The Ter-Mikayelian Effect on QCD Radiative Energy Loss

Magdalena Djordjevic and Miklos Gyulassy

Dept. Physics, Columbia University, 538 W 120-th Street,
New York, NY 10027, USA

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Abstract

The color dielectric modification of the gluon dispersion relation in a dense QCD medium suppresses both the soft and collinear gluon radiation associated with jet production. We compute both the longitudinal and transverse plasmon contributions to the zeroth order in opacity radiative energy loss. This QCD analog of the Ter-Mikayelian effect in QED leads to $\sim 30\%$ reduction of the energy loss of high transverse momentum charm quarks produced in a QCD plasma with a characteristic Debye mass $\mu \sim 0.5 \text{ GeV}$.

1 Introduction

Jet tomography in ultra-relativistic nuclear collisions can be used to map out the density of the produced QCD plasma from the suppression pattern of high transverse momentum hadrons \cite{1}-\cite{8}. Jet quenching \cite{9, 10} is mainly due to the medium induced radiative energy loss of high energy partons propagating through ultra-dense QCD matter. Even if final state multiple elastic and inelastic interactions are neglected, the gluon radiation associated with hard QCD vertex softens the lowest order jet spectra. In elementary particle collisions, this can be taken into account through the $Q^2$ (DGLAP) evolution of the hadronic fragmentation functions. In a medium, this radiation is modified by the dielectric properties of the plasma as discussed first by Ter-Mikayelian \cite{11, 12}. The non-abelian QCD analog of the Ter-Mikayelian effect is the subject of this paper. A summary of our main results has been published in paper I\cite{13}. In this paper, the details of how those results were obtained are presented.

This work is motivated by the surprising observation of PHENIX \cite{14} that “prompt” single electron spectrum from open charm production in $Au+Au$ collisions at $\sqrt{s} = 130 \text{ AGeV}$ shows no sign of heavy quark energy loss \cite{15}. In contrast, a dramatic suppression (by a factor $\sim 5$) was observed at RHIC for pions with $p_T > 3 \text{ GeV}$ originating from the fragmentation of light quark and gluons \cite{17}-\cite{22}. See a recent review of light quark and gluon tomography in A+A in ref.\cite{16}. The suppression of light hadrons is consistent with the expected large radiative induced energy loss of light quark and gluon jets in an ultra-dense plasma of density approximately 100 time higher than in ground state nuclei.

The spectrum of induced radiation depends on the optical thickness or opacity $\chi = L/\lambda$ of the plasma, and has been computed to arbitrary order in $\chi^n$ for massless partons in GLV\cite{3, 16}. It was expected \cite{23}-\cite{25} that a similar quenching pattern should also be observed for heavy quark ($c$ or $b$) jet fragmentation. However, in \cite{26} it was pointed out that the a large quark mass would lead to a “dead cone” effect, reducing induced radiation inside the cone $\theta < M/E$ and that this should reduced radiative energy loss of heavy quarks as compared to light partons. Numerical estimates indicated that the quenching of charm quarks may be approximately about a half that of light quarks.
Experimentally, PHENIX however, suggest that the charm quark energy loss could be significantly smaller than that.

In the letter [13] we showed that the apparent null effect observed for heavy quark energy loss via single electrons could be due to the cancelation of two important medium effects, i.e. Ter-Mikayelian (plasmon) effect and medium induced radiative energy loss. Here we provide a detailed derivation of the Ter-Mikayelian effect.

In [26] the suppression of radiation below a plasma frequency cutoff was estimated to be only \( \sim 10\% \) effect on the induced energy loss. However, the Ter-Mikayelian effect on the zeroth order in opacity \((L/\lambda)^0\) radiation was not considered up to now. The first estimates of the influence of a plasma frequency cutoff in QCD plasmas were reported in ref. [32] using a constant plasmon mass \( \omega_0 \) [33]-[35].

The \( k \) dependence of the gluon self energies and the magnitude of longitudinal radiation were not investigated in that work. We extend those results by taking both longitudinal as well as transverse modes consistently into account via the frequency and wavenumber dependent hard thermal (1-loop HTL) self energy \( \Pi^\mu\nu(\omega, \vec{k}) \) [27]-[31].

As noted in I[13], the dielectric properties of an isotropic plasma lead to a transverse gluon self energy \( \Pi_T(\omega, \vec{k}) \) with \( \Pi_T(\omega_{pl}(0), 0) = \omega_{pl}^2(0) \approx \mu^2/3 \), where \( \mu \approx gT \) is the Debye screening mass of a plasma at temperature \( T \) in the deconfined phase. In addition, long wavelength collective longitudinal gluon modes arise with \( \Pi_L(\omega_{pl}(0), 0) = \omega_{pl}^2(0) \). This dynamical gluon mass suppresses the radiation of soft \( \omega < \omega_{pl}(\vec{k}) \) gluons and shields the collinear \( k_\perp \to 0 \) singularities that arise for massless quarks. In this paper we study under what conditions the assumption of using a \( k \) independent effective plasmon mass and neglecting longitudinal modes may be adequate. We show below that this simplifying assumption is surprisingly accurate \( \sim 10\% \) if the asymptotic mass [30], \( \omega_\infty = m_E/\sqrt{2} \) rather than the \( k = 0 \) \( \omega_{pl} = \sqrt{2/3} \omega_\infty \) is employed. The accuracy of the approximation also improves dramatically for heavy quark jets.

## 2 Jet Production in the Vacuum

In order to introduce the method that we use to compute the QCD analog of the Ter-Mikayelian [11]-[12] effect on the zeroth order in opacity radiation, we consider first the well known case of radiation in the vacuum.

![Fig.1a](image)

Fig.1a illustrates the bare jet vacuum to vacuum amplitude

\[
-iM_0 = \frac{d_J}{2} \int \frac{d^4 p}{(2\pi)^4} J(p) \Delta_M(p) J(-p)
\]  

(1)

where \( \Delta_M(p) = (p^2 + M^2 + i\epsilon)^{-1} \) is the jet propagator for a spinless parton of mass \( M \) in the \( d_J \) dimensional representation of \( SU(N_c) \). We will ignore spin effects throughout since they are irrelevant in the soft radiation limit.

The effective jet source current, \( J \), creates an invariant distribution of jets as given by

\[
2\text{Im} M_0 = \int d^3 N_J = \int \frac{d^3 \vec{p}}{2E_p} E_p \frac{d^3 N_J}{d^3 \vec{p}}
\]  

(2)

where \( E_p d^3 N_J / d^3 \vec{p} = d_J |J(E_p, \vec{p})|^2/(2(2\pi)^3) \) with \( E_p^2 = M^2 + \vec{p}^2 \).
The one gluon radiative correction amplitude, \( iM_1 \), to the jet spectrum is illustrated in Fig.1b

\[
-iM_1 = -\frac{g^2 C_J d_J}{2} \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} |J(p + k)|^2 \Delta_M(p + k)^2 \Delta_M(p)(2p + k)^\mu D^{(0)}_{\mu\nu}(k)(2p + k)^\nu. \tag{3}
\]

The free gluon propagator in the axial \((u_\mu A^\mu = 0)\) gauge is

\[
D^{(0)}_{\mu\nu}(k) = -\left( g^{\mu\nu} - \frac{u^\mu k^\nu + k^\mu u^\nu}{(uk)^2} + u^2 \frac{k^\mu k^\nu}{(uk)^2} \right) \Delta_m(k), \tag{4}
\]

where \( m \) is an infrared mass in the massive gluon scheme introduced by need to study the \( m = 0 \) limit. In the next section, the dynamic polarization tensor will replace \( m \).

The imaginary part of this amplitude contains real and virtual radiation corrections to \( iM_0 \). The inelastic one gluon radiation contribution is obtained by the Cutkosky rule [39]

\[
\Delta_M(p)\Delta_m(k) \rightarrow (-2\pi i)^2 \delta(k^2 - m^2)\delta(p^2 - M^2). \tag{5}
\]

This gives a contribution

\[
2\text{Im}\, M_1 |_{rad} = \int \frac{d^4p}{(2\pi)^4} \delta(p^2 - M^2)d_J \int \frac{d^4k}{(2\pi)^4} \delta(k^2 - m^2)|J(p + k)|^2 \times \frac{C_J g_s^2}{(Q^2 - M^2)^2} \left\{ (Q^2 - M^2)^4 \frac{(pu)}{(uk)^2} - (Q^2 - M^2)^2 \frac{u^2}{(uk)^2} - 4M^2 + m^2 \right\}, \tag{6}
\]

where

\[
Q^2 = (p + k)^2 = 2(pk) + k^2 + M^2. \tag{7}
\]

In general, the result depends on the gauge parameter \( u^\mu \) because the external color source \( J \) breaks gauge invariance. However, in the soft gluon limit, \( k_\perp \ll k^+ \ll p^+ \), where \( k_\perp \) is measured relative to the \( \vec{p} \) axis of the jet, one can extract the familiar DGLAP soft radiation spectrum with any \( u^\nu \) for which \((uk)/(up) \approx k^+/p^+ = x \ll 1\). For typical light cone kinematics of interest
\[
\begin{align*}
   k^\mu & = \left[ xE^+, (k_\perp^2 + m^2)/xE^+, k_\perp \right] \\
   p^\mu & = \left[ (1 - x)E^+, (M^2 + k_\perp^2)/(1 - x)E^+, -k_\perp \right] \\
   2pk & = \frac{k_\perp^2 + (1 - x)^2m^2 + x^2M^2}{x(1 - x)} \\
   Q^2 & = pk + m^2 + M^2 = \frac{k_\perp^2 + (1 - x)m^2 + xM^2}{x(1 - x)}.
\end{align*}
\]

Note that the vertex factor \{...\} in Eq. (6) reduces to \((2pk + m^2)/x\) in the \(x \to 0\) limit in both the \(A^+ = 0\) light cone and temporal \(A^0 = 0\) gauges.

We assume that \(J\) is slowly varying \(J(p + k) \approx J(p)\) for soft radiation, and therefore, in soft radiation approximation the spectrum can be extracted as

\[
2\text{Im}M_1|_{\text{rad}} \approx \int d^3N_J \int d^3N_g^{(0)} ,
\]

leading to the finite mass generalization of the small \(x\) invariant DGLAP radiation spectrum

\[
\omega \frac{dN_g^{(0)}}{d^3k} \approx x \frac{dN_g^{(0)}}{dx d^2k_\perp} \approx \frac{C_J\alpha_s}{x\pi^2(Q^2 - M^2)} = \frac{C_J\alpha_s}{\pi^2} \frac{1}{k_\perp^2 + (1 - x)m^2 + xM^2}.
\]

3 Ter-Mikayelian effect

In this section we want to compute zeroth order medium induced radiative quark energy loss. In soft gluon limit, the result should not depend on the choice of gauge, as long as \((uk)/(up) \approx x \ll 1\) is satisfied. We simplify our calculations by choosing the temporal axial gauge. However, we have explicitly checked that, in soft gluon limit, the same result is obtained using the light cone gauge.

The radiative heavy quark energy loss in hot dense medium involves both transverse and longitudinal gluon radiation. In temporal axial gauge gluon propagator has the following form (see appendix A):

\[
D_{\mu\nu} = -\frac{P_{\mu\nu}}{\omega^2\epsilon_T - \frac{k^2}{\omega^2\epsilon_L}} - \frac{Q_{\mu\nu}}{\omega^2\epsilon_L},
\]

where transverse \((P_{\mu\nu})\) and longitudinal \((Q_{\mu\nu})\) projectors are given in terms of \(\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\) and \(\bar{u}_\mu = \bar{g}_{\mu\nu} u_\nu\), by

\[
P_{\mu\nu} = \bar{g}_{\mu\nu} - \frac{\bar{u}_\mu \bar{u}_\nu}{\bar{u}^2},
\]

\[
Q_{\mu\nu} = (g_{\mu\nu} - \frac{u_\mu k_\nu + k_\mu u_\nu}{u \cdot k}) + \frac{u^2 k_\mu k_\nu}{(u \cdot k)^2} - \frac{u \cdot k^2}{k^2 u^2} P_{\mu\nu}
\]

and

\[
\epsilon_T = 1 - \frac{\mu^2}{2k^2}(1 - \frac{\omega^2 - \bar{k}^2}{2\omega|k|} \log(\frac{\omega + \bar{k}}{\omega - |\bar{k}|}))
\]
\[ \epsilon_L = 1 + \frac{\mu^2}{k^2} \left( 1 - \frac{\omega}{2|k|} \log\left( \frac{\omega + |k|}{\omega - |k|} \right) \right) \]  

(15)

are transverse and longitudinal dielectric functions respectively \[31\].

In order to obtain the main order radiative energy loss we want to compute the squared amplitude of Feynman diagram diagram \(|M_{rad}|\), which represents the source \(J\) that produces an off-shell jet with momentum \(p'\), which subsequently radiates an on-shell gluon with momentum \(k\). The jet emerges with momentum \(p\) and mass \(m\). From momentum conservation we have that \(p' = p + k\). To solve this diagram we will use optical theorem, i.e.:

\[
\int |M_{rad}|^2 \frac{d^3\vec{p}}{(2\pi)^3 2E} \frac{d^3\vec{k}}{(2\pi)^4 2\omega} = 2\text{Im}M_{TM} = \int d^3N_J \int d^3N_g^{TM},
\]

(16)

where, in axial gauge, \(M_{TM}\) is the amplitude of the following diagram:

\[
\text{Im}M_{TM}
\]

FIG. 2 shows the parts of the diagram \(M\) which will contribute to the radiative energy loss (i.e. the parts in which momentum \(p + k\) is off-shell, and momenta \(p\) and \(k\) are on-shell). Therefore, the dashed line shows which propagators are to be put on-shell. The HTL blob in the diagram means using Eq. (11) with (14, 15).

We assume that \[3\] \(J\) varies slowly with change of \(p\), i.e. that \(J(p + k) \approx J(p)\). Neglecting the spin in the problem, the amplitude \(M\) becomes:

\[
-iM = -\frac{C_R D_R}{2} \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} |J(p)|^2 g_s^2 \frac{1}{(Q^2 - M^2)^2} \frac{1}{p^2 - M^2 + i\epsilon} (2p + k)^\mu D_{\mu\nu}(2p + k)^\nu.
\]

(17)

The contribution to on-shell radiation is

\[
2\text{Im}M_{TM} = iC_R D_R \int \frac{d^4p}{(2\pi)^4 2E} |J(p)|^2 \frac{d^4k}{(2\pi)^4} g_s^2 \frac{1}{(Q^2 - M^2)^2} (2p + k)^\mu D_{\mu\nu}(2p + k)^\nu,
\]

(18)

where we have used \[39\] \(1/(p^2 - M^2 + i\epsilon) \rightarrow (-2\pi i)\delta(p^2 - M^2)\) for the cuted quark propagator in Fig. 2. Also, as shown in Appendix B, the contour integration over \(\omega\) for on-shell quark and glue is equivalent to replacing \(D_{\mu\nu}\) by:
\[ D_{\mu\nu} \rightarrow -P_{\mu}(2\pi i)\delta(\omega^2\epsilon_T - \vec{k}^2) - \frac{Q_{\mu\nu}}{\omega^2}(-2\pi i)\delta(\epsilon_L), \]  

where we only keep positive energy contribution.

\[ \int d^3N_\perp \int d^3N_g^{TM} = \int \frac{d^3\vec{P}}{(2\pi)^32E} |J(p)|^2 \int C_{RG} \frac{1}{(2\pi)^3(Q^2 - M^2)^2} \times \]

\[ \times (2p + k)^\mu(-P_{\mu\nu}\delta(\omega^2\epsilon_T - \vec{k}^2) - \frac{Q_{\mu\nu}}{\omega^2}\delta(\epsilon_L))(2p + k)^\nu \]

For this calculation it is more convenient to choose coordinate system such that \( p = (E, |\vec{p}|, 0, 0) \), and \( k = (\omega, |\vec{k}|\cos \theta, |\vec{k}|\sin \theta, 0) \).

With this kinematics we get:

\[ (2p + k)^\mu P_{\mu\nu}(2p + k)^\nu = -(2p + k)^\mu(\delta_{ij} - \frac{k_i k_j}{k^2})(2p + k)^j = -4\vec{p}^2 \sin^2 \theta \]

\[ (2p + k)^\mu Q_{\mu\nu}(2p + k)^\nu = -(2p + k)^\mu(\frac{k_i k_j}{k^2})(2p + k)^j = -(2|\vec{p}| \cos \theta + |\vec{k}|)^2. \]

The integrated radiation yield than becomes:

\[ \int d^3N_g^{TM} = \int d\omega d\cos \theta \vec{k}^2 d|\vec{k}| \frac{C_{R\alpha S}}{\pi} \frac{1}{(Q^2 - M^2)^2} \times \]

\[ \times \left\{ 4\vec{p}^2 \sin^2 \theta \delta(\omega^2\epsilon_T - \vec{k}^2) + \frac{(2|\vec{p}| \cos \theta + |\vec{k}|)^2}{\omega^2} \delta(\epsilon_L) \right\} \]

where \( Q^2 - M^2 = \omega^2 - \vec{k}^2 + 2(E\omega - |\vec{p}| |\vec{k}| \cos \theta) \), as in Eq. (7).

The Ter-Mikayelian modified \( 0^{th} \) order in opacity radiative energy loss is defined as \( dI_g = \omega d^3N_g^{TM} \). Using this, the transverse and longitudinal contribution to the \( 0^{th} \) order radiated energy loss per wave number is given by the following:

\[ \frac{dI_T}{d|\vec{k}|} = \frac{C_F}{\pi} \frac{4\vec{p}^2 \omega_T^2(\omega_T^2 - \vec{k}^2)}{\omega_T^2 \mu^2 - (\omega_T^2 - \vec{k}^2)^2} \int_0^1 d\cos \theta \frac{\alpha_s(Q^2 - M^2)}{(Q^2 - M^2)^2} \sin^2 \theta \]

\[ \frac{dI_L}{d|\vec{k}|} = \frac{C_F}{\pi} \frac{4\vec{p}^2 \omega_L^2(\omega_L^2 - \vec{k}^2)}{\mu^2 - (\omega_L^2 - \vec{k}^2)^2} \int_0^1 d\cos \theta \frac{\alpha_s(Q^2 - M^2)}{(Q^2 - M^2)^2} \left( \cos \theta + \frac{\vec{k}^2}{2|p|^2} \right)^2 \]

where we keep only the forward, \( \theta > 0 \), emission to isolate the energy loss of the nearside jet. In Eq. (23) \( \omega_T \) and \( \omega_L \) are positive zeros of \( (\omega_T^2 - \vec{k}^2) \) and \( \epsilon_L \) respectively.

The angular integration can be performed analytically if \( \alpha_s \) does not run, but it is not particularly useful. We perform the integration using

1) Zero momentum cutoff
2) "Frozen \( \alpha \) model" [40]

\[ \alpha_s(Q^2 - M^2) = \text{Min}\{0.5, \frac{4\pi}{\beta_0 \log(\frac{Q^2 - M^2}{\Lambda_{QCD}^2})}\}, \]  

(24)
where $\beta_0 = \frac{2\pi}{3}$ for effective number of flavors $n_f \approx 2.5$ and $\Lambda_{QCD} \approx 0.2$ GeV.

Figures 3a and 3b show transverse an longitudinal contribution to the charm and bottom radiative energy loss at different Debye masses.

FIG 3a. The 0$^{th}$ order in opacity contribution to Charm quark radiated energy loss for 15 GeV jet is shown as a function of wave number. The dashed-dotted curve shows what would the energy loss be if gluons were treated as massless and transversely polarized. From top to bottom (left to right) solid (dashed) curves show medium modified transverse (longitudinal) contribution to the energy loss for Debye mass 0.25 GeV, 0.5 GeV and 1 GeV respectively.

FIG 3b. The 0$^{th}$ order in opacity contribution to Bottom quark radiated energy loss for 15 GeV jet is shown as a function of wave number. The dashed-dotted curve shows what would the energy loss be if gluons were treated as massless and transversely polarized. From top to bottom (left to right) solid (dashed) curves show medium modified transverse (longitudinal) contribution to the energy loss for Debye mass 0.25 GeV, 0.5 GeV and 1 GeV respectively.

We see that longitudinal contribution to the energy loss is significant only in the $\omega$ region around Debye mass. As temperature $T$ of the medium increases the transverse contribution to the energy loss decreases, while longitudinal contribution increases. Therefore, we can conclude that for lower Debye
masses (lower temperatures) the longitudinal contribution to the energy loss is negligible, but at high enough temperatures it may become comparable to the transverse contribution.

Fig. 4 shows 0th order fractional energy loss for charm and bottom quarks in hot medium with Debye mass $\mu = 0.5$ GeV, and zero momentum cutoff.

![Graph](image)

FIG 4. The 0th order in opacity fractional energy loss for charm and bottom quarks in hot medium with Debye mass $\mu = 0.5$ GeV, and zero momentum cutoff is shown as a function of the charm quark energy. The upper (lower) solid curve shows transverse fractional energy loss for charm (bottom) quark. The dashed curves show the negligible additional effect of longitudinal plasmons.

However, the kinematic mass effects on the transverse contribution are important (as shown on Fig. 5 and 6).

![Graph](image)

FIG 5. One loop transverse plasmon mass $m_g(|\vec{k}|) \equiv \sqrt{\omega^2 - |\vec{k}|^2}$ is shown as a function of quark momentum $|\vec{k}|^2$. We see that $m_g$ starts with the value $\omega_{pl} = \mu/\sqrt{3}$ at low $|\vec{k}|$, and that as $|\vec{k}|$ grows, $m_g$ asymptotically approaches the value of $\omega_{\infty} = \mu/\sqrt{2}$ in agreement with [30].
FIG 6. Medium modified zeroth order in opacity charm quark fractional energy loss is shown as a function of quark energy. Full curve shows the transverse energy loss using Eq. (23). Dot-dot-dashed, dashed and dot-dashed curves show what would be the transverse energy loss if we define gluon mass as $\omega_{pt}$, $m_{inf}$ and $\mu$ respectively.

From Fig. 5 and 6 we can conclude that we can approximate the Ter-Mikayelian effect by simply taking $m_g \approx m_\infty$ (see also Appendix B1).

FIG 7. The reduction of the vacuum energy loss for charm and bottom quark due to the QCD Ter-Mikayelian plasmon effect is shown as a function of the quark energy. The upper (lower) dashed-dotted curve shows the vacuum energy loss for charm (bottom) quark if gluons are treated as massless and transversely polarized. The upper (lower) solid curve shows medium modified (but zeroth order in opacity) transverse fractional energy loss for charm (bottom) quark. We take $\mu = 0$ GeV for vacuum and $\mu = 0.5$ GeV for medium case.

We see that for charm quark medium effect is significant since it leads to $\approx 30\%$ suppression of the vacuum radiation. The Ter-Mikayelian effect thus enhances the yield of high transverse charm quarks relative to the vacuum case. On the other hand, for bottom quark Ter-Mikayelian effect is negligible, since it leads only to the small suppression of the vacuum radiation.
FIG 8. Medium modified zeroth order in opacity fractional energy loss is shown as a function of Debye mass $\mu$. We see that medium effect is important for light (dot-dashed curve) and charm (dashed curve) quark energy loss, since it leads to the strong suppression of the vacuum radiation. On the other hand, bottom quark (dot-dot-dashed curve) energy loss shows insignificant dependence on the medium.

So far, we have discussed the Ter-Mikayelian effect for only heavy quarks. The generalization of the plasmon effect to light quarks is not trivial due to the fact that the light quark vacuum energy loss is infrared divergent. However, on Fig. 8 we see that in a QCD medium, dynamic polarization naturally regulates infrared divergences for light quarks, since quark [41, 42] and gluon acquire a finite self energy. Confinement in the vacuum naturally limits the effective screening scale to $\mu_{\text{vac}} = \Lambda_{QCD}$. The Ter-Mikayelian effect on both heavy and light quarks would then be the difference between radiative energy loss at $\mu_{\text{vac}}$ and $\mu_{\text{medium}}$.

4 Conclusion

In this paper we computed both the transverse and longitudinal contributions to the zeroth order in opacity radiative quark energy loss. We have shown that longitudinal contribution can be neglected for the energy range of experimental interest. We have also seen that transverse polarization can be, for moderate range of temperatures ($0.5 \leq \mu \leq 1$ GeV), approximated by simple form $D_{\mu\nu} \approx -P_{\mu\nu}/k^2 - m_g^2$, where $P_{\mu\nu}$ is transverse projector and $m_g \approx \mu/\sqrt{2}$. Therefore, it is surprising how well the effect of medium polarization $\Pi_{\mu,\nu}(\omega, \vec{k})$ can be approximated by a simple effective mass $m_g$.

As reported in I [13], the Ter-Mikayelian reduces the $\mu_{\text{vac}} = 0$ GeV vacuum energy loss by $\approx 30\%$ for a $\mu = 0.5$ GeV medium. If $\mu_{\text{vac}} > 0$ GeV is set then the reduction would, of course, be smaller. The interplay between the enhanced, medium induced first order in opacity contribution, $\Delta E^{(1)}$, to the reduced zeroth order energy loss, $\Delta E^{(0)}$, as reported in I [13] will be discussed in more detail in the subsequent paper [38].

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A Gluon propagator in temporal axial gauge

We recall here some of the basic tensorial properties of the gluon propagator, in axial gauges \((u^\mu A_\mu(\vec{x}, t) = 0, u^\mu\) a fixed four vector).

The transverse projector \(P_{\mu\nu}\) (with respect to both \(k^\mu\) and \(u^\mu\)), is given by Eq. (12). Note that \(\Pi^2 = u^2 - (ku)^2/k^2\) is well defined even in light cone \((u^2 = 0)\) gauge.

The orthogonal longitudinal projector, \(Q_{\mu\nu}\) (with respect to \(k^\mu\)), is given by

\[
\begin{align*}
Q_{\mu\nu} &= (g_{\mu\nu} - u^\mu k_{\nu} + k^\mu u_{\nu}) + P_{\mu\nu} = \left(\frac{u^\mu u_{\nu}}{u^2} (uk)^2 + k^\mu k^\nu u^2 - (u^\mu k_{\nu} + k^\mu u_{\nu})(uk)\right) \frac{1}{u^2 k^2 - (uk)^2} \\
&= \left(\frac{u^\mu u_{\nu}}{u^2} (uk)^2 + k^\mu k^\nu u^2 - (u^\mu k_{\nu} + k^\mu u_{\nu})(uk)\right) \frac{1}{u^2 k^2 - (uk)^2}
\end{align*}
\]

In terms of these projectors the free gluon propagator in the axial gauge is

\[
D^{(0)}_{\mu\nu}(k) = -\Delta(k) (P_{\mu\nu} + \beta(k, u) Q_{\mu\nu}) = -\frac{1}{k^2 + i\epsilon} \left(g_{\mu\nu} - \frac{u^\mu k_{\nu} + k^\mu u_{\nu}}{(uk)} + u^2 \frac{k^\mu k_{\nu}}{(uk)^2}\right)
\]

where

\[
\beta(k, u) = \frac{k^2 u^2}{(uk)^2}.
\]

is a kinematic factor. In the temporal axial gauge, \(u = (1, 0, 0, 0), \beta = 1 - k^2/\omega^2\) and the projectors reduce to

\[
P_{\mu\nu} = Q_{\mu\nu} = 0, \text{ if } \mu, \nu = 0
\]

\[
P_{ij} = -\delta_{ij} + \frac{k_i k_j}{k^2}, \quad Q_{ij} = -\frac{k_i k_j}{k^2}, \text{ if } i, j = 1, 2, 3.
\]

In a medium with four velocity \(u^\mu\) the gluon acquires a temperature dependent self energy \(\Pi_{\mu\nu}\) in addition to its vacuum self energy. The one-loop (Hard Thermal Loop [30] (HTL) or equivalently, eikonal linear response [31]) self energy can be decomposed as

\[
\Pi_{\mu\nu} = \Pi_L R_{\mu\nu} + \Pi_T P_{\mu\nu}
\]

where

\[
R_{\mu\nu} = \frac{\Pi_{\mu\nu}}{\Pi^2}
\]

is the longitudinal projector in covariant gauges with respect to \(u^\mu\) but transverse with respect to \(k^\mu\). Therefore, the HTL self energy is transverse with respect to \(k, k\Pi = \Pi k = 0\). Note that \(RP = PR = 0, R^2 = R,\) and

\[
QRQ = Q/\beta(k, u).
\]

where \(\beta\) is given by (27).

The HTL medium modified gluon propagator \((D_{\mu\nu})\) can be obtained by solving the Dyson equation [41]

\[
D_{\mu\nu} = D_L Q_{\mu\nu} + D_T P_{\mu\nu} = D_{\mu\nu}^{(0)} - D_{\mu\nu}^{(0)} \Pi^{a\beta} D_{\alpha\nu} = -\frac{P_{\mu\nu}}{\omega^2 \epsilon_T - \mathbf{k}^2} - \frac{Q_{\mu\nu}}{\omega^2 \epsilon_L}.
\]
Note that, in general [31], we would have one extra term $\eta \frac{k\cdot k}{k^2}$ in the gluon propagator, where $\eta$ is a gauge parameter. However, in temporal axial gauge [41], $D_{\mu\nu} = 0$, if $\mu, \nu = 0$.

Therefore, in temporal axial gauge $\eta$ has to be equal to zero, i.e. extra term vanishes.

B Cutkosky rules for gluon propagator in the medium

In this appendix we want to justify using the Cutkosky rules for gluon propagator in the medium, which is represented by Eq. (19) in our computations.

Diagram $M$

corresponds to $\sum M_n$, where $M_n$ is the amplitude of the following diagram:

Here,

Diagram 2
\((iM_n) = Tr\left\{ \sum_{a\sigma} \int \frac{d^4p_J}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} |J(p+k)|^2 \frac{1}{((p_J+k)^2-M^2+i\epsilon)^2} \frac{i}{p_J^2-M^2+i\epsilon} \right. \\
\left. \times (-ig_s(2p_J+k)^\mu T_{a0}) \frac{-i\gamma_{\nu\mu\alpha}}{k^2+i\epsilon} \frac{i\gamma_{\nu\mu\alpha}}{k^2+i\epsilon} \frac{i\gamma_{\nu\mu\alpha}}{k^2+i\epsilon} \frac{-i\gamma_{\nu\mu\alpha}}{k^2+i\epsilon} \frac{-i\gamma_{\nu\mu\alpha}}{k^2+i\epsilon} \frac{-i\gamma_{\nu\mu\alpha}}{k^2+i\epsilon} \right\}\) (33)

where \(\pi^{\mu\nu}(k)\) is the amplitude of the diagram 2, averaged over thermal momentum distribution \(n_{eq}(p)\) given by Eq. (35).

\[
i\pi^{\mu\nu}(k) = \int d^4p_{eq}(p) \left\{ \frac{i}{(p+k)^2+i\epsilon} (-ig_s(2p+k)^\mu)(-ig_s(2p+k)^\nu) + \right. \\
\left. + \frac{i}{(p-k)^2+i\epsilon} (-ig_s(2p-k)^\mu)(-ig_s(2p-k)^\nu) + 2ig^{\mu\nu} \sum_j jT_aT_bj^\dagger \right\} \) (34)

where \(\sum_j jT_aT_bj^\dagger = \frac{1}{2}\delta_{ab}\), and \(n_{eq}(p)\) is the equilibrium momentum distribution [31] at temperature \(T\) including both quarks and gluons

\[
n_{eq}(p) = \frac{1}{2}(Q^-_{eq} + Q^+_{eq}) + NG_{eq}. \) (35)

Here

\[
Q^\pm_{eq} = \frac{2N_f}{(2\pi)^3} 2\theta(p^0)\delta(p^2)(\exp(\pm \frac{p_0}{T}) + 1)^{-1} \) (36)

and

\[
G_{eq} = \frac{2N}{(2\pi)^3} 2\theta(p^0)\delta(p^2)(\exp(\frac{p_0}{T}) - 1)^{-1} \) (37)

are quark (antiquark) and gluon distributions respectively. \(N_f\) is the number of flavors, and \(N\) is the number of colors.

In the soft gluon limit \(\pi^{\mu\nu}\) becomes

\[
\pi^{\mu\nu} = -g_s^2 \int d^4p \frac{p^\mu k_\alpha (p^\rho \partial_\rho p^\alpha - p^\alpha \partial_\rho p^\rho)}{(pk) + i\epsilon} n_{eq}(p). \) (38)

in agreement with [31].

It is easy to prove that \(\pi^{\mu\nu}\) is transverse, i.e. \(\pi^{\mu\nu}k_\nu = k_\mu \pi^{\mu\nu} = 0\).

Therefore, in covariant gauge \(\pi^{\mu\nu}\) can be decomposed:

\[
\pi^{\mu\nu} = \Pi_T P^{\mu\nu} + \Pi_L R^{\mu\nu}, \) (39)

where \(P^{\mu\nu}\) and \(R^{\mu\nu}\) are given by Eqs. (12) and (30) respectively.

Using Eqs. (38, 39, 12, 30) we get

\[
\Pi_L = Q_{\mu\nu}\pi^{\mu\nu} = -g_s^2 \frac{k^2}{u^2k^2 - (ku)^2} \int d^4p (pu)(ua\partial_n) - (pu)(ka\partial_n) \) (40)
\[
\Pi_T = \frac{1}{2} P_{\mu\nu} \pi^{\mu\nu} = -\frac{1}{2} g_s^2 \int d^4p \frac{1}{(pk)} \{(k\partial n)(\frac{k^2(pu)^2}{u^2k^2 - (ku)^2} - p^2) + (pk)((p\partial n) - \frac{k^2(pu)(u\partial n)}{u^2k^2 - (ku)^2})\} \tag{41}
\]

By replacing \( n_{eq}(p) \) from Eq. (35) we get

\[
\Pi_L = -\frac{(\omega^2 - \bar{k}^2)\mu^2}{k^2} (1 + \frac{\omega}{2|\bar{k}|} \log |\omega - |\bar{k}||) \tag{42}
\]

and

\[
\Pi_T = \frac{\mu^2}{2} + \frac{(\omega^2 - \bar{k}^2)\mu^2}{k^2} (1 + \frac{\omega}{2|\bar{k}|} \log |\omega - |\bar{k}||) \tag{43}
\]

where \( \mu = g_s T \sqrt{1 + \frac{N_f}{6}} \).

These results are in agreement with [31], and they lead to \( \epsilon_L \) and \( \epsilon_T \) which we use in this paper.

Using \( \pi^{\mu\nu} \) from Eq. (39) \( M \) finally becomes:

\[
M = (-i) \int D_R |J(p_J)|^2 \frac{d^4p_J}{(2\pi)^4} \frac{1}{p_J^2 - M^2 + i\epsilon} \times \frac{1}{k^2 - \Pi_T + i\epsilon} \tag{44}
\]

where \( D_{\mu\nu} = -\frac{p_{\mu\nu}}{k^2 - \Pi_T + i\epsilon} - \frac{Q_{\mu\nu}}{k^2 - \Pi_L + i\epsilon} \), and we assume that \( \epsilon \) is positive.

Let's now compute

\[
\int d\omega \frac{1}{((p_J + k)^2 - M^2 + i\epsilon)^2} (2p_J + k)^\mu D_{\mu\nu}(2p_J + k)^\nu = \int d\omega (\frac{f_T(p_J, k)}{k^2 - \Pi_T + i\epsilon} - \frac{f_L(p_J, k)}{k^2 - \Pi_L + i\epsilon}) \tag{45}
\]

Since we are interested only in the radiative energy loss, we assume that initial jet is off-shell, and therefore we have that \( f_T(p_J, k) \) and \( f_L(p_J, k) \) are analytic functions.

From Eqs. (40, 41) we see that \( \Pi_T \) and \( \Pi_L \) can be written as \( \sum_p \xi_T(L)(p,k) \) where \( \xi_T(L)(p,k) \) are analytic functions, and where we have assumed that we have discrete set of \( p' \)s.

Since discussion for both parts of the integral (45) is the same, we will consider only one part:

\[
\int d\omega \frac{f(p_J, k)}{\omega^2 - \bar{k}^2 - \sum_p \xi(p,k) + i\epsilon} \tag{46}
\]

Note that \( \xi(p,k) \) is infinite when \( \omega = \frac{p^2 - k^2}{p^2} \).

For simplicity, suppose that all \( \frac{p^2 - k^2}{p^2} \) are different. We can order them in such a way that \( \omega_1 < \omega_2 < ... < \omega_n \leq |k| \).

Then, for fixed \( |k| \) zeros, \( \pm (\Omega_i - i\epsilon) \), of \( (\omega^2 - \bar{k}^2 - \sum_p \xi(p,k) + i\epsilon) \) can be found graphically:
FIG. B1. shows schematically the zeros $\Omega_i$ for $\omega^2 - \vec{k}^2 - \sum_p \xi(p,k)/(pk)$ at fixed $|\vec{k}|$.

We see that we have $n' < n$ different solutions $\Omega_i \leq |\vec{k}|$, and exactly one solution $\Omega_0 > |\vec{k}|$. Then, if we close the contour in Eq. (46) in clockwise direction, we will pick up only positive poles, i.e. $(\Omega_i - i\epsilon)$.

Therefore,

$$\int d\omega \frac{f(p_I,k)}{\omega^2 - \vec{k}^2 - \sum_p \xi(p,k)/(pk) + i\epsilon} = (-2\pi i) \sum_{i=1}^{n'} \text{Res}(\Omega_i) + (-2\pi i) \text{Res}(\Omega_0).$$

(47)

In the following subsection, we will test the relative magnitude of the two contributions for the case of the dominant transverse excitations. Solutions $\Omega_i < |\vec{k}|$ correspond to particle hole excitation, and we will prove that this contribution is negligible.

B.1 Simplifying the gluon propagator in hot dense medium

The transverse response

$$\rho_T(k) \equiv \frac{1}{2\pi} \text{Disc}\left[\frac{1}{k^2 - \Pi_T(k)}\right]$$

(48)

obeys the sum rule [42]

$$\int_{-\infty}^{\infty} d\omega \omega \rho_T(\omega, |\vec{k}|) = 1$$

(49)

We can write

$$\rho_T(\omega, |\vec{k}|) = \beta(\omega, |\vec{k}|) + \delta(\omega^2 \epsilon_T - \vec{k}^2),$$

(50)

where first part in this formula corresponds to the particle hole excitation, and the second part corresponds to the delta function contribution.
It is easy to show that

$$\delta(\omega^2 \epsilon_T - \vec{k}^2) = \delta(\omega^2 - \omega_T^2) Z_T(k)$$  \hspace{1cm} (51)

where \(\omega_T\) is the transverse plasmon spectrum, and

$$Z_T(k) = \frac{2 \omega_T^2 (\omega_T^2 - \vec{k}^2)}{\omega_T^4 \mu^2 - (\omega_T^2 - \vec{k}^2)^2}. \hspace{1cm} (52)$$

The sum rule reduces to

$$Z_T(k) + \int_{-\infty}^{\infty} d\omega \beta(\omega,|\vec{k}|) = 1 \hspace{1cm} (53)$$

FIG B2. shows \(Z_T(k)\) as a function of \(|\vec{k}|\). Full curve shows exact \(Z_T(\omega_T,|\vec{k}|)\), while dot-dashed curve shows \(Z_T(\sqrt{\vec{k}^2 + m_g^2},|\vec{k}|)\), in which \(\omega_T\) is approximated by \(\sqrt{\vec{k}^2 + m_g^2}\).

In Fig. B2 we see that \(Z_T(\omega_T,|\vec{k}|)\) is approximately equal to 1 for whole region of \(|\vec{k}|\). Therefore, we can conclude that the contribution from the particle hole excitation is negligible.

On Fig.5 we saw that \(\omega_T \approx \sqrt{\vec{k}^2 + m_g^2}\). Dot-dashed curve on Fig. B2 represents \(Z_T(\sqrt{\vec{k}^2 + m_g^2},|\vec{k}|)\) as a function of \(|\vec{k}|\). Again, with the exception of the small \(|\vec{k}| < 1\) GeV region, we see that \(Z_T(\sqrt{\vec{k}^2 + m_g^2},|\vec{k}|)\) is approximately equal to 1 for whole region of \(|\vec{k}|\). Thus, we can approximate

$$\delta(\omega^2 \epsilon_T - \vec{k}^2) \approx \delta(\omega^2 - (\vec{k}^2 + m_g^2)). \hspace{1cm} (54)$$

Since longitudinal contribution is negligible, we can conclude that a gluon propagator in a hot dense medium can be approximated by

$$D_{\mu \nu} \approx -\frac{P_{\mu \nu}}{k^2 - m_g^2 + i\epsilon}. \hspace{1cm} (55)$$

References


[38] Djordjevic M, Gyulassy M, Radiative heavy quark energy loss in QCD matter, to be published