Chiral Symmetry Aspects of Positive and Negative Parity Baryons

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Chiral symmetry aspects for baryon properties are studied. After a brief discussion on general framework, we introduce two distinctive chiral group representations for baryons: the naive and mirror assignments. Using linear sigma models, nucleon properties are studied in both representations. Finally, we propose an experiment to distinguish the two assignments in the reactions of pion and eta productions.

§1. Introduction

Chiral symmetry is one of fundamental symmetries of QCD and is considered to be very important in hadron physics. The fact that the observed particles do not fall into degenerate chiral multiplets (parity doublets) is an indication that the symmetry is broken spontaneously. Consequently, the almost massless Nambu-Goldstone bosons (pions and kaons) appear and their interactions are dictated by low energy theorems. Hadron properties are then governed by chiral symmetry with its spontaneous break down. In particular many of their properties are influenced by the pattern and strength of the symmetry breaking. Recent interests in the change in hadron properties at finite baryonic densities and/or temperatures are also related to the investigation of QCD phase properties.

Particle spectrum is the most fundamental issue in order to obtain the information on realization of chiral symmetry. If chiral symmetry is manifest, the particle states are classified by irreducible representations of the chiral symmetry group. If not, however, the states are in general written as superposition of infinitely many terms of irreducible representations. Some time ago, by imposing suitable consistency conditions for scattering amplitudes of pions computed by using the low energy chiral lagrangian, Weinberg showed that observed hadrons may be classified into linear representations of the chiral symmetry group\(^1\),\(^2\). He also discussed examples of (1) \(\pi, \sigma, \rho\) and \(a_1\) and (2) \(N\) and \(\Delta\), where they form a larger representation of the chiral symmetry group\(^2\). Such an investigation is interesting, since some observed quantities such as the axial coupling constants which can take any number in the non-linear scheme can be determined. Recently investigations based on such chiral representation theories were also made in Refs.\(^3\)–\(^5\) These algebraic methods are similar to the method of sum rule in the dispersion theory\(^6\), but here the integrated states are saturated by one particle states. The method of chiral representations is also useful for the discussion of property changes of hadrons when chiral symmetry is

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In particular, the role of the sigma meson as a Higgs particle in the chiral theory can be studied in an explicit manner. The change in sigma meson properties at finite temperature and densities should be a clear indication of the importance of the chiral dynamics.\(^7\)

In this paper we discuss the role of chiral symmetry especially in the baryon sector. Besides the general remarks given in the above, we would like to clarify the meaning of the chiral representations for the positive and negative parity baryons. Since naively chiral symmetry transformations may relate states of opposite parities, it is natural to expect that some positive and negative parity states (parity doublet) form chiral representations. We will show that such identifications should be done carefully.

First we discuss what chiral representations are possible for baryons. Among (in principle) infinitely many representations, we group them into two classes; one is what we call the naive assignment and the other the mirror assignment.\(^8\),\(^9\) It is then shown that only in the mirror assignment, the positive and negative parity baryons fall into a single chiral representation. In contrast, in the naive assignment, the baryons with opposite parities are no longer related by chiral symmetry but are just independent particles. The two classes of chiral symmetry assignments for baryons may be distinguished by the sign of the axial coupling. We propose an experiment to observe the sign difference in pion and eta productions.

\[\text{\S 2. Naive and mirror assignments of baryons}\]

Chiral symmetry is defined as a flavor symmetry for positive and negative chirality states. Here we consider for simplicity isospin symmetry. Therefore, the chiral symmetry is expressed as \(SU(2) \times SU(2)\). In QCD, it is for the (approximately) massless \(u,d\) quark fields of two chiralities:

\[
q_r = \frac{1 + \gamma^5}{2} q, \quad q_l = \frac{1 - \gamma^5}{2} q. \tag{2.1}
\]

In the massless limit, the chirality states are identified with the helicity states. Since hadrons are composite, their chiral representations can be, in principle, any one as labeled by \((p,q)\), where \(p\) and \(q\) are isospin values of the \(SU(2)\) representations. In this notation, the quark field can be expressed as

\[
q_r \sim (1/2, 0), \quad q_l \sim (0, 1/2). \tag{2.2}
\]

Due to parity invariance, if \(h_r\) (a hadron with positive = right-handed chirality) \(\sim (p,q)\), then \(h_l \sim (q,p)\), and physical states contain their equal weighted superposition \(h = h_r \pm h_l\). Moreover, when spontaneous breaking occurs, the states are superposition of infinitely many terms, \(\sum_{pq} c_{pq} (p,q)\). According to the Weinberg’s argument,\(^1\) however, there is a good reasoning to believe that hadrons belong to simpler (low dimensional) representations. In the following discussions, we consider such situations.

Let us consider two nucleons, \(N_1\) and \(N_2\), where we assume that \(N_1\) and \(N_2\) carry positive and negative parities. Naively, we expect that they both belong to the
same chiral representation;
\[ N_{1r}, N_{2r} \sim (p, q) \], \[ N_{1l}, N_{2l} \sim (q, p) \].
(2-3)

However, as pointed out previously, it is also possible to consider an assignment as
\[ N_{1r}, N_{2l} \sim (p, q) \], \[ N_{1l}, N_{2r} \sim (q, p) \],
(2-4)
where for the second nucleon the role of the right and left handed components are interchanged. We call the first one of (2-3) naive assignment and the second one of (2-4) mirror assignment.

There are two significant differences in the two assignments. First, the sign of the axial coupling \( g_A \) differs for \( N_1 \) and \( N_2 \). In the naive assignment, both \( N_1 \) and \( N_2 \) carry the same axial coupling, while in the mirror assignment, \( g_A \) of \( N_2 \) has opposite sign of that of \( N_1 \). Second point concerns the mass parameters. In general two types are possible;\(^1\) one behaves as the 0-th component of a chiral 4-vector and the other behaves as a chiral scalar. The mass parameter of chiral 4-vector may be related to the scale of chiral symmetry breaking \( \langle \sigma \rangle \equiv \sigma_0 \), while the chiral scalar mass parameter could be independent of it. In the naive assignment, only mass terms of chiral 4-vector are allowed, while in the mirror assignment mass terms of both the chiral 4-vector and chiral scalar are allowed.

§3. Linear sigma models

Let us consider linear sigma models for the naive and mirror models. For simplicity we consider the nucleon of the fundamental representations; \( N \sim (1/2, 0) + (0, 1/2) \). Furthermore, \( \sigma \) and \( \pi \) meson fields are introduced as in the representation \( (1/2, 1/2) \). Their transformation rules are then given by
\[ N_r \rightarrow g_R N_r, \quad N_l \rightarrow g_L N_l \]
(3.1)
and
\[ \sigma + i\vec{\tau} \cdot \vec{\pi} \rightarrow g_L (\sigma + i\vec{\tau} \cdot \vec{\pi}) g_R^\dagger. \]
(3.2)
where \((g_R, g_L)\) are elements of \( SU(2) \times SU(2) \).

3.1. Naive model

In the naive assignment, the chiral invariant lagrangian up to order (mass)\(^4\) is given by:\(^8\),\(^9\)
\[ L_{\text{naive}} = \bar{N}_1 i\partial N_1 - g_1 \bar{N}_1 (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) N_1 + \bar{N}_2 i\partial N_2 - g_2 \bar{N}_2 (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) N_2 \\ - g_{12} \{ \bar{N}_1 (\gamma_5 \sigma + i\vec{\tau} \cdot \vec{\pi}) N_2 - \bar{N}_2 (\gamma_5 \sigma + i\vec{\tau} \cdot \vec{\pi}) N_1 \} + L_{\text{mes}}, \]
(3.3)
where the coupling constants \( g_1 \), \( g_2 \) and \( g_{12} \) are free parameters. The terms of \( g_1 \) and \( g_2 \) are ordinary chiral invariant coupling terms of the linear sigma model. The term of \( g_{12} \) is the mixing of \( N_1 \) and \( N_2 \). Since the two nucleons have opposite parity,
\( \gamma_5 \) appears in the coupling with \( \sigma \), while it does not in the coupling with \( \pi \). The meson lagrangian \( L_{\text{mes}} \) in (3.3) is not important in the following discussion.

Chiral symmetry breaks down spontaneously when the sigma meson acquires a finite vacuum expectation value, \( \sigma_0 \equiv \langle 0 | \sigma | 0 \rangle \). This generates masses of the nucleons as a chiral 4-vector. From (3.3), the mass term can be expressed by a \( 2 \times 2 \) matrix in the two nucleon space of \( N_1 \) and \( N_2 \). The mass matrix can be diagonalized by the rotated states,

\[
\begin{pmatrix}
N_+ \\
N_-
\end{pmatrix} = \begin{pmatrix}
\cos 2\theta & \gamma_5 \sin 2\theta \\
-\gamma_5 \sin 2\theta & -\cos 2\theta
\end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix},
\]

(3.4)

where the mixing angle and mass eigenvalues are given by

\[
\tan 2\theta = \frac{2g_1}{g_1 + g_2}, \quad m_\pm = \frac{\sigma_0}{2} \left( \sqrt{(g_1 + g_2)^2 + 4\mu^2} \pm (g_1 - g_2) \right). \quad (3.5)
\]

In the naive model, since the interaction and mass matrices take the same form in the space of \( 2 \times 2 \), the physical states, \( N_+ \) and \( N_- \), decouple completely; the lagrangian becomes as usual the \( N_+ \) and \( N_- \) parts. Because of this, the coupling between the positive and negative parity baryons disappears, which was considered as a part of the reasons for the small coupling \( g_{N(1535)} \to \pi N \sim 1 \). Thus, chiral symmetry does not relate \( N_+ \) and \( N_- \) in the naive model. The role of chiral symmetry in the naive model is nothing special. When chiral symmetry is restored and \( \sigma_0 \to 0 \), both \( N_+ \) and \( N_- \) become massless and degenerate. However, the degeneracy is trivial as they are independent.

3.2. Mirror (Chiral doublet) model

Let us turn to the mirror assignment. The lagrangian is given as\(^{8,9}\)

\[
L_{\text{mirror}} = \bar{N}_1 i\sigma_0 \gamma_5 N_1 - g_1 \bar{N}_1 (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) N_1 + \bar{N}_2 i\sigma_0 \gamma_5 N_2 - g_2 \bar{N}_2 (\sigma - i\gamma_5 \vec{\tau} \cdot \vec{\pi}) N_2 - m_0 (\bar{N}_1 \gamma_5 N_2 - \bar{N}_2 \gamma_5 N_1) + L_{\text{mes}}. \quad (3.6)
\]

Here we can introduce a mass term through the coupling between \( N_1 \) and \( N_2 \). We can verify that due to the transformation rule associated to (2.4), this term is invariant under chiral symmetry transformations. Therefore, the mass parameter \( m_0 \) behave as a chiral scalar. This lagrangian (3.6) was first formulated by DeTar and Kunihiro.\(^{12}\)

The mass matrix of the lagrangian (3.6) can be diagonalized as in the naive model by a linear transformation. The mixing angle and mass eigenvalues are given by

\[
\tan 2\theta = \frac{2m_0}{\sigma_0 (g_1 + g_2)}, \quad m_\pm = \frac{\sigma_0}{2} \left( \sqrt{(g_1 + g_2)^2 + 4\mu^2} \pm (g_1 - g_2) \right), \quad (3.7)
\]

where \( \mu = m_0 / \sigma_0 \). Physical masses are written in terms of the two parameters; the chiral 4-vector \( \sigma_0 \) and the chiral scalar \( m_0 \). The nucleon masses can take a finite value \( m_0 \) when chiral symmetry is restored (\( \sigma_0 = 0 \)).
The axial couplings can be computed from the commutation relations between the axial charge operators $Q_5^a$ and the nucleon fields. They take a matrix form as

$$g_A = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix}. \quad (3.8)$$

From this we see that the signs of the diagonal components $g_A^{++}$ and $g_A^{--}$ have opposite signs as expected. The absolute value is, however, smaller than one in contradiction with experiment, $g_A \sim 1.25$. Since the physical states $N_{\pm}$ are the superposition of $N_1$ and $N_2$, whose axial couplings are $\pm 1$, it follows that $|g_A^{++}|, |g_A^{--}| < 1$. If we consider a mixing with higher representation such as $(1, 1/2)$, the $g_A$ value can be larger.\(^1\)

It is interesting to see how the mirror model becomes consistent with experimental data. For this purpose, let us consider $N(939)$ and $N(1535)$ for the two nucleons $N_+$ and $N_-$. In order to fix the four parameters, we use the inputs $m_+ = 939$, $m_- = 1535$ MeV, $\sigma_0 = f_\pi = 93$ MeV and $g_{\pi NN_{\pm}} \sim 0.7$. The last relation is extracted from the partial decay width of $N(1535)$\(^{13}\) (although large uncertainties for the width have been reported\(^{14,15}\)),

$$\Gamma_{N^* \rightarrow \pi N} \sim 75 \text{ MeV}, \quad (3.9)$$

and the formula

$$\Gamma_{N^* \rightarrow \pi N} = \frac{3g (E_N + M)}{4\pi M^*} g_{\pi NN^*}^2, \quad (3.10)$$

where $q$ is the relative momentum in the final state. We find

$$\sigma_0 = 93 \text{ MeV}, \quad m_0 = 270 \text{ MeV},$$

$$g_1 = 9.8, \quad g_2 = 16. \quad (3.11)$$

From these parameters, we obtain the mixing angle

$$\theta = 6.3^\circ, \quad (3.12)$$

giving the diagonal value of the axial charges

$$g_A^{++} = -g_A^{--} = 0.98. \quad (3.13)$$

Hence the mixing of the two states $N_{1,2}$ are not very large; in this scheme the nucleon $N(939)$ is dominated by the ordinary chiral component $N_1$, while $N(1535)$ by the mirror component $N_2$.

3.3. Properties under chiral symmetry restoration

The change in chiral symmetry is expressed by the order parameter $\sigma_0$, which is identified with the pion decay constant $f_\pi$. It decreases (increases) as temperature or density increases (decreases). As $f_\pi$ varies, physical quantities receive the following changes:
mixing angles and masses (Fig. 1): When chiral symmetry is restored \((\sigma_0 = 0)\), the only source of mixing is the off-diagonal mass term in \((3.6)\). Therefore, the two degenerate states \(N_1\) and \(N_2\) mix with equal weight \((\theta = 45^\circ)\), having equal masses \(m_\pm = m_0\). As the interaction is turned on and chiral symmetry starts to break spontaneously, the mixing angle decreases monotonically and reaches \(6.3^\circ\) in the real world \((\sigma_0 = 93\ MeV)\). Also, as \(\sigma_0\) is increased, the masses increase, with the degeneracy being resolved. Eventually, masses take their experimental values \(m_+ = 939\ MeV\) and \(m_- = 1535\ MeV\) when \(\sigma_0 = 93\ MeV\).

\(g_A\) (Fig. 2)
It is interesting to see the behavior of the diagonal \(g_A^{++} = -g_A^{-}\) and off-diagonal \(g_A^{+-}\) as functions of \(\sigma_0\). In particular, \(g_A^{+-}\) increases as chiral symmetry begins to be restored. Since the axial charges are related to pion couplings, the increase of the off-diagonal couplings \(g_A^{+-} \sim g_{\pi N_+ N_-}\) has been considered as a cause of the increase in the width of \(N(1535)\) in nuclear medium.\(^{16–18}\) A more detailed calculation is reported in Ref.\(^{16}\)

Such changes in masses and \(g_A\) might be observed in nuclear reactions with threshold or bound \(\eta\) produced in a nucleus. For instance, the change in the mass of \(N(1535)\) leads to a characteristic change in the potential shape of \(\eta\) in the nucleus. Expected \((d, ^3\text{He})\) cross sections are recently calculated by Jido, Nagahiro and Hirenzaki.\(^{19}\)

\[\begin{array}{c}
\text{Naive} \\
\text{Mirror}
\end{array}\]

\[\begin{array}{c}
\text{Masses of the positive and negative parity nucleons}
\end{array}\]

(a) Mixing angle of the mirror model

(b) Masses of the positive and negative parity nucleons, as functions of the chiral condensate \(\sigma_0\).

§4. Magnetic moments in the mirror model

In this section we discuss magnetic moments of the nucleons. The main interest stems from the fact that the anomalous term of \(\sigma_{\mu
u}\) mixes chirality just as the (Dirac) mass term does. In contrast, the normal \((\gamma_\mu)\) term conserves chirality. Explicitly, the \(\gamma NN\) coupling can be written as

\[
\mathcal{L}_{\gamma NN} = -\bar{N} \left( \gamma_\mu Q + i r \frac{\sigma_{\mu
u} q^\nu}{2M} \right) N A^\mu
\]
Fig. 2. Diagonal and off-diagonal axial charges in the naive and mirror models, as functions of the chiral condensate $\sigma_0$.

\[
= - \left[ (\bar{N}_l\gamma_\mu QN_l + \bar{N}_r\gamma_\mu QN_r) + i \left( \bar{N}_l \frac{\sigma_{\mu\nu}\eta_\nu}{2M} \kappa N_r + \bar{N}_r \frac{\sigma_{\mu\nu}\eta_\nu}{2M} \kappa N_l \right) \right] A^\mu. \tag{4.1}
\]

Here $Q$ is the charge of the nucleons and

\[
\kappa = \kappa_S + \kappa_V T_3, \tag{4.2}
\]

where $\kappa_S$ and $\kappa_V$ are isoscalar and isovector anomalous magnetic moments.

Now let us assume the linear representation of chiral symmetry for the nucleon again. In the spirit of chiral symmetry, the electromagnetic coupling is regarded as a part of the chiral invariant coupling with the external chiral vector fields. In order for the anomalous coupling to be chirally symmetric, it should contain the chiral field $\sigma + i \vec{r} \cdot \vec{\pi}\gamma_5$. Such a term becomes finite when chiral symmetry is broken spontaneously and $\langle \sigma \rangle \neq 0$. Hence in this case, the anomalous magnetic moment should be zero in the chirally symmetric limit.

Another possibility is that we take the mirror model and construct a chiral invariant term for the anomalous magnetic moment:

\[
\mathcal{L}_{\text{anomalous}} = - \frac{i}{2M} \left( \bar{N}_1 \sigma_{\mu\nu}\gamma_5 \kappa N_2 + \bar{N}_2 \sigma_{\mu\nu}\gamma_5 \kappa N_1 \right) q^\nu A^\mu. \tag{4.3}
\]

This is the lagrangian to the lowest order ($n=0$) in powers of $\langle \sigma \rangle^n$ and is a dominant term as chiral symmetry is getting restored, $\langle \sigma \rangle \to 0$. In the following discussion, we consider only this leading order term of $\mathcal{O}(\langle \sigma \rangle^0)$, in order to reduce the number of free parameters. Hence we consider the following photon coupling term:

\[
\mathcal{L}_{\gamma NN} = - \left( \bar{N}_1 \gamma_\mu QN_1 + \bar{N}_2 \gamma_\mu QN_2 + (\bar{N}_1 \Gamma_\mu N_2 + \bar{N}_2 \Gamma_\mu N_1)q^\nu \right) A^\mu. \tag{4.4}
\]

where we have introduced the notation $\Gamma_\mu = (i\kappa/2M)\sigma_{\mu\nu}q^\nu$.

In terms of the physical field $N_\pm$, the coupling term takes on the form:

\[
\mathcal{L}_{\gamma NN} = - \left( \bar{N}_+ \gamma_\mu QN_+ + \bar{N}_- \gamma_\mu QN_- \right) A^\mu \\
- \sin 2\theta (\bar{N}_+ \Gamma_\mu N_+ + \bar{N}_- \Gamma_\mu N_-) A^\mu \\
+ \cos 2\theta (\bar{N}_+ \Gamma_\mu \gamma_5 N_- + \bar{N}_- \Gamma_\mu \gamma_5 N_+) A^\mu. \tag{4.5}
\]
From this expression, we find that the anomalous magnetic moments of \( N^+ \) and \( N^- \) take the same value in units of nuclear magneton. Furthermore, there remain transition moments between the two nucleons, the \( \gamma N^+ N^- \) vertex.\

Let us now briefly discuss the transition moments. Note that the transition term has the structure of \( E1 \) because of the parity. In the previous section, the mixing angle was estimated to be \( \theta \sim 6.3^\circ \). We can then use the proton and neutron magnetic moments to fix the \( \kappa \)'s (including the mixing angle): \( \kappa_S \sin 2\theta = -0.06 \) and \( \kappa_V \sin 2\theta = 1.85 \). Using these numbers, we find for the transition moments: \( \mu_{pp^*} = 8.42 \) and \( \mu_{nn^*} = -8.99 \). The isovector dominance in these quantities is consistent with experimental data, but their magnitudes are too large. Empirically, \( |\mu_{pp^*}| \sim |\mu_{nn^*}| \sim 1 \) in units of the nuclear magneton, as extracted from the helicity amplitudes: \( A_{1/2}^p \sim 9.5 \times 10^{-3} \text{GeV}^{-1/2}, \quad A_{1/2}^n \sim -80 \times 10^{-3} \text{GeV}^{-1/2} \). (4.6)

In summary of this section, we have shown that the introduction of the anomalous magnetic in the linear representation is different for the naive and mirror models. In the naive model, the insertion of the chiral field is necessary in order to make the anomalous term chirally invariant. Consequently, the spontaneous breaking gives finite values of anomalous magnetic moment. In contrast, in the mirror model, the anomalous term itself is chiral invariant. To the lowest order of \( \langle \sigma \rangle \) in the mirror model, we obtain \( \mu_N \sim \mu_{N^*} \) and the transition moment as a function of the mixing angle. The relatively small transition moments as observed in experiment may suggest a larger mixing angle, as opposed to the result obtained from the pion couplings previously in eq. (3.12). Both facts could be an indication that higher order terms in \( \langle \sigma \rangle \) could be important. In any event, magnetic moments of the nucleon as well as of its excited state provide useful information of chiral symmetry of baryons.

§5. π and η productions at threshold region

As discussed in the preceding sections, one of the differences between the naive and mirror assignments is the relative sign of the axial coupling constants of the positive and negative parity nucleons. Here we consider meson production reactions to observe such a sign difference, by making identification once again \( N^+ \sim N(939) \equiv N \) and \( N^- \sim N(1535) \equiv N^* \). From experimental point of view, \( N(1535) \) has a nice feature that it has a strong coupling to an \( \eta \), which can be used as a filter for the resonance production. In the reactions, we can observe the pion couplings which are related to the axial couplings through the Goldberger-Treiman relation

\[ g_{\pi N \pm N \pm} f_\pi = g_A^\pm M_\pm. \]

Let us consider the following two reactions:

\[ \pi^- + p \rightarrow \pi^- + \eta + p, \quad \gamma + p \rightarrow \pi^0 + \eta + p. \] (5.1)

There are many diagrams that contribute to these reactions. However, suppose that the first two diagrams of Fig. 3 are dominant. Then, depending on the relative sign of the \( \pi N N \) and \( \pi N^* N^* \) couplings, the two graphs are added either constructively or destructively. This is the essential idea to discuss the relative sign of the couplings.
Fig. 3. Twelve diagrams for $\pi$ and $\eta$ productions. The incident wavy line is either a pion of photon. For $\pi^- + p \to \pi^- + \eta + p$, the first six diagrams contribute, while for $\gamma + p \to \pi^0 + \eta + p$, all twelve diagrams do.

In actual computation, we take phenomenological lagrangians:

\begin{align}
L_{\pi NN} &= g_{\pi NN} \bar{N} i \gamma_5 \vec{r} \cdot \vec{N}, \quad L_{\eta NN^*} = g_{\eta NN^*} (\bar{N} \eta N^* + \bar{N}^* \eta N), \\
L_{\pi NN^*} &= g_{\pi NN^*} (\bar{N} \tau \cdot \pi N^* + \bar{N}^* \tau \cdot \pi N), \\
L_{\pi N^*N^*} &= g_{\pi N^*N^*} (\bar{N}^* i \gamma_5 \tau \cdot \pi N^*). \quad (5.2)
\end{align}

We use these interactions both for the naive and mirror cases with empirical coupling constants for $g_{\pi NN} \sim 13$, $g_{\pi NN^*} \sim 0.7$ and $g_{\eta NN^*} \sim 2$. The coupling constants $g_{\pi NN^*} \sim 0.7$ and $g_{\eta NN^*} \sim 2$, are determined from the partial decay widths, $\Gamma_{N^*(1535)\to\pi N} \approx \Gamma_{N^*(1535)\to\eta N} \sim 70$ MeV. The unknown parameter is the $g_{\pi N^*N^*}$ coupling. One can estimate it by using the theoretical value of the axial coupling $g_A^*$ and the Goldberger-Treiman relation for $N^*$. When $g_A^* = \pm 1$ for the naive and mirror assignments, we find $g_{\pi N^*N^*} = g_A^* m_N^*/f_{\pi} \sim \pm 17$. Here, just for simplicity, we use the same absolute value as $g_{\pi NN}$.

The photon coupling takes on the form

\[ L_{\gamma NN} = -e \bar{N} \gamma_{\mu} 1 + \tau_3 2 N A^\mu + e 4 M \bar{N} (\kappa_S + \kappa_V \tau_3) \sigma^{\mu\nu} N F_{\mu\nu} + h.c. , \quad (5.3) \]

where $\kappa_S$ and $\kappa_V$ are the physical isoscalar and isovector anomalous magnetic mo-
ments *),
\[ \kappa_S = -0.12 \quad \kappa_V = 3.7. \tag{5.4} \]

For \( \gamma N^*N^* \), we assume the same form as (5.3) but here the nucleon mass is replaced by the resonance mass. In this calculation, the unknown magnetic moment of \( N^* (\equiv \kappa^{**}) \) is taken to be the same as that of the nucleon. The uncertainty resulting from this assumption is, however, not very important, since the diagrams which contain the \( \gamma N^*N^* \) coupling (Figs. 3 (5) and (8)) play only a minor role. For the \( \gamma NN^* \) vertex, we employ the tensor form that is compatible with gauge invariance:

\[ \mathcal{L}_{\gamma NN^*} = i e \kappa^V_{**} 2 (m_{N^*} + m_N) \bar{N}^* \gamma_5 \sigma^{\mu \nu} N F_{\mu \nu} + h.c. \tag{5.5} \]

Here we have used isovector dominance, and the coupling constant is given by

\[ \kappa^V_{**} = 0.9, \tag{5.6} \]

which is determined from analyses of eta photoproduction.\(^{20}\)

![Fig. 4. Background contributions to the present reaction](image)

Several remarks are in order:

1. We have assumed resonance \((N^*)\) pole dominance. This is considered to be good particularly for the \( \eta \) production at the threshold region, since \( \eta \) is dominantly produced by \( N^* \).
2. For the pion induced reaction, there are several background contributions as shown in Fig. 4 (a - d). The diagrams (a - c) are suppressed due to G-parity conservation. The diagram (d) is shown to be negligibly small by explicit computation.
3. Among various diagrams of Fig. 3, the three diagrams (1 - 3) are dominant in for the pion induced process, while two (1 - 2) are important for the photon induced reaction.

We show various cross sections for the pion and photon induced processes in Fig. 5. We observe that

1. Total cross sections are of order of micro barn, which are well accessible by experiments.
2. In the photon induced process, the two dominant diagrams (1) and (2) interfere. Hence for the naive model, the cross sections are enhanced, while they are suppressed for the mirror model. In the pion induced case, due to the momentum dependence of the initial vertex the third term (3) becomes dominant as well as (1) and (2).

\(^{*}\) The anomalous magnetic moments here are physical ones and are different from those in section 4.
3. In the pion induced reaction, the angular distribution of the final state pion differs crucially. They reflect the difference in the sign of the $\pi NN$ and $\pi N^* N^*$ couplings.

§6. Summary

In this report, we have investigated chiral symmetry aspects for baryon properties. Our investigation here is based on several simple linear representations of the chiral symmetry group. By assigning suitable representations, we can compute matrix elements of such as the axial coupling $g_A$, magnetic moments and masses, and put some constraints on them. As one of possibilities which was not considered before, we have studied the two different schemes of the naive and mirror chiral representations for the baryons, using linear sigma models. We have also proposed an experiment to distinguish the two chiral assignments in pion and eta production reactions. By detailed study of interfering processes, it would be possible to extract information on the sign of the axial coupling. The possibility of the negative axial coupling and the chiral scalar mass $m_0$ of baryons are interesting to investigate further.

The description based on the linear chiral representations may be extended to
other higher resonances, once we assume suitable chiral representations so as to include relevant resonance states. In Ref. 3 a larger chiral multiplet $(1,1/2)$ was investigated. Since this representation contains isospin $I = 1/2$ and $3/2$ component, considering their parity partners, the four particles, $\Delta(1232), \Delta(1700), N(1720)$ and $N(1520)$ take participates in the model. Using the similar argument shown here, they have obtained interesting constraints on the masses and decay rates of these resonances, which are quite consistent with the observed properties of spin $J = 1/2, 3/2, 5/2$ sectors. Another scheme where the nucleon $N(939)$, the delta $\Delta(1232)$ and the Roper $N(1440)$ are included using a larger chiral multiplet, $(1/2,0)+(1,1/2)$, was also considered. It would be interesting to investigate further baryon properties from the theory of linear chiral representations.

References

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