies that $M_D$ can be as low as TeV, for values of the compactification scale $r$ of order eV$^{-1}$ up to keV$^{-1}$, depending on the number of extra dimensions $N$.

In the original ADD model, only gravity propagates in the extra dimensions, while matter is confined to a 4D brane. One has also the option of having the ordinary matter fields propagate in the bulk. There are a variety of models of this type. In some of them just a certain set of fields (usually the gauge bosons or the Higgs fields) propagate in extra dimensions [3]; in other, called Universal Extra Dimensions (UED) models, all Standard Model fields propagate into the bulk [4]. They also differ in the number of dimensions in which matter propagates; usually this is taken to be 5 or 6, but can conceivably be higher.

Note, however, that having matter in the extra dimensions introduces a certain asymmetry in the model. Experimental non-observation of KK matter excitations requires that the radius of the space on which matter is compactified be of order of at least inverse TeV. This is much smaller than the eV$^{-1}$ (or keV$^{-1}$) size needed to achieve the natural scale for gravity as in the ADD scenario. One can think of several ways in which to accommodate such bound: first, one can assume that in such a model the space is asymmetric; the one or two extra dimensions in which matter propagate have radius of inverse TeV size, while the dimensions in which only gravity propagates are of inverse eV size [5]. Another possibility is to take the size of the whole space to be of the order inverse eV, but to assume that the matter fields are confined by some mechanism close to the 4D brane. This is known as the fat brane scenario [6], where the length $R$ in which matter can propagate into the bulk is the width of the brane.

Besides answering some long standing theoretical questions, models where gravity and possibly matter propagate in extra dimensions may also have very interesting phenomenological consequences. Although the probability of radiating a specific KK graviton is very small, there is a large number of them, which can lead to measurable effects in collider processes [7, 8, 9]. In models with matter propagating in extra dimensions, gravity can play additional roles. For example, in UED-type models, it can mediate the decays of stable first level KK excitations, thus leading to interesting collider signals [10, 11].

The purpose of this paper is to derive the interactions of gravity with matter in the 4D effective theory for the case when both propagate in extra dimensions. These interactions have been derived for the case when matter is restricted to a 4D brane [12, 13]. We generalize here these results for models where matter propagates in one compact extra dimension. Although our results are applicable for a larger class of models, for the sake of specificity, we
formulate them in an UED scenario with matter living on a fat brane. We then analyze some phenomenological implications of the gravity-matter interactions in this particular model.

The paper is organized as follows: in the next section we describe concisely the model under consideration, i.e. we separately review the UED scenario for matter propagating in one extra dimension, as well as the KK reduction of linearized gravity on an $N$-dimensional torus. This allows us to introduce our conventions and notations. In Section 3 we derive the form of the interaction terms between matter and gravity in the 4D effective theory and also present the corresponding Feynman rules. (The Feynman rules for the interactions involving more than two matter fields are presented in the Appendix). Section 4 deals with phenomenological applications, including the computation of gravitational decay widths of KK excitations of matter, and the possibility of producing single KK excitations at colliders through gravity mediated interactions. At the end we present our conclusions.

2 Gravity and matter in extra dimensions

We start by specifying the parameters of our model. Gravity is assumed to propagate in $N$ 'large' extra dimensions compactified on a torus, which have common size of inverse eV up to inverse keV. We denote the radius of these dimensions by $r/2\pi$; then the linearized metric field has a KK expansion:

$$\hat{h}_{\hat{\mu}\hat{\nu}}(x, y) = \sum_{\vec{n}} \hat{h}_{\vec{n}}(x) \exp \left( \frac{2\pi \vec{n} \cdot \vec{y}}{r} \right).$$

With respect to gravity, mostly the notations introduced in [12] are used; that is, the ‘hat’ denotes quantities which live in $4+N$ dimensions: $\hat{\mu}, \hat{\nu} = 0, \ldots, 3, 5, \ldots 4+N$, while $\mu, \nu = 0, \ldots, 3$. The $(4+N)$D graviton field is decomposed into 4D tensor, vector and scalar components by:

$$h_{\hat{\mu}\hat{\nu}} = V_N^{-1/2} \left( h_{\mu\nu} + \eta_{\mu\nu}\phi \ A_{\mu i} \phi_{ij} \right)$$

where $V_N = r^N$ is the volume of the $N$-dimensional torus. Also, $i, j = 5, 6, \ldots 4+N$ and $\phi = \phi_{ij}$. The fields $h_{\mu\nu}, A_{\mu i}$ and $\phi_{ij}$ are decomposed in KK modes as in Eq. (1).

We take the matter to propagate into one extra dimension, considered to be the fifth one. Since keV spaced excitations of the SM fields have not been observed, we have to restrict the matter on a fat brane [6]; that is, the matter fields can go only a limited length $\pi R$ in the fifth dimension, with $R$ of order inverse TeV. Moreover, in order to obtain chiral 4D fermions, it is necessary to impose additional constraints on the extra-dimensional fields. Usually this is achieved by placing the higher dimensional fermions on an orbifold $S^1/\mathbb{Z}_2$ of length $\pi R$. This is also the case for the fermions in the UED model. The matter fields are expanded in KK modes as follows [10]:

$$(\Phi, B^a_{\mu}) = \frac{1}{\sqrt{\pi R}} \left[ (\Phi_0, B^a_{\mu,0}) + \sqrt{2} \sum_{n=1}^{\infty} (\Phi_n, B^a_{\mu,n}) \cos \left( \frac{n\pi y}{R} \right) \right]$$
\[ Q = \frac{1}{\sqrt{\pi R}} \left\{ Q_L + \sqrt{2} \sum_{n=1}^{\infty} \left[ Q_L^n \cos \left( \frac{ny}{R} \right) + Q_R^n \sin \left( \frac{ny}{R} \right) \right] \right\} \]
\[ q = \frac{1}{\sqrt{\pi R}} \left\{ q_R + \sqrt{2} \sum_{n=1}^{\infty} \left[ q_R^n \cos \left( \frac{ny}{R} \right) + q_L^n \sin \left( \frac{ny}{R} \right) \right] \right\}. \tag{3} \]

Here, \( \Phi \) and \( B_{\mu}^a \) are scalar and vector boson fields; the gauge for the latter ones can be chosen such that \( B_{\mu}^5 = 0 \) [14]. \( Q \) and \( q \) are the two fermion fields in 5D corresponding to each fermion field in the SM, which are doublets (\( Q \)) respectively singlets (\( q \)) under the \( SU(2) \) gauge group, and whose zero modes are the left, respectively right-handed components of the SM fermion fields.

The KK decomposition of the fields in Eq. (3) ensures that the 5D fields have definite parity with respect to reflections \( y \to -y \) of the co-ordinate \( y : -\pi R \leq y \leq \pi R \) that parameterizes the circle \( S^1 \). Alternately, as it was noted in [4], the decompositions (3) correspond to the KK expansion of a 5D field restricted to the interval \( 0 \leq y \leq \pi R \) with the field and its derivatives satisfying certain boundary conditions on the end-points \( y = 0, \pi R \). The above discussion suggests the following possibilities for the embedding of the fat brane into the fifth dimension: The first one is to take the interval \( 0 \leq y \leq \pi R \) to be a (small) segment of the circle that parameterizes the fifth dimension. The matter fields obey boundary conditions at the end-points of the interval, and are then decomposed as in Eq. (3). Gravity is unaffected by the orbifolding of the matter, i.e. it contains both even and odd modes to which the matter couples. Such an approach was used in [8, 10] and we shall adopt it in this paper as well. The other possibility is that the whole fifth dimension is an orbifold of length \( r/2 \) and the matter is embedded as an interval of length \( \pi R \). The other extra dimensions that are generally assumed to form a torus may or may not be subject to such additional discrete symmetry. The resulting model describes the same matter content as the first one but now the gravity components will also have specific parity i.e. the decomposition in (1) as well as the gravity-matter interactions will be altered.

Since in this paper we do not attempt to address the question of the origin of such fat brane construction, it is appropriate to make the following short comment: although it may be less realistic, the first model is easier to deal with in practice because gravity is not affected by orbifolding. Hence, we will use this scenario in the following computations. However, our results can be applied for the second scenario as well; one needs in addition to project out the gravity modes with unwanted parity and to modify the form-factors introduced in Eq. (15) below.

The decomposition in Eqns. (3) ensures that the kinetic terms of the matter KK excitations in the effective theory have their canonical form in terms of the \( \Phi_n, \psi_n \) and \( B_n \) fields. This does not hold for the gravity Lagrangian when written in terms of the fields \( h_{\mu\nu}^\parallel, A_{\mu i}^\parallel \) and \( \phi_{ij}^\parallel \). It is then necessary to redefine the fields in the gravitation sector and to work in terms of ‘physical’ fields \( \tilde{h}_{\mu\nu}^\parallel, \tilde{A}_{\mu i}^\parallel \) and \( \tilde{\phi}_{ij}^\parallel \) that have canonical kinetic and mass terms. The details of this redefinition are worked out in [12, 13]. Here we shall just review the results.

The \( D(D-3)/2 \) internal degrees of freedom (d.o.f) of a \( D \) dimensional massless graviton \( (D = 4 + N) \) will appear, after dimensional reduction, as the components of a massive spin 2 graviton \( \tilde{h}_{\mu\nu}^\parallel \) (5 d.o.f), \( N - 1 \) massive vector bosons \( \tilde{A}_{\mu i}^\parallel \), with \( n_i \tilde{A}_{\mu i}^\parallel = 0 \) (3 d.o.f each), and
$N(N-1)/2$ massive scalars $\tilde{\phi}_{ij}^\eta$, with $n_i \tilde{\phi}_{ij}^\eta = 0$ and $\tilde{\phi}_{ij}^\eta = \tilde{\phi}_{ji}^\eta$ (1 d.o.f. each). All of these fields have the same mass $m_\eta^2 = 4\pi^2 n^2/r^2$. The gauge of the spin-two and spin-one physical fields is fixed by $\partial^\mu \hat{h}_{\mu\nu}^\eta = 0$, $\hat{h}_{\mu\nu}^\eta = 0$, and $\partial^n \hat{A}_{\mu}^\eta = 0$.

The definition of the physical ‘tilde’ fields in terms of the initial fields $h_{\mu\nu}, A_{\mu i}$ and $\phi_{ij}$ is given in [12]. We will need in the following to express the initial fields in terms of the $N$ modes we obtain:

$$\hat{h}_{\mu\nu}^\eta = h_{\mu\nu}^\eta + \omega \left( \frac{\partial_\mu \partial_\nu}{m_\eta^2} - \frac{1}{2} \eta_{\mu\nu} \right) \tilde{\phi}_{ij}^\eta$$

$$A_{\mu i}^\eta = A_{\mu i}^\eta + \tilde{A}_{\mu i}^\eta$$

$$\phi_{ij}^\eta = \frac{1}{\sqrt{2}} \tilde{\phi}_{ij}^\eta - \frac{3}{2} \omega a \left( \delta_{ij} - \frac{n_i n_j}{n^2} \right) \tilde{\phi}_{ij}^\eta$$

and $\phi_{ij}^\eta = (3\omega/2) \tilde{\phi}_{ij}^\eta$. Here $\phi_{ij}^\eta$ (same for the tilde fields), $\omega = \sqrt{2/3(N+2)}$, and $a$ is a solution of the equation $3(N-1)a^2 + 6a = 1$.

Now we have all the ingredients necessary to compute the interactions of gravity with matter. Following the notations in [12], the $D$ dimensional action is:

$$S_{int} = -\frac{\kappa}{2} \int d^Dx \delta(x^0) \ldots \delta(x^N) \hat{h}_{\mu\nu}^\eta T_{\mu\nu}^\eta$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the 5D matter. Expanding the gravity field in KK modes we obtain:

$$S_{int} = -\frac{\kappa}{2} \int d^Dx \int_0^{\pi R} dy \sum_\eta \left[ (h_{\mu\nu}^\eta + \eta_{\mu\nu} \phi_{ij}^\eta) T_{\mu\nu}^\eta - 2 A_{\mu 5}^\eta T_5^\eta + 2 \phi_{55}^\eta T_5^\eta \right] e^{2\pi i n_5 y/\pi R}.$$ (10)

In terms of the physical fields, the effective 4D Lagrangian is:

$$L_{int} = -\frac{\kappa}{2} \sum_\eta \int_0^{\pi R} dy \left\{ \left[ \hat{h}_{\mu\nu}^\eta + \omega \left( \eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{m_\eta^2} \right) \tilde{\phi}_{ij}^\eta \right] T_{\mu\nu} - 2 A_{\mu 5}^\eta T_5^\eta + \left( \sqrt{2} \tilde{\phi}_{55}^\eta - \xi \tilde{\phi}_{ij}^\eta \right) T_5^\eta \right\} e^{2\pi i n_5 y/\pi R}.$$ (11)

where $\xi = 3\omega a(1 - n_5^2/n^2)$. Defining the projections of the matter energy momentum tensor on the $\bar{n}$-th graviton state by

$$T_{MN}^{n_5}(x) = \int_0^{\pi R} dy \ T_{MN}(x,y) \ e^{2\pi i n_5 y/\pi R}$$
we can write

\[ \mathcal{L}_{int} = -\frac{\kappa}{2} \sum_{\vec{n}} \left\{ \left[ \hat{h}^{\vec{n}}_{\mu \nu} + \omega \left( \eta_{\mu \nu} + \frac{\partial_{\mu} \partial_{\nu}}{m_{\vec{n}}^2} \right) \tilde{\phi}^{\vec{n}} \right] T^{\mu \nu}_{\vec{n} \vec{n}^\prime} - 2\tilde{\alpha}^{\vec{n}}_{\mu} T^\mu_{\vec{n} \vec{n}^\prime} + \left( \sqrt{2} \tilde{\phi}^{\vec{n}}_{55} - \xi \tilde{\phi}^{\vec{n}} \right) T_{55}^{\vec{n}} \right\} . \]  

(12)

In the following section, we will give the explicit form of the energy-momentum tensor for different types of matter, and the corresponding Feynman rules for the interaction of matter with gravity.

### 3 Gravity-matter interactions

#### 3.1 Scalar matter

The energy momentum tensor in 5D for a complex scalar field is:

\[ T^{S}_{\mu \nu} = -\eta_{MN} (D^R \Phi^\dagger D_R \Phi - m_{\Phi}^2 \Phi^\dagger \Phi) + D_M \Phi^\dagger D_N \Phi + D_N \Phi^\dagger D_M \Phi , \]

(13)

where \( M, N, R \) are indices which run from 0 to 5, and \( D_M = \partial_M + ig B_M^a T^a \) is the covariant derivative. We can expand the fields in KK modes, and integrate over the fifth dimension, to obtain the projections of the energy-momentum tensor:

\[
T^{S n}_{\mu \nu} = \sum_{l, m} \left\{ \left[ -\eta_{\mu \nu} (\partial^\rho \Phi^\dagger m m \partial_\rho \Phi^\dagger - m_{\Phi}^2 \Phi^\dagger m m \Phi^\dagger) + \partial_{(\mu} \Phi^\dagger m m \partial_{\nu)} \Phi^\dagger \right] \mathcal{F}^{(cc)}_{lmnn} + \eta_{\mu \nu} \frac{m^2}{R^2} \Phi^\dagger m m \Phi^\dagger \mathcal{F}^{(ss)}_{lmnn} \right\}
\]

\[
= -ig \sum_{l, m, k} \left[ -\eta_{\mu \nu} \Phi^\dagger m m \tilde{B}^{k k} \partial_\rho \Phi^\dagger + \Phi^\dagger m m \tilde{B}^{k k} \partial_\rho \Phi^\dagger \right] \mathcal{F}^{(ccc)}_{lmknn} + g^2 \sum_{l, m, k, j} \left[ \Phi^\dagger m m \left( -\eta_{\mu \nu} \tilde{B}^{k k} \tilde{B}^{j j} + \tilde{B}^{k k} \tilde{B}^{j j} \right) \Phi^\dagger \right] \mathcal{F}^{(ccc)}_{lmkjnn}
\]

\[
T^{S n}_{\mu 5} = \sum_{l, m} \left\{ (\partial^\rho \Phi^\dagger m m \partial_\rho \Phi^\dagger) \mathcal{F}^{(sc)}_{lmnn} + \Phi^\dagger m m (\partial_{(\mu} \Phi^\dagger) \frac{m}{R} \mathcal{F}^{(cs)}_{lmnn} \right\}
\]

\[
= +ig \sum_{l, m, k} \Phi^\dagger m m \tilde{B}^{k k} \partial_\rho \Phi^\dagger \left( -\frac{l}{R} \mathcal{F}^{(sc)}_{lmknn} + \frac{m}{R} \mathcal{F}^{(cs)}_{lmknn} \right)
\]

\[
T^{S n}_{55} = \sum_{l, m} \left\{ (\partial^\rho \Phi^\dagger m m \partial_\rho \Phi^\dagger - m_{\Phi}^2 \Phi^\dagger m m \Phi^\dagger) \mathcal{F}^{(cc)}_{lmnn} + \frac{m^2}{R^2} \Phi^\dagger m m \Phi^\dagger \mathcal{F}^{(ss)}_{lmnn} \right\}
\]

\[
= -ig \sum_{l, m, k} \Phi^\dagger m m \tilde{B}^{k k} \partial_\rho \Phi^\dagger \mathcal{F}^{(ccc)}_{lmknn} + g^2 \sum_{l, m, k, j} \Phi^\dagger m m \tilde{B}^{k k} \tilde{B}^{j j} \Phi^\dagger \mathcal{F}^{(ccc)}_{lmkjnn}
\]  

(14)

In the above, \( l, m, k, j \) are the KK excitation numbers of the matter fields, with values starting from zero. We also use the following notations: \( X_{(\mu} Y_{\nu)} = X_{\mu} Y_{\nu} + X_{\nu} Y_{\mu} \), \( \partial Y = X (\partial Y) - (\partial X) Y \) (in (14) the derivatives act only on the scalar fields \( \Phi \)), and \( \tilde{B} = B^a T^a \), with sum over the gauge index \( a \). (We do not write explicitly the color or SU(2) indices; terms like \( \Phi^\dagger m m \tilde{B}^{k k} \Phi^\dagger \) should be read like \( \Phi^\dagger m m T^a_{\alpha \beta} \Phi^\dagger B^a_{\alpha \beta} \)). The form factors \( \mathcal{F}^{(c...)}_{lmnn} \) quantify
the superposition of the wave functions of the interacting fields (or their derivatives) in the fifth dimension:

\[ F_{\ldots l_{i}... n} = \int_{0}^{R} \prod_{i} c_{i} f_{i} \left( \frac{l_{i} y}{R} \right) \exp \left( \frac{2\pi i n y}{r} \right), \]

where the functions \( f_{i}(\cdot) \) can be \( \sin(\cdot) \) or \( \cos(\cdot) \), and \( c_{i} = \sqrt{2/\pi R} \) if \( l_{i} \neq 0 \) or \( c_{i} = \sqrt{1/\pi R} \) for \( l_{i} = 0 \).

### 3.2 Vector boson matter

In 5D the energy momentum tensor for a vector boson field is:

\[ T_{V}^{MN} = \eta_{MN} \left( \frac{1}{4} F_{aRS} F_{RS} - \frac{m_{B}^{2}}{2} B_{aR} B_{R}^{a} \right) - \left( F_{M}^{aR} F_{NR}^{a} - m_{B}^{2} B_{aM}^{a} B_{N}^{a} \right) \]

with \( F_{aRS}^{\nu} = \partial_{R} B_{S}^{a} - \partial_{S} B_{R}^{a} + g f^{abc} B_{R}^{b} B_{S}^{c} \) the field tensor in five dimensions (\( a \) is the gauge index of the field). For purposes of generality, we added a 5D mass term in our expressions.

Expanding (16) in KK modes and integrating over the fifth dimension:

\[ T_{\mu\nu}^{Vn} = \sum_{l,m} \left\{ \left[ \eta_{\mu\nu} \left( \frac{1}{4} \tilde{F}^{a} \tilde{F}_{a} - \frac{m_{B}^{2}}{2} B_{a} B_{a} \right) - \left( \tilde{F}_{\mu}^{a} \tilde{F}_{\nu}^{a} - m_{B}^{2} B_{\mu} B_{\nu} \right) \right] F^{(cc)} \right\} \]

\[ T_{\mu 5}^{Vn} = \sum_{l,m} \left( \frac{1}{R} \tilde{F}_{\mu}^{a} B_{\rho}^{a} \right) F^{(sc)} + g f_{ab} \sum_{l,m,k,j} \left( \frac{1}{R} B_{\mu}^{a} B_{\rho}^{b} B_{\sigma}^{c} B_{\rho}^{d} \right) F^{(sc)} \]

\[ T_{55}^{Vn} = - \sum_{l,m} \left( \frac{1}{4} \tilde{F}^{a} \tilde{F}_{a} - \frac{m_{B}^{2}}{2} B_{a} B_{a} \right) F^{(cc)} + \frac{1}{2} \frac{m_{B}^{2}}{R} B_{a} B_{a} F^{(ss)} \]

\[ \times \sum_{l,m,k,j} \left( B_{\mu}^{a} B_{\rho}^{c} \tilde{F}_{\rho}^{a} \right) F^{(cc)} + \frac{g^{2}}{4} f_{ab} f_{ade} \sum_{l,m,k,j} \left( B_{\mu}^{a} B_{\rho}^{c} B_{\sigma}^{d} B_{\rho}^{e} \right) F^{(cc)} \]

Here \( l, m, k, j \) are KK indices, and \( \mu, \nu, \sigma, \rho \) are 4D space-time indices. Also, \( \tilde{F}_{\rho}^{a} = \partial_{\rho} B_{\sigma}^{a} - \partial_{\sigma} B_{\rho}^{a} \).

### 3.3 Fermionic matter

The energy momentum tensor for a fermion field in 5D is:

\[ T_{MN}^{F} = - \eta_{MN} \left( \bar{\psi} i \Gamma^{R} D_{R} \psi - \frac{1}{2} \partial^{R} (\bar{\psi} i \Gamma_{R} \psi) - m_{\psi} \bar{\psi} \psi \right) \]

\[ + \left( \frac{1}{2} \bar{\psi} i \Gamma_{M} D_{N} \psi - \frac{1}{2} \partial_{M} (\bar{\psi} i \Gamma_{N} \psi) + (M \leftrightarrow N) \right) \]

(18)
with the covariant derivative $D_M = \partial_M + igB^a_M T^a$. A 5D mass term is also added in here for purposes of generality. The $\Gamma_M$ matrices in five dimensions are: $\Gamma \mu = \gamma_\mu, \Gamma_5 = i\gamma_5$. Expanding in KK modes for the fields in (3), we get:

$$T_{\mu\nu}^{F} = \sum_{l,m} \bar{\psi}_m \left[ \frac{i}{4} \left[ \gamma_\mu \partial_\nu - 2\eta_\mu \gamma^\sigma \partial_\sigma \right] G^{(c)\pm}_{l,m|n} + \frac{\eta_\mu}{2R} \left( lG^{(c)\pm}_{l,m|n} + mG^{(c)\pm}_{l,m|n} \right) \right] \psi^l \pm \eta_\mu m_Q G^{(s)\pm}_{l,m|n} \psi^l + g \sum_{l,m,k} \bar{\psi}_m \left[ \left( \eta_\mu \gamma^\rho \hat{B}_\rho - \frac{1}{2} \gamma(\mu \hat{B}_\nu) \right) G^{(c)\pm}_{l,m|n} \right] \psi^f$$

$$T_{\mu}^{F} = \sum_{l,m} \bar{\psi}_m \left[ \frac{1}{4} \left[ \gamma_\mu \partial_l - i\gamma_\mu \right] G^{(s)\pm}_{l,-m|n} + \frac{i\gamma_\mu}{R} \left( lG^{(s)\pm}_{l,-m|n} + mG^{(s)\pm}_{l,-m|n} \right) \psi^l \right]$$

$$-ig \sum_{l,m,k} \bar{\psi}_m \left[ \frac{1}{2} \left[ \hat{B}_\mu G^{(sc)\pm}_{l,m|n} \right] \psi^l \right]$$

$$T_{55}^{F} = \sum_{l,m} \bar{\psi}_m \left[ \frac{1}{2} \gamma^\rho \bar{\gamma}_\rho G^{(cc)\pm}_{l,m|n} + m_Q G^{(s)\pm}_{l,m|n} \right] \psi^l - g \sum_{l,m,k} \bar{\psi}_m \left[ \gamma^\rho \hat{B}_\rho G^{(cc)\pm}_{l,m|n} \right] \psi^l. \quad (19)$$

Here $\psi$ stands for either the doublet or singlet fields in Eqs. (3); the upper sign applies for the doublet case (that is, for $\psi^m = Q_L^m + Q_R^m$) and the lower sign applies for the singlet case ($\psi^m = -q_L^m + q_R^m$, the minus sign here being due to a $\gamma_5$ rotation necessary to obtain the canonical form for the mass terms). For the Standard Model in extra dimensions, we have a doublet ($u_L, d_L$) and two singlet set of fields $u_R, d_R$ for each generation. (Note that $\gamma_5$ and $m_Q$ both change their signs when going from the doublet to the singlet fields). In the limit where the Yukawa interactions are negligible [4, 10], the fields $\psi^m$ are the mass eigenstates for the fermion KK excitations.

The following form factors are used in the above expressions:

$$G^{(c)\pm}_{l,m|n} = (F^{(c)}_{l-m|n} \pm F^{(c)}_{m+l|n} \gamma_5)/2, \quad G^{(cc)\pm}_{l,m|n} = (F^{(cc)}_{l-m|n} \pm F^{(cc)}_{m+l|n} \gamma_5)/2$$

$$G^{(s)\pm}_{l,m|n} = (F^{(s)}_{m+l|n} \pm F^{(s)}_{l-m|n} \gamma_5)/2, \quad G^{(sc)\pm}_{l,m|n} = (F^{(sc)}_{m+l|n} \pm F^{(sc)}_{l-m|n} \gamma_5)/2 \quad (20)$$

In a slight abuse of notation, the form factors $F^{(c),(s)}_{l+m|n}, F^{(cc),(sc)}_{l+m|n}$ used here contain the coefficients $c_l c_m, c_l c_m c_k$, rather than $c_l \pm c_m, c_l \pm c_m c_k$ as implied by Eq. (15).

For the case of SM fermions, the terms with $l = 0, m = 0$ in the Eqs. (19) simplify to

$$T_{\mu\nu}^{FSM} = \bar{\psi}^0 \left\{ \frac{i}{4} \left[ \gamma_\mu \partial_\nu - 2\eta_\mu \gamma^\sigma \partial_\sigma \right] F^{(cc)}_{00|n} + g \sum_k \left[ \eta_\mu \gamma^\rho \hat{B}_\rho - \frac{1}{2} \gamma(\mu \hat{B}_\nu) \right] F^{(ccc)}_{00k|n} \right\} \psi^0$$

$$T_{\mu}^{FSM} = 0$$

$$T_{55}^{FSM} = \bar{\psi}^0 \left\{ \frac{i}{2} \gamma^\rho \bar{\gamma}_\rho F^{(cc)}_{00|n} - g \sum_k \gamma^\rho \hat{B}_\rho F^{(ccc)}_{00k|n} \right\} \psi^0 \quad (21)$$

where $\psi^0$ is either the left-handed or right-handed part of the fermion, and $\hat{B} = T^a B^a$ are the corresponding gauge fields. As expected, in this case the results are equal to the
expressions obtained for the case when matter is restricted to four dimensions, multiplied by an appropriate form factor. The Feynman rules for the interactions of the SM fermions with gravity can then be obtained directly from [12].

3.4 Feynman rules

It is convenient to express the vertex Feynman rules for the gravity-matter interactions in terms of the components of the energy-momentum tensor in the momentum representation. For example, define

\[
T^S_{MN}^{lmn}(k_1, k_2) = \langle \Phi^m(k_2)|T^{Sn}_{MN}|\Phi^l(k_1) \rangle
\]

\[
\epsilon_1^\alpha \epsilon_2^\beta T^{V_{MN,\alpha\beta}}_{MN}(k_1, k_2) = \langle V^m(k_2, \epsilon_2)|T^{V_n}_{MN}|V^l(k_1, \epsilon_1) \rangle
\]

\[
\bar{u}(k_2)T^{F_{MN}}_{MN}^{lmn}(k_1, k_2)u(k_1) = \langle \psi^m(k_2)|T^{F_{MN}}|\psi^l(k_1) \rangle
\]

(22)

Note that only the terms bilinear in matter fields contribute to these quantities. They will describe the radiation of a KK graviton from a scalar, vector boson or fermion line (with the potential change of the KK excitation number of the matter as well). From Eq. (12) we then obtain the following vertex interaction rules:

\[
\tilde{h}^{\mu\nu} P^m(k_2)P^l(k_1) : -\frac{i\kappa}{2} T^{P\ lmn}_{\mu\nu}(k_1, k_2)
\]

\[
\tilde{A}^{\mu}_{i\mu} P^m(k_2)P^l(k_1) : i\kappa \delta_{i5} T^{P\ lmn}_{\mu5}(k_1, k_2)
\]

\[
\tilde{\phi}^{ij}_{i\mu} P^m(k_2)P^l(k_1) : -\frac{i\kappa}{2} \left[ \delta_{ij}\omega \left( \eta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{m^2_{\tilde{\phi}}} \right) T^{P\ lmn}_{\mu\nu}(k_1, k_2) \right.
\]

\[
\left. + (\sqrt{2}\delta_{i5}\delta_{j5} - \xi\delta_{ij})T^{P\ lmn}_{55}(k_1, k_2) \right] \]

(23)

The convention here is that a KK level-\(l\) excitation \(P\) with initial momentum \(k_1\) radiates a graviton with momentum \(p = k_1 - k_2\), becoming a KK level-\(m\) excitation with momentum \(k_2\) (as represented pictorially in Fig. 1).
### Table 1: The bilinear terms of the matter energy-momentum tensor in momentum representation.

<table>
<thead>
<tr>
<th></th>
<th>$T_{\mu\nu}$</th>
<th>$T_{5\mu}$</th>
<th>$T_{55}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$(C_{\mu\nu,\rho\sigma} k_1^\rho k_2^\sigma + m_5^2 \eta_{\mu\nu}) F^{(cc)}_{lm</td>
<td>n} + \frac{ml}{R^2} \eta_{\mu\nu} F^{(ss)}_{lm</td>
<td>n}$</td>
</tr>
<tr>
<td>V</td>
<td>$\left( (m_B^2 - k_1 k_2) C_{\mu\nu,\alpha\beta} - D_{\mu\nu,\alpha\beta}(k_1, k_2) \right) F^{(cc)}_{lm</td>
<td>n} + \frac{lm}{R^2} C_{\mu\nu,\alpha\beta} F^{(ss)}_{lm</td>
<td>n}$</td>
</tr>
<tr>
<td>F</td>
<td>$\frac{1}{8} C_{\mu\nu,\rho\sigma} \gamma^\rho (k_1 + k_2) \gamma^\sigma G^{(c)\mp}_{lm</td>
<td>n} + \eta_{\mu\nu} A^{\pm}_{lm</td>
<td>n}$</td>
</tr>
</tbody>
</table>
The expressions for the quantities defined in Eqs. (22) are given in Table 1, with the following notations being used (see also [12]):

\[
\begin{align*}
C_{\mu\nu,\rho\sigma} & = -\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} \\
C'_{\mu\nu,\rho\sigma} & = -2\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} \\
D_{\mu\nu,\rho\sigma}(k_1, k_2) & = \eta_{\mu\nu}(k_1\rho k_1\sigma + k_2\rho k_2\sigma + k_1\rho k_2\sigma) - \left[ \eta_{\nu\sigma}k_1\rho k_1\rho + \eta_{\mu\rho}k_2\rho k_2\sigma + (\mu \leftrightarrow \nu) \right] \\
D'_{\rho\sigma,\alpha\beta} & = \eta_{\rho\sigma}\eta_{\alpha\beta} - \eta_{\rho\beta}\eta_{\alpha\sigma} \\
A_{lmn}^\pm & = \frac{1}{2R} \left( lG^{(c)\pm}_{lmn} + mg^{(c)\mp}_{lmn} \right) \pm mQG^{(s)\pm}_{lmn} \\
B_{lmn}^\pm & = \frac{1}{R} \left( lG^{(s)\mp}_{lmn} + mg^{(s)\pm}_{lmn} \right) 
\end{align*}
\]

The upper (lower) sign in the definition of the \( A, B \) factors correspond to the case of doublet (singlet) fermions participating in the interaction. The Feynman rules for vertices with more that three particles are given in the Appendix.

The interaction rules for the 0-level KK excitations (the SM fermions) are easily obtained from Eqs. (23) by setting \( T_{\mu 5}, T_{5 5} \) equal to zero. One then obtains the vertex rules given in [12, 13], with one difference: [12, 13] make use of the conservation of energy-momentum tensor \( p^\mu T_{\mu\nu} = 0 \). Therefore they do not have the term proportional to \( p^\mu p^{\nu}/m_5^2 \) in the interaction vertices with the scalar gravitons. However, energy–momentum conservation holds only for on-shell particles; our results are then applicable also for the case when the SM fermions in the vertex are off-shell.

4 Phenomenological Implications

The results obtained in the previous section are applicable in all cases when matter and gravity propagate in 5D. For example, they are also valid in models in which only the gauge bosons or the Higgs propagate in extra dimensions. These results can also be used even if we are not in a fat brane scenario, but the matter fields propagate all the way in the fifth dimension; in that case, many of the form factors will be zero (due to KK number conservation), but the general formulas still work.

However, the most interesting scenario in terms of its phenomenological consequences may be the UED-type model with matter on a fat brane. In such a model, in the absence of KK number violating interactions, the first level KK excitations are stable [10]; or, if the masses of these excitations are split by loop corrections, the lightest KK particle is stable [15]. However, in the presence of gravity, these particles can decay by radiating KK gravitons [10]. The collider signals are strongly dependent on the decay branching ratios (gravitational versus electroweak or strong decay) [11], so it is necessary to evaluate the decay width to gravitons. This can be done using the results of the previous section. Moreover, the interactions of the SM matter with gravity are affected compared to the case when the matter is restricted to the 4D brane; and finally, KK number violating graviton exchange can mediate the production of single KK excitations at hadron or linear colliders.
### 4.1 Gravity-mediated decays of KK excitations

In a fat brane scenario, the gravity-matter interactions do not respect the KK number conservation rules which hold for matter interactions in UED models. Gravity interactions will therefore mediate the decay of the first level KK excitations of matter (or the lightest KK particle), which otherwise would be stable.

The decay widths to a single graviton are given below. For the decay of a KK fermion:

\[
\Gamma(q^l \rightarrow q h^{\vec{\nu}}) = |\mathcal{F}_{l n}^c|^2 \frac{\kappa^2}{2 \times 384\pi} \frac{M^3}{x^4} \left[(1 - x^2)^4 (2 + 3 x^2)\right]
\]

\[
\Gamma(q^l \rightarrow q A^{\vec{\nu}}) = |\mathcal{F}_{l n}^c|^2 \frac{\kappa^2}{2 \times 256\pi} M^3 \left[(1 - x^2)^2 (2 + x^2)\right] \times P_{55}
\]

\[
\Gamma(q^l \rightarrow q \phi^{\vec{\nu}}) = |\mathcal{F}_{l n}^c|^2 \frac{\kappa^2}{2 \times 256\pi} M^3 (1 - x^2)^2 \left[c_{11} \frac{(1 - x^2)^2}{x^4} + 2 c_{12} \frac{1 - x^2}{x^2} + c_{22}\right]
\]

(25)

Here \(M\) is the mass of the KK particle \(M = l/R\), \(m_{\vec{\nu}}\) is the mass of the graviton, and \(x = m_{\vec{\nu}}/M\). The coefficients \(P_{55}\) and \(c_{ij}\) appear because not all \(\vec{A}_i, \vec{\phi}_{ij}\) fields are independent. To eliminate the spurious degrees of freedom we can introduce the (extra dimensional) polarization vector \(e_i^k\) and tensor \(e_{ij}^a\) as in [12], and we have:

\[
P_{55} = \sum_{k=1}^{N-1} e_i^k e_j^{k*} \delta_{i5} \delta_{j5} = 1 - n_{55}^2
\]

\[
c_{11} = \sum_{s=1}^{N(N-1)/2} e_i^{s*} e_j^{s*} \omega^2 \delta_{ij} \delta_{kl} = \omega^2 (N - 1)
\]

\[
c_{12} = \sum_{s=1}^{N(N-1)/2} e_i^s e_j^{s*} \omega \delta_{ij} (\sqrt{2} \delta_{k5} \delta_{l5} - \xi \delta_{kl}) = -\frac{2}{N + 2} P_{55}
\]

\[
c_{22} = \sum_{s=1}^{N(N-1)/2} e_i^s e_j^{s*} (\sqrt{2} \delta_{i5} \delta_{j5} - \xi \delta_{ij})(\sqrt{2} \delta_{k5} \delta_{l5} - \xi \delta_{kl}) = \frac{2(N + 1)}{N + 2} P_{55}^2
\]

(26)

For the decay of a KK gauge boson excitation, the following results are obtained:

\[
\Gamma(B^l \rightarrow Bh^{\vec{\nu}}) = |\mathcal{F}_{l n}^c|^2 \frac{\kappa^2}{3 \times 96\pi} \frac{M^3}{x^4} \left[(1 - x^2)^3 (1 + 3 x^2 + 6 x^4)\right]
\]

\[
\Gamma(B^l \rightarrow BA^{\vec{\nu}}) = |\mathcal{F}_{l n}^c|^2 \frac{\kappa^2}{3 \times 32\pi} \frac{M^3}{x^2} \left[(1 - x^2)^3 (1 + x^2)\right] \times P_{55}
\]

\[
\Gamma(B^l \rightarrow B \phi^{\vec{\nu}}) = |\mathcal{F}_{l n}^c|^2 \frac{\kappa^2}{3 \times 32\pi} M^3 (1 - x^2)^2 \left[c_{11} \frac{1}{x^4} + 2 c_{12} \frac{1}{x^2} + c_{22}\right]
\]

(27)

The above formulas apply for the gluon as well as the electroweak gauge boson excitations, but in the later case the mass \(M\) of the boson acquires a correction coming from the SM gauge boson mass term.
Figure 2: Decay widths for KK fermions (left) and bosons (right) as a function of the particle mass. Straight lines corresponds to $N = 2$ extra dimensions, dashed lines to $N = 4$, and dotted lines to $N = 6$.

In order to obtain the total gravitational decay width for a KK excitation, a sum over KK gravitons with mass smaller than the mass of the decaying particle has to be performed. Since the masses of the graviton KK excitations are closely spaced, this sum can be replaced

Figure 3: Mass distribution (left) and energy distribution (right) for the graviton radiated in the decay of one matter KK excitation with mass 1 TeV. Straight lines corresponds to $N = 2$ extra dimensions, dashed lines to $N = 4$, and dotted lines to $N = 6$. 

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by an integral over the graviton density of states (for more details, see [12, 10]). The results obtained for some values of the model parameters are presented in Fig. 2.

The distributions of the mass and energies of gravitons contributing to the decay of a KK excitations follow the pattern described in [10]. That is, for \( N = 2 \), the decay is mediated mostly by light gravitons, and the missing energy (the energy taken away by the graviton) is about half the particle mass. For higher \( N \), more massive gravitons contribute to the total decay width, and the missing energy increases to values close to the KK particle mass, as in Fig. 3.

4.2 Effects of the fat-brane form factor

If gravity propagates in large extra dimensions, SM physics may be affected, either by graviton radiation from the final (or initial) state of SM processes, or by contributions to these processes due to virtual exchange of graviton states. The fact that the graviton coupling constant to matter is proportional to \( 1/M_{Pl}^2 \) is compensated by the large number of gravitons which can play a role in these interactions. Consequently, there is a large body of literature dealing with the phenomenological consequences of KK gravity in large extra dimensions [7, 8, 9].

It is natural then to ask how the fat-brane scenario discussed here affects this phenomenology. The only difference between the fat-brane and thin-brane cases (where matter is confined in 3 spatial dimensions) is the appearance of the form-factor:

\[
F_{0in5} = \frac{1}{\pi R} \int_0^{\pi R} dy \exp \left( \frac{2\pi in_5y}{r} \right)
\]

which multiplies the relevant interaction vertex. Since the absolute value of this form factor is smaller than 1, we can conclude that in this scenario, the width of the brane has an effect of softening the gravity-SM matter interaction.

In order to better quantify this effect, let us consider the contribution to four-fermion SM processes due to the exchange of virtual gravitons [12, 7, 8]. The amplitude for such a contribution can be written as (see, for example [12]):

\[
\mathcal{M}(f_1f_1 \rightarrow G \rightarrow f_2f_2) = \frac{\kappa^2}{16\pi} \frac{i}{s - m_{\vec{n}}^2 + i\epsilon} \mathcal{A}
\]

where \( \mathcal{A} \) is an amplitude which does not depend on the mass of the graviton being exchanged. (For the moment we assume that matter is restricted to 4D). Summing over all graviton states:

\[
\mathcal{M}(f_1f_1 \rightarrow \sum_{\vec{n}} G \rightarrow f_2f_2) = \frac{\kappa^2}{16\pi} D(s) \mathcal{A},
\]

with

\[
D(s) = \sum_{\vec{n}} \frac{i}{s - m_{\vec{n}}^2 + i\epsilon}.
\]  

The function \( D \) can be evaluated by changing the sum into an integral over the graviton density of states. However, for the number of extra dimensions in which gravity propagates
\( N \geq 2 \), this integral is divergent; hence the need to impose an ultraviolet cutoff \( M_S \) on the graviton mass \( m_{\tilde{g}} \).

Assuming now that the CM energy of the experiment is much lower than the cutoff scale \( M_S \), we have:

\[
D(s) = \left( \frac{r}{2\pi} \right)^N V_N \frac{-iM_S^{N-2}}{N-2} \left( 1 + \mathcal{O}\left( \frac{s}{M_S^2} \right) \right)
\]

where \( V_N = \frac{2\pi^{N/2}}{\Gamma(N/2)} \) is the volume of the unit sphere in \( N \) dimensions. The above formula is valid for \( N > 2 \); if \( N = 2 \), the \( M_S^{N-2}/(N-2) \) is replaced by \( \log(M_S^2/s) \). Note that there is a strong dependence on the cutoff mass, which may mean that the details of physics at the scale where the theory becomes nonpertubative will have a large impact on the physics at low energies.\(^4\)

In the case when matter propagates in one extra dimension, expression (28) becomes:

\[
D_0(s) = \sum_{\vec{n}} \mathcal{F}_{0|n_5} \frac{i}{s - m_{\vec{n}}^2 + i\epsilon} (\mathcal{F}_{0|n_5})^*
\]

with:

\[
\mathcal{F}_{0|n_5} = \frac{i}{\pi m_5} \left( e^{i\pi m_5 \frac{5}{M}} - 1 \right)
\]

where \( M = 1/R \) is the mass of the matter first level KK excitations, and \( m_5 = 2\pi n_5/r \) is the part of the graviton mass due to excitation along the \( y \) dimension. Noting that the form factor depends only on \( n_5 \), we can integrate along the other extra dimensions to obtain:

\[
D_0(s) = \left( \frac{r}{2\pi} \right)^N V_{N-1} \int_0^{M_s} dm_5 |\mathcal{F}_{0|n_5}|^2 \frac{-i(M_S^2 - m_5^2)^{N-3}}{N-3} \left( 1 + \mathcal{O}\left( \frac{s - m_5^2}{M_S^2 - m_5^2} \right) \right)
\]

(here we take \( N > 3 \)). Due to the \( 1/m_5 \) behavior of the form factor, the main contribution to the above integral comes from small values of \( m_5 \); therefore, the terms of order \( m_5^2/M_S^2 \) can be neglected. With the change of variables: \( x = \pi m_5/M \), we can rewrite:

\[
D_0(s) = \left( \frac{r}{2\pi} \right)^N V_{N-1} \frac{-iM_S^{N-3}}{N-3} \frac{M}{\pi} \int_0^{\pi M_S/M} e^{ix} - 1 \right|^2 \frac{\left( e^{ix} - 1 \right)^2}{x^2} dx.
\]

The integral in the above expression has an asymptotic limit \( \pi \) (when \( M_S \gg M \)). Therefore, the effective strength of the four-fermion interaction operator is reduced in the fat-brane scenario by a factor \( \sim M/M_S \).

From a phenomenological point of view, this will make the observation of virtual graviton exchange effects at colliders more difficult. Therefore, the limits set in [7, 8] on the string scale \( M_D \) can be somewhat lowered. We estimate that this softening effect will be less visible in processes with radiation of real gravitons, since in that case the energy of the collider provides a natural cutoff scale for the sum over graviton excitations which is less than \( M \).\(^5\)

\[^4\]For an example of how string-scale physics can affect low scale phenomenology, see, for example, [16].

\[^5\]Conversely, if the collider energy is larger than \( M \), KK excitations of matter can be observed directly.
From a theoretical point of view, this behavior is interesting because it reduces the dependence of the strength of the effective interactions mediated by gravity at low energies on the details of the theory at the scale \( M_D \). This can be seen from the fact that in this model, where matter propagates in one extra dimension, for \( N = 2 \) the cutoff scale can be let go to infinity (the sum over KK graviton propagators is convergent), while for \( N = 3 \) the dependence on the cutoff scale is logarithmic. In a model where the brane is ‘fat’ in all \( N \) extra dimensions, sums like (30) are finite for any value of \( N \). In this manner, the non-zero thickness of the brane provides a softening factor for the coupling of gravity to matter (for a different mechanism, see [17]), in effect replacing the string scale \( M_D \) by the inverse thickness of the brane \( M \) as a cutoff factor.

### 4.3 Production of single KK excitations

Finally, we will consider the effect which gravity interactions may have in the production of a single matter KK excitation. Due to KK number conservation in UED models, matter KK excitations can be produced only in pairs. However, the gravity-matter interaction does not obey the KK number conservation rule, therefore graviton exchange may mediate the production of a single KK excitation at linear or hadron colliders. The relevant propagator sum for such a process is:

\[
D_1(s) = \sum_{\vec{n}} \mathcal{F}_{0|n_5} \left( \frac{i}{-M_N^2} \right) (\mathcal{F}_{c|n_5}^*)^*.
\]

Since we assume that the experiment energy is much smaller than the string scale \( M_D \) (or the cutoff scale \( M_S \)), we have neglected it in the above sum, which therefore describes both \( s \)-channel and \( t \)-channel contributions. Integrating over the graviton density of states like in the previous section we then obtain:

\[
D_1(s) = (\frac{r}{2\pi})^N V_{N-1} \frac{-i M_S^{N-3}}{N-3} \frac{M - 2i \sqrt{2}}{\pi^2} \int_0^{\pi M_s/M} \frac{\sin x}{1 - x^2/\pi^2} dx.
\]

The integral above has the asymptotic value \( \pi \text{SinIntegral}(\pi) \simeq 5.81 \).

The amplitude for the production of a single KK excitation is therefore of order \( \kappa^2 s D_1 \sim s^2M/M_D^5 \) (we take the cutoff scale to be \( M_S \sim M_D \)). On the other hand, the amplitude for the production of two KK states is of order \( g^2 \), where \( g \) is the coupling constant for the force which mediates the interaction (strong at a hadron collider, electroweak at a linear collider). Then it can be expected that the ratio of cross-sections for single KK production versus double KK production should be of order \( (s^2M/g M_D^5)^2 \) which seems to be quite small. However, the fact that greater CM energy is needed to produce two KK excitations has also to be taken into account. If \( 2M > \sqrt{s} \) at a linear collider, production of double KK excitations cannot take place. Also, at a hadron collider, the effective luminosity decreases rather fast with \( s \).

For illustration, we present in Fig. 4 the production cross-section for a single KK excitation of the \( b \) quark (process mediated by the \( s \)-channel exchange of a KK tower of gravitons), and the production cross-section for the double KK state \( b^* b^* \). We take \( M_D = 2 \text{ TeV for} \)
Figure 4: $q\bar{q} \rightarrow b\bar{b}^*$ (straight line) and $q\bar{q} \rightarrow b^*\bar{b}^*$ (dotted line) production rates at Tevatron Run II (left) and LHC (right).

the Tevatron cross-section, and $M_D = 5$ TeV for the LHC cross-section\(^6\). Note that, up to values of $M$ of about 1/3 of the total collider CM energy, the cross-section for single KK production increases with mass. This is due to the $s^2M/M_D^2 \sim (M/M_D)^5$ behavior of the amplitude. (For higher values of $M$, the decrease in effective luminosity will bring the cross-section down). This behavior may allow for the observation of a single KK state even if the compactification scale $M$ is too large for the production of two excited states. However, the cross-section for single KK production will be large enough to be observable only in the case when $M_D$ is not much larger than $M$, since every doubling of the string scale will reduce this cross-section by about three orders of magnitude.

5 Conclusions

If there are large extra dimensions, gravity may have an important role to play in the phenomenology of present day colliders. Almost all analyses of the gravitational effects so far concentrate on scenarios with matter being restricted to the 4-dimensional brane. However, gravity can also have an important role to play in models where matter also propagate into extra dimensions. In this paper, we analyze the interactions of matter with gravity in such models.

For the gravitational field, we use the linearized lagrangian which describes small perturbations around a flat background metric. The reduction to KK modes and the gauge fixing for the gravitons is done in a manner identical to [12]. The results obtained for the gravity-matter interaction are valid for a large class of models where matter fields propagate into one extra dimension. For the sake of specificity, we consider a case where all matter fields

\(^6\)For $M_D$, we use the definition: $M_{Pl}^2 = (r/2\pi)^{N+2}M_D^N$. 

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propagate a reduced distance along the fifth dimension (UED on the fat brane scenario). We then compute the energy-momentum tensor of the matter scalar, vector and fermion fields in five dimensions, perform the reduction to KK modes, and derive the general Feynman rules for interaction between the KK excitations of matter and gravity.

We then make use of these results for some phenomenological analysis. First, we compute the gravitational decay widths of first level KK excitations of matter. If such excitations are produced at hadron collider, their decay modes will play an important role in their detection and identification, as discussed in [11]. Second, we discuss the effect which the thickness of the brane has on the interactions of SM matter with gravity. We find out that having a fat brane reduces the strength of the effective gravitational coupling by a factor proportional to the superposition of the wave functions of matter and gravity along the fifth direction. Perhaps more interesting, we find out that the thickness of the brane can act as an effective cutoff scale in the evaluation of sums over propagators of virtual KK gravitons.

Finally, we consider the case of production of single KK excitations of quarks and gluons at hadron colliders. Due to KK number conservation in UED-type models, KK excitations of matter are usually produced in pairs, thus requiring a large CM energy even for relatively small compactification scale. However, the gravitational interactions break KK number conservation (in models with matter on a fat brane), therefore they can mediate single KK production. We show that the cross-section for such a process is reasonably large, as long as the string scale $M_D$ is not much larger than the mass of the matter KK excitations.

Acknowledgments

This work was supported in part by the U.S. Department of Energy Grant Numbers DE-FG03-98ER41076, DE-FG02-01ER45684, and DE-FG02-91ER40685.

Appendix

We present here the components of the matter energy-momentum tensor which are trilinear and quadrilinear in the matter fields. For simplicity, these components are given in momentum representation. The Feynman rules for the gravity interactions can then be easily derived (as in Eqs. (23)).

Fig. 5 contains the trilinear terms in the matter energy-momentum tensor, while Fig. 6 contains the quadrilinear terms. The following notations are used:

$$F_{\mu\nu,\rho\sigma\lambda}(k_1, k_2, k_3) = \eta_{\mu\rho} \eta_{\sigma\lambda}(k_2 - k_3)_\nu + \eta_{\mu\sigma} \eta_{\rho\lambda}(k_3 - k_1)_\nu + \eta_{\mu\lambda} \eta_{\rho\sigma}(k_1 - k_2)_\nu + (\mu \leftrightarrow \nu)$$

$$G_{\mu\nu,\rho\sigma\lambda\delta} = \eta_{\mu\nu} D'_{\rho\sigma,\lambda\delta} + \left( \eta_{\mu\rho} \eta_{\nu\delta} \eta_{\lambda\sigma} + \eta_{\mu\lambda} \eta_{\nu\sigma} \eta_{\rho\delta} - \eta_{\mu\rho} \eta_{\nu\delta} \eta_{\lambda\sigma} + (\mu \leftrightarrow \nu) \right)$$

The quantities $C_{\mu\nu,\rho\sigma}, C'_{\mu\nu,\rho\sigma}, D_{\mu\nu,\rho\sigma}(k_1, k_2)$, and $D'_{\rho\sigma,\lambda\delta}$ are defined in Eqs. (24), $g$ is the
Figure 5: The trilinear components of the matter energy-momentum tensor in momentum representation. Symbols are defined in the text.

gauge coupling constant, $f^{abc}$ are the structure constants of the gauge group and $T^a$ are the generators of the representation of the gauge group associated with the corresponding matter fields. Also, $j, k, l, m$ are the KK indices of the matter fields. In the definition of the fermion–fermion–gauge-boson vertex, the upper sign holds for the case of doublet fields $Q^l, Q^m$ while the lower sign holds for the case of singlet fields $q^l, q^m$.

References


\[ T_{\mu\nu} = - g^2 C_{\mu\nu, \rho\sigma} \{ T^a_k, T^b_l \} F_{lmkj\in}^{(cccc)} \]

\[ T_{\mu\nu} = 0 \]

\[ T_{\mu5} = 0 \]

\[ T_{55} = - g^2 \eta_{\rho\sigma} \{ T^a_k, T^b_l \} F_{lmkj\in}^{(cccc)} \]

\[ T_{\mu\nu} = g^2 \left( f^{eac}_{\mu\nu} f^{ebd}_{\rho\sigma, \lambda\delta} + f^{eab}_{\mu\nu} f^{ecd}_{\rho\sigma, \lambda\delta} \right) G_{\mu\nu, \rho\sigma, \lambda\delta} \]

\[ T_{\mu5} = 0 \]

\[ T_{55} = g^2 \left( f^{eac}_{\mu\nu} f^{ebd}_{\lambda\sigma, \rho\delta} + f^{eab}_{\mu\nu} f^{ecd}_{\lambda\sigma, \rho\delta} \right) D'_{\rho\sigma, \lambda\delta} \]

Figure 6: The quadrilinear components of the matter energy-momentum tensor in momentum representation. Symbols are defined in the text.


