The Tully-Fisher Relation of Barred Galaxies

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ABSTRACT

1Visiting Astronomer, Kitt Peak National Observatory, National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc. (AURA) under cooperative agreement with the National Science Foundation.
We present new data exploring the scaling relations, such as the Tully-Fisher relation (TFR), of bright barred and unbarred galaxies. A primary motivation for this study is to establish whether barredness correlates with, and is a consequence of, virial properties of galaxies. Various lines of evidence suggest that dark matter is dominant in disks of bright unbarred galaxies at 2.2 disk scale lengths, the point of peak rotation for a pure exponential disk. We test the hypothesis that the TF plane of barred high surface brightness galaxies is offset from the mean TFR of unbarred galaxies, as might be expected if barred galaxies are “maximal” in their inner parts. We use existing and new TF data to search for basic structural differences between barred and unbarred galaxies. Our new data consist of 2-dimensional Hα velocity fields derived from SparsePak integral field spectroscopy (IFS) and V,I-band CCD images collected at the WIYN Observatory for 14 strongly barred galaxies. Differences may exist between kinematic and photometric inclination angles of barred versus unbarred galaxies. These findings lead us to restrict our analysis to barred galaxies with $i > 50^\circ$. We use WIYN/SparsePak (2-D) velocity fields to show that long-slit (1-D) spectra yield reliable circular speed measurements at or beyond 2.2 disk scale lengths, far from any influence of the bar. This enables us to consider line width measurements from extensive Tully-Fisher surveys which include barred and nonbarred disks and derive detailed scaling relation comparisons.

We find that for a given luminosity, barred and unbarred galaxies have comparable structural and dynamical parameters, such as peak velocities, scale lengths, or colors. In particular, the location of a galaxy in the TF plane is independent of barredness. In a global dynamical sense, barred and unbarred galaxies behave similarly and are likely to have, on average, comparable fractions of luminous and dark matter at a given radius.

*Subject headings:* galaxies: bars — galaxies: formation — galaxies: kinematics — galaxies: photometry — galaxies: spirals — galaxies: structure

1. Introduction

Based on the flatness of rotation curves in spiral galaxies and the density profiles inferred from X-ray temperatures and stellar velocity dispersion profiles in ellipticals, it is widely believed that galaxies are embedded in non-dissipative massive dark halos. More than 90% of the total mass of a galaxy would be in the form of dark matter. Less appreciated is the fact that we still have a rather muddled picture of the mass distribution of luminous and
dark matter in the luminous part of a galaxy. This is unfortunate since the final distribution of baryons in a galaxy is a tell-tale sign of its formation and evolution. Numerical and analytical models of disk formation in a dissipationless dark matter halo predict, for realistic total fractions of baryonic to dark matter, that spiral disks should live in dark halos that dominate the mass fraction at nearly all radii (e.g. Mo, Mao, & White 1998), beyond about a disk scale length. This ratio may quite possibly be different for barred and unbarred galaxies of a given total mass or luminosity (Courteau & Rix 1999; hereafter CR99).

Recent debates about the Cold Dark Matter paradigm (e.g. Weinberg & Katz 2002; Sellwood 2003; Courteau et al. 2003a) and galaxy structural properties inferred from new infrared surveys (e.g. Eskridge et al 2002; MacArthur et al. 2003) have brought barred galaxies to the fore. Bar perturbations in galaxies, far from just being dynamical curiosities, actually play a fundamental role in shaping galaxies into the structures we see today (see Buta, Crocker, & Elmegreen 1996 for reviews). For instance, the early dynamical evolution of a massive rapidly rotating gaseous bar could provide enough energy and angular momentum to significantly modify the inner CDM halo (Silk 2002). Dynamical and structural studies of barred galaxies are however few, due in part to the complexity in interpreting their velocity fields (e.g. Weiner et al. 2001) and their surface brightness profiles (e.g. MacArthur et al. 2003). Many large-scale flow studies of spiral galaxies have also excluded disturbed or barred galaxies to minimize scatter, as previously believed, in the distance-measuring technique. The latter studies have enabled extensive scaling relation studies of unbarred galaxies, but little attention has been paid to their barred cousins. This is again deplorable as a comparative study of the scaling relations for barred and unbarred galaxies would potentially unravel clues about the structure and origin of bars and the role of dynamical processes in establishing the Hubble sequence of disk galaxies.

The body of numerical simulations of barred galaxies is comparatively richer and has recently reached new heights with the availability of superior N-body realizations with more than $10^6$ particles (post 2010 readers may enjoy a moment of laughter). Until just recently, it was believed that bar instabilities in a disk might be suppressed by a massive halo. Thus only low concentration halos, or equivalently systems of very high surface brightness (HSB) or low angular momentum per unit luminosity, would be prone to generating a non-axisymmetric (bar/oval) structure in their center (Ostriker & Peebles 1973; accounts of the misconceptions surrounding this argument are presented in Bosma 1996 and Sellwood & Evans 2001).

The suggestion that barred galaxies would have an especially high ratio of baryons-to-dark matter within the optical disk (“maximal disk”) might imply that these systems define their own sequence in the luminosity-line width diagram, if one assumes that unbarred galaxies are, on average, sub-maximal (CR99). Thus, for a given absolute magnitude, a galaxy
with higher baryon fraction, or disk mass-to-light ($M/L$) ratio, would have a shorter disk scale length and rotate faster. Verification of this important, though tentative, suggestion should be easily obtained from a large sample of uniformly selected barred galaxies that are part of a well-calibrated, self-consistent luminosity-line width survey. The current study was largely motivated by this question.

In discussing the mass distribution in spiral galaxies, we shall use the definition that a disk is “maximal” if it contributes more than 75% of the total rotational support of the galaxy at $R_{\text{disk}} \equiv 2.2 h_{\text{disk}}$, the radius of maximum disk circular speed (Sackett 1997). Thus, for a maximal disk, $V_{\text{disk}}/V_{\text{total}} \gtrsim 0.75$, where $V_{\text{total}}$ is the total amplitude of the rotation curve at $R_{\text{disk}}$ and $V_{\text{disk}} = V(R_{\text{disk}})$. Note that for $V_{\text{disk}}/V_{\text{total}} = 0.7$ the disk and halo contribute equally to the potential at $R_{\text{disk}}$. Large bulges for late-type galaxies make little difference for the computation of this quantity at $R_{\text{disk}}$ (CR99).

The pattern speeds of bars have been considered as a potential indicator of the relative fraction of dark matter in galaxy disks. N-body simulations of bar formation in stellar disks suggest that dynamical friction from a dense dark matter halo dramatically slows the rotation rate of bars in a few orbital periods (Debattista & Sellwood 1998, 2000; hereafter DS00). Because bars are observed to rotate quickly, DS00 proposed that dark matter halos in HSB galaxies must have a low central density; thus, their disks ought to be maximal. These simulations were revisited by Valenzuela & Klypin (2003; hereafter VK03) with similar N-body simulations (no gas) but with an order of magnitude improvement in the force resolution. VK03 found that dynamical friction from transfer of angular momentum of the bar to the halo does play a role but, contrary to DS00, that effect appears to be small. In addition, VK03 find that bars can form even in the presence of strong halos, and that stellar disks make a negligible contribution to the inner rotation curve (at $R_{\text{disk}}$). The bars modeled in DS00 also span nearly the entire disk whereas the observed bar-to-disk scale length ratio seldom exceeds 1.5, as also pointed out by VK03. These authors find that mass and force resolution are critical for modeling the dynamics of bars, and the contentious results from DS00 would stem primarily from numerical resolution effects. However, the higher force resolution of VK03 induces numerical viscosity which may bring their results into question (J. Sellwood 2003; priv. comm.)! Free from the vagaries of numerical simulations, Athanassoula (2003) uses analytical calculations to warn against the use of bar slowdown rate to set limits on the baryonic to dark matter fraction within the optical radius$^4$, in agreement with Sellwood (2003). This point is however moot since one cannot observe bar slowdown from a single snapshot. Athanassoula (2003) further suggests that the ratio of corotation to bar

$^4$Athanassoula (2003) finds that the bar slowdown rate depends not only on the relative halo mass at a given radius, but also on the velocity dispersion of both the bulge and disk components.
length may not be an adequate estimator of halo fraction, but the model ratios (see her Figs. 12 & 14) upon which these conclusions are drawn are inconsistent with observations.

A complete picture of bar dynamics awaits a self-consistent treatment of both the stars and gas embedded in a cosmologically motivated halo. These simulations should include dynamical friction and ultimately reproduce the fraction of strong bars detected in the infrared and predict the rate of bar slowdown and dissolution as a function of bulge/total brightnesses, time, and environment.

The model-independent quest of the relative matter distribution in barred and unbarred galaxies is by no means straightforward either, but is most significant as it provides a necessary constraint for the shape and amplitude of the dark matter density profile in the luminous part of a galaxy. Whether disks are maximal or not at $R_{\text{disk}}$, the inner 1-2 kpc may be dominated by baryons in most galactic systems, including early and late-type HSB barred and unbarred spirals (e.g. Broeils & Courteau 1997; Corsini et al. 1999), low surface brightness (LSB) galaxies (Swaters 1999; Swaters, Madore, & Trehwellia 2000; Fuchs 2002) and ellipticals (e.g. Brighenti & Mathews 1997; see also Ciotti 2000). Maximally massive disks in LSB galaxies may however require unrealistically high disk $M/L$ ratios (Swaters et al. 2000; Fuchs 2002), based on stellar population synthesis models.

Also troublesome is our lack of knowledge about the distribution of matter in our own Milky Way. Whether it has a maximal disk (Gerhard 2002) or not (Dehnen & Binney 1998; Klypin, Zhao, & Somerville 2002) is still a matter of debate. Crucial elements for local mass density estimates include the precise contribution of the massive central bar (e.g. Zhao, Rich, & Spergel 1996) or elongated bulge (Kuijken 1995), an accurate measure of the disk scale length, and constraints from microlensing towards the bulge.

The determination of the relative fraction of visible and dark matter in external barred and unbarred galaxies relies on our ability to determine stellar $M/L$ ratios accurately. The modeling of disk dynamical mass in barred galaxies relies heavily on the interpretation of the non-axisymmetric motions of ionized gas around the bar within the context of a hydrodynamical model. This model does have a local potential, and hence the bar and disk $M/L$ are parameters of the model. It is certainly a more complicated approach than using collisionless particles as dynamical tracers, as with stellar velocity dispersions, but the latter has its own complications as well (e.g. Swaters et al. 2003). Significant improvements in mass modeling techniques for individual galaxies are expected with the development of stellar population synthesis models (Bell & de Jong 2001) and dynamical constraints (Weiner, Sellwood, & Williams 2001) to yield realistic $M/L$ ratios, and further constraints from cosmological simulations of dark halos to curtail disk-halo degeneracies (Dutton, Courteau, & de Jong 2003).
Various lines of circumstantial evidence for external systems favor dark matter halos that dominate the mass budget within $R_{\text{disk}}$. Arguments based on the stellar kinematics of galactic disks (Bottema 1997), gas kinematics (Kranz, Slyz, & Rix 2003), the stability of disks (Fuchs 2001) and the lack of correlated scatter in the Tully-Fisher relation (hereafter TFR; Tully & Fisher 1977) of unbarred LSB and HSB galaxies (CR99) suggest that, on average, disks with $V_{\text{max}} < 200$ km s$^{-1}$ are sub-maximal. The two very different analyses by Bottema and CR99 both yield $V_{\text{disk}}/V_{\text{total}} = 0.6 \pm 0.1$, or $M_{\text{dark}}/M_{\text{total}} = 0.6 \pm 0.1$ for HSB galaxies at $R_{\text{disk}}$. The geometry of gravitational lens systems, coupled with rotation curve measurements, can also be used to decompose the mass distribution of a lensing galaxy. This promising technique, pioneered by Maller et al. (2000), has been applied to the galaxy-lens system 2237+0305 by Trott & Webster (2002) who find $V_{\text{disk}}/V_{\text{total}} = 0.57 \pm 0.03$, in excellent agreement with the studies above and predictions from analytical models of galaxy formation (e.g. Dalcanton et al. 1997; Mo, Mao, & White 1998).

While a consistent picture of galaxy structure is emerging in which a dark halo dominates with $M_{\text{dark}}/M_{\text{total}} \geq 0.6$ well into the optical disk, a number of pro-maximal disk arguments are still found in the literature, citing evidence from the shapes and extent of rotation curves and mass modeling (see, e.g., Bosma 2002). The match between pure disk mass models and H$\alpha$ rotation curves (e.g. Broeils & Courteau 1997; Seljak 2002; Jimenez et al. 2003) is usually satisfactory for spiral galaxies of different surface brightnesses and morphologies and has often been invoked as evidence for a maximal concentration of baryons relative to the dark matter inside the optical disk (Buchhorn 1992; Palunas & Williams 2000). However, mass modeling with H$\alpha$ rotation curves alone is not a uniquely determined problem. The equivalence of, or degeneracy between, the two descriptions – pure disk versus sub-maximal disk + dark halo – was demonstrated in Broeils & Courteau (1997) and CR99 for a sample of 300 disk galaxies; residuals for the maximal or sub-maximal fits are indistinguishable. Without an accurate estimate of $M/L_{\text{disk}}$, or external constraints on $V_{\text{disk}}/V_{\text{virial}}$ at $R_{\text{disk}}$, mass modeling cannot discern a maximal and sub-maximal disk models.

Our study of the dynamical structure of barred and unbarred galaxies will offer new insights in the debate of the maximal disk hypothesis in barred and unbarred galaxies. However, we plan to revisit this controversial issue in a future presentation (Courteau et al. 2003a). Here we pursue our comparison of barred and unbarred galaxies in the context of global scaling relations.
1.1. Available galaxy samples

The study of scaling relations of barred galaxies, and tracing their location in the TFR, requires that we utilize “fundamental plane” surveys of an ensemble of galaxies. The “Shellflow” and “SCII” all-sky Tully-Fisher surveys of Courteau et al. (2000) and Dale et al. (1999) are useful in that respect. These surveys were designed to map the convergence of the velocity field on $\sim 60 h^{-1} \text{Mpc}$ scales while minimizing calibration errors between different telescopes in different hemispheres; state-of-the-art TF calibrations are thus available in both cases. Both surveys include line width and luminosity measurements for a small fraction of barred galaxies that can be used to study structural trends, provided the presence of bar does not bias these measurements. More details about the surveys will be given in §3.

In order to calibrate existing long-slit spectra of barred galaxies and initiate a comprehensive study of barred galaxy velocity fields, we have collected new deep V and I-band images and integral field H$\alpha$ velocity fields of 14 strongly barred galaxies at the WIYN 3.5-m telescope. We present the new data and velocity field analysis in §2 and discuss possible limitations of the data, such as due to inclination uncertainties and non-circular motions. We then examine the location of barred and unbarred galaxies in the TF samples discussed above in §3. We find that barredness does not play a role in the luminosity-line width and luminosity-size planes of spiral galaxies. In §4, we discuss future programs that may benefit the study of scaling relations in barred and unbarred galaxies.

2. A New WIYN Survey of Barred Galaxies

2.1. Observations

In March 2002, we obtained 2-D H$\alpha$ velocity maps and deep V and I-band photometry at the WIYN 3.5-m (3 nights) and WIYN 0.9-m (2 nights), respectively for 14 strongly barred bright galaxies (SBb-SBc; $m_B \lesssim 15$; see Table 1) and one unbarred spiral galaxy (NGC 3029). The galaxies were selected according to the same criteria as the TF Shellflow survey of spiral galaxies, save the emphasis on the bar-like morphology. Ultimately, we aim to calibrate our new data on the same system as Shellflow, a survey deficient in barred galaxies, to enable direct comparisons between barred and unbarred systems.

Integral field spectroscopy (IFS), which is lacking in Shellflow and SCII, is required to fully characterize the velocity amplitudes of the bulge, bar, and underlying disk, especially if non-circular velocities are conspicuous. We have obtained 2-D velocity maps with the
SparsePak integral field unit (Bershady et al. 2003a). The SparsePak IFU is a fiber-optic array of 82 fibers mounted at the Nasmyth f/6.3 focus imaging port on the WIYN 3.5-m telescope. SparsePak has 75 fibers arranged in a sparsely filled grid sub-tending an area of $72'' \times 73''$. Each fiber has an active core diameter of $4.69$ (500 $\mu$m); cladding and buffer increase the total fiber diameter to $5.69$. The filling factor for the grid is $\sim 25\%$ on average, but rises to $\sim 55\%$ in the inner $16''$ where the fibers are more densely packed. In addition to the 75 fibers arranged in a square, another 7 fibers are spaced around the square roughly $70'' - 90''$ from the center and are used to measure the “sky” flux. An example of the SparsePak footprint is shown in Fig. 1.

SparsePak feeds the WIYN Bench Spectrograph, a fiber–fed spectrograph designed to provide low to medium resolution spectra. We used the Bench Spectrograph camera (BSC) and 316 lines mm$^{-1}$ echelle grating in order 8 to cover $6500\AA < \lambda < 6900\AA$, with a dispersion of $0.2\AA$ pix$^{-1}$ ($8.8$ km s$^{-1}$ pix$^{-1}$) and an instrumental FWHM of $0.6\AA$ ($26.5$ km s$^{-1}$). The BSC images the spectrograph onto a T2KC thinned SITe $2048 \times 2048$ CCD with 24 $\mu$m-pixels. The chip has a read noise of $4.3$ e$^-$ and was used with the standard gain of $1.7$ e$^-$/ADU. The peak system throughput for this setup is roughly $5.5\%$, estimated from standard-star observations (Bershady et al. 2003b).

Given SparsePak’s $\sim 15''$ center-to-center fiber spacing and total area, we used 3 pointings along the galaxy’s position angle to maximize spatial coverage and filling factor. Typical pointing offsets were $\sim 6''$. The observed galaxies have moderate sizes ($a \sim 20'$) and their velocity field can thus be mapped from center-to-edge. Total Sparsepak integrations consisted of 3 pointings $\times 2900s$ exposures per pointing, for a total of 1.75 hours per galaxy. Multiple exposures at each position were used to identify and remove cosmic rays.

Spectra obtained from SparsePak closely resemble WIYN Densepak or Hydra spectra (i.e. multi-fiber spectral data). Thus basic spectral extraction, flattening, wavelength calibration and sky subtraction were done using the NOAO IRAF$^5$ package dohydra. After basic reductions, we used a Gaussian line–fitting algorithm to measure Gaussian fluxes, widths, centers and centroid errors for H$\alpha$ emission lines (Andersen et al. 2003). We rejected any line with a $S/N < 5$. More than 70% of measured H$\alpha$ lines, even at the edge of the field, had significantly higher signal-to-noises, with $S/N \gtrsim 20$, yielding a mean centroiding error of only $2.4$ km s$^{-1}$ for these 15 galaxies.

The V and I images were acquired at the WIYN 0.9-m telescope in f/13.5 mode

$^5$IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.
integrations were 600s in each filter. Isophotal brightness errors are $\lesssim 0.1$ mag arcsec$^{-2}$ at, or below, 26.5 mag arcsec$^{-2}$ in V and I. The imaging was obtained in non-photometric conditions (thin wisps covered the Arizona desert sky) and thus cannot readily be merged into the Shellflow imaging data base. Structural parameters can still be measured accurately, down to deep levels, as we discuss below.

Three previously observed SB (NGC 2540, UGC 5141, UGC 8229) and two SAB (NGC 3029, UGC 6895) Shellflow galaxies with available long-slit Hα spectra and V,I photometry were duplicated at the WIYN telescopes for comparison. These observations enable us to tie the SparsePak velocity field information with the Shellflow long-slit spectra obtained with the KPNO & CTIO 4m telescopes + RC Spec (Courteau et al. 2003b).

2.2. Data Analysis

Azimuthally-averaged surface brightness profiles were extracted for all the galaxies using ellipse fitting with a fixed center. To ensure a homogeneous computation of structural parameters and color gradients, we use the position angles and ellipticities of our I-band isophotal maps to determine the SB profiles in the V-band. The position angle and ellipticity are allowed to vary at each isophote. Please refer to Courteau (1996) for details about our surface brightness extraction technique.

Reduction techniques for the extraction of rotation curves from long-slit spectra are described in Courteau (1997). We shall simply state that the 1-D rotation curve is constructed by measuring an intensity-weighted centroid at each resolved major-axis Hα emission feature above a noise threshold. For the 2-D SparsePak data, a single, inclined, differentially rotating, circular disk model with a fixed center is used to fit the Hα velocity fields (Andersen & Bershady 2003). Briefly, we assume a radially symmetric rotation curve and an axisymmetric velocity field. Using this smooth functional representation of the velocity field, we compared the model velocity field to observations. Parameters are varied using a multi-dimensional down-hill simplex method (Press et al. 1992) to minimize a $\chi^2$ statistic. Our velocity field model has nine free parameters: seven for the rotation curve (see next paragraph), and two for inclination and position angle. Two additional parameters account for positional offsets from differential telescope pointing errors for each SparsePak position, yet in practice these parameters were consistent with zero and were thereafter not allowed to vary.

We parameterize the model used to fit the rotation curves of both the 1-D (long slit) and
2-D (SparsePak) velocity field data with the following empirical function (Courteau 1997):

\[ v(r) = v_0 + v_a \frac{(1 + x)^\beta}{(1 + x^\gamma)^{1/\gamma}}, \]  

(1)

where \( x = 1/R = r_t/(r - r_0) \), \( v_0 \) and \( r_0 \) are the velocity and spatial centers of rotation, \( v_a \) is an asymptotic velocity, and \( r_t \) is a transition radius between the rising and flat part of the rotation curve. Solid-body rotation, or \( v(r) \propto r \) (with \( \partial v/\partial r \sim v_a/r_t \)), is recovered for \( |r-r_0| \ll r_t \), and flat rotation, or \( v(r) \propto v_a \), is achieved for \( |r-r_0| \gg r_t \). The term \( \gamma \) governs the degree of sharpness of turnover, and \( \beta \) can be used to model the drop-off or steady rise of the outer part of the rotation curve.

Table 1 gives velocity field and structural parameters for the SparsePak data collected at WIYN in March 2002. Listed are the number \( N \) of velocity data points, the kinematic and photometric inclinations, the kinematic and photometric position angles, the velocity fit parameters, \( v_a, r_t, \beta, \) and \( \gamma \) (see Eq. 1), the bar radius, \( R_{\text{bar}} \) in the plane of the galaxy, the I-band scale length \( h \) of the disk, and the recessional velocity of the galaxy, \( v_0 \). The bar radius is defined as the location where the I-band surface brightness drops and/or position angle changes abruptly. Disk scale lengths were determined as in MacArthur et al. (2003). No photometric parameters are listed for IC 0784 which could not be observed at the telescope due to time and weather constraints.

Appendix A contains rotation curves and extracted velocity fields (spider diagrams) for the WIYN/SparsePak galaxies. The model rotation curves, based on Eq. (1), are a decent match to most extracted integral field velocity data points. These models are shown mostly for illustrative purposes and for comparison with similar fits to rotation curves derived from long-slit spectra. They can also be used for future dynamical modeling.

The overall impression from the comparison of velocity data for the 5 Shellflow galaxies with long-slit 1-D and SparsePak 2-D rotation curves in Appendix A is very favorable. For NGC 2540 (Fig. 9), the 1-D and 2-D velocity models are indistinguishable, owing in part to the very similar position angles and inclinations used to extract the velocity amplitudes. The unbarred galaxy, NGC 3029 (Fig. 10), was re-observed for consistency check; again the velocity data and models agree very well within the measurement uncertainty. NGC 5141 shows only slight differences in the modeled RCs, and UGC 6895 and UGC 8229 show slightly larger differences in the inner slopes, perhaps caused by a misaligned slit. While the data distributions agree within their respective scatter, the RC models predict different maximum rotation speeds, at the 10-20 km s\(^{-1}\) level. However, the basic impression to retain for this comparison is that long-slit and IFS rotation curves agree well within their measurement errors and intrinsic scatter and it can be assumed that line widths from 1-D rotation curves are a fair representation of the overall velocity field, even for barred galaxies. Close agreement...
between 1-D rotation curves from Hα long-slit spectra and major-axis rotation curves from Fabry-Perot (2-D) velocity fields was also demonstrated by Courteau (1997).

Another concern, when mapping the kinematic and dynamic structure of barred galaxies, is whether our diagnostics are affected by non-circular velocities, radial flows, and/or isophotal distortions. In order to assess the importance of non-circular motions, we have examined minor-axis rotation curves (not shown here for simplicity) and spider diagrams in Appendix A (see also Swaters et al. 2003). The minor-axis rotation curves are consistent with 10–20 km s$^{-1}$ velocity dispersions of the turbulent gas with little hint of systematic deviations. The spider diagrams do show signs of non-circular motions, especially within $\sim 1.2R_{\text{bar}} (\simeq 1.5h_{\text{disk}})$. However, beyond the extent or reach of the bar, most position-velocity diagrams are symmetric about the major kinematic axis. With the exception of IC 2104 (Fig. 8), a symmetric velocity pattern is recovered for all galaxies at, and beyond, $R_{\text{disk}}$.

The good match between 1-D and 2-D velocity fields and lack of significant non-circular motions at or beyond $R_{\text{disk}}$ suggests that we can compare raw rotation speeds of barred and unbarred galaxies, all other quantities being equal, without significant bias. This is what we do in §3 for the Shellflow and SCII data. Any putative offset of the barred galaxies in the TF plane should not be due to systematic effects in the line widths.

Deprojection of velocity fields requires an inclination estimate. TF studies usually make use of photometric inclinations determined in the outer disk, away from a bar or spiral distortions, where ellipticities and position angles do not vary appreciably (e.g. Courteau 1996, Beauvais & Bothun 2001). We compare our SparsePak kinematic and I-band photometric inclination and position angle estimates in Fig. 2 and Table 1. A position angle offset would systematically lower the observed long-slit rotation, and inclination differences could displace a galaxy in the TF plane. We find that galaxies with $i_{\text{kin}} > 45^\circ$ show no appreciable inclination offset (within 3$^\circ$ rms) and a mild position angle offset (10$^\circ$ rms) between kinematic and photometric estimates. Position angle differences can be large for more face-on galaxies but our sample is too small to isolate systematic trends.

For galaxies with $i < 35^\circ$, photometric inclination angles are, on average, $\sim 12\%$ larger (more edge-on) than kinematic estimates. Inclination offsets for the low-inclination unbarred galaxy NGC 3029 are large and can only be explained by model fitting (kinematic vs isophotal) differences, whereas excellent agreement is found for UGC 6895, a higher inclination ($i = 45^\circ$) unbarred galaxy.

Note that our velocity model assumes circular, instead of elliptical, orbits. Kinematic inclinations are still precise enough to construct a TFR with small scatter ($\sigma_{\text{TF}} \simeq 0^\circ.3$) even at very low inclinations (Andersen & Bershady 2003). It is however unclear which of
the kinematic or photometric inclination is more “representative” of the disk projection on the sky. The inclination offset may result from a combination of kinematic modeling that favors more circular orbits and great sensitivity of the isophotal mapping technique to $m = 2$ brightness perturbations. Spiral arms typically originate at the ends (ILR) of bars and retain a small pitch angle, highly noticeable in the brightness distribution, hence the plausible bias towards higher photometric inclinations. These effects are especially acute when spiral arms are fully resolved.

In a similar study, Sakai et al. (2000; H$_0$ Key Project) find that photometric and kinematic (radio synthesis mapping) inclination angles differ for barred galaxies. Among the 21 calibrator galaxies in their TF sample, 7 are barred and their kinematic inclination angles are $\sim 10$-15% smaller than photometric inclinations. Their barred galaxies all have $i_{\text{phot}} > 45^\circ$. However, inclination offsets for their unbarred galaxies are nearly absent. Peletier & Willner (1991) give radio and infrared inclination angles for 13 barred and unbarred nearby spirals with $27^\circ < i < 70^\circ$. Radio synthesis inclinations are also $\sim 12^\circ$ smaller than photometric estimates, but for all inclinations.

To illustrate this potentially confusing situation, we plot in Fig. 3 the inclination difference, $\Delta i$ (kin – phot), against kinematic inclination for the galaxy samples considered above, plus a sample of nearby, face-on, unbarred spiral galaxies (Andersen 2001). At low inclinations, kinematic inclinations appear to be systematically lower (more face-on) than photometric inclinations, with a trend of increasing differences with decreasing inclination. This is made very clear by examination of Andersen’s data. At high inclinations, both barred and unbarred galaxies have smaller inclinations offsets, apparently independent of inclination. At these high inclinations, the effect on the velocity deprojection is negligible ($< 5\%$). It may be that SparsePak and photometric inclinations in these inclined galaxies are affected by extinction as higher opacity would naturally bias high optically-determined inclinations. However, the radio synthesis inclinations compiled in Sakai et al. are insensitive to dust and the inclination difference is most likely explained by modeling differences; 2-D velocity fields are modeled under the assumption of circular orbits and the larger kinematic inclinations at large inclination may result from an underestimate of the disk thickness. In general, with increasing inclination, photometric inclinations become increasingly sensitive to the estimated disk thickness while velocity fields (especially radio velocity fields) become increasingly affected by warps and other non-circular motions. In any event, the inclination differences at $i_{\text{kin}} > 50^\circ$ are small ($< 5^\circ$) and do not affect our study. Barnes & Sellwood (2003) find a similar result for a sample of inclined galaxies with inferred photometric and kinematic (Fabry-Perot) inclinations.

Opposite trends are found in the compilation of Peletier & Willner (1991) if all their
data are considered. Inclination offsets are large even at high inclinations. This discrepancy however hinges on three galaxies, NGC 4178, 4192, and 4216, that display various pathologies. NGC 4178 is a very late type system, NGC 4192 has a strong warp in the outer disk, and NGC 4216 has a very pronounced dust-lane; these all make photometric measurements uncertain. If we ignore the Peletier & Willner data (bottom panel; Fig. 3), we find that the transition threshold where kinematic inclinations becomes significantly lower than photometric inclinations depends on type: $i_{\text{kin}} = 50^\circ$, $40^\circ$, and $30^\circ$ for barred, weakly-barred, and unbarred galaxies.

Clearly, a more extensive 2-D spectroscopic survey of barred and unbarred galaxies in the near-infrared and radio will help address our general concerns about their dynamical structure and the limitations of our modeling techniques. Infrared imaging should also be secured for extinction-free inclination measurements. The measurement of a “true” inclination of a galaxy is certainly ill-defined as it depends on the bandpass, dust extinction, the detector, reduction methods, and assumptions concerning the galaxy structure (e.g. presence of warps). Yet, inclination angles from radio synthesis mapping may come closest to the most representative tilt angle of a galaxy on the sky.

As we await more detailed comparisons of radio and optically determined inclinations, systematic differences between barred and unbarred galaxies can be avoided if we restrict our Shellflow and SCII samples to galaxies with $i_{\text{phot}} \gtrsim 50^\circ$. Fortunately, all barred galaxies in our samples (Shellflow, SCII) already meet this criterion. We pursue our TF analysis with a discussion of Shellflow and SCII galaxies below. Our SparsePak sample will be reconsidered for TF analysis when calibrated imaging is available.

### 3. The Tully-Fisher Relation of Barred Galaxies

We use the “Shellflow” and “SCII” all-sky TF surveys to map the location of barred galaxies in the TF plane. Shellflow includes 300 bright spiral (Sab-Scd) field galaxies in a shell bounded at $4500 < cz < 7000$ km s$^{-1}$, and SCII has 441 cluster spirals (Sa-Sd) spanning $5000 < cz < 19000$ km s$^{-1}$.

Shellflow galaxies were drawn from the Optical Redshift Survey sample of Santiago et al. (1995) with inclinations in the range $[45^\circ, 78^\circ]$, $m_B \leq 14.5$, and $|b| \geq 20^\circ$. Interacting, disturbed, and some barred galaxies were rejected. Rotation speeds from resolved H$\alpha$ rotation curves were measured at 2.2 disk scale lengths; the upper inclination limit ($i < 78^\circ$) reflects a desire to minimize extinction effects in the inner parts of the rotation curve (e.g. Courteau & Faber 1988; Giovanelli & Haynes 2002). Deep I-band and V-band
images were collected for each Shellflow galaxy. Disk scale lengths were obtained from B/D decompositions of the azimuthally-averaged I-band surface brightness (SB) profile (Courteau et al. 2003b).

The SCII cluster galaxies were selected from CCD I-band images taken at the KPNO and CTIO 0.9-m telescopes and classified by eye and by their bulge-to-disk ratio or concentration index. These galaxies have inclinations in the range [32°,90°] and I-band magnitudes 12 ≤ m_I ≤ 17. SCII line widths were measured from both Hα long-slit spectra and HI line profiles. SCII disk scale lengths were obtained by “marking the disk”, or fitting the exponential part of the SB profile from ~21 I-mag arcsec^-2 to ~25 I-mag arcsec^-2 (Dale et al. 1999).

Shellflow and SCII galaxies have −20 ≤ M_I^Shell ≤ −24 and −18 ≤ M_I^{SCI} ≤ −24, respectively. Both TF calibrations are based on digital I-band imaging; V − I colors, to test for M/L variations and extinction effects, are available for the Shellflow sample only. De-projection of velocity widths uses photometric inclinations measured in the outer disk where ellipticities and position angles do not vary appreciably. Shellflow and SCII magnitudes are corrected for Galactic and internal extinction and distances account for a Hubble expansion, bulk flow model, and effects of incompleteness. The exact choice of distance scale does not affect our conclusions.

According to the RC3, 37% of the Shellflow sample is barred (SB types only). In general, the proportion of galaxies with bars of all sizes is even higher (Eskridge et al. 2002) but we are here only concerned with galaxies with the strongest bars; i.e. those with potentially the highest central baryon fraction. Visual examination of the Shellflow galaxies revealed only 6 strongly barred systems (at I-band); these have R_{bar}/h_{disk} ≥ 1.2, where R_{bar} and h_{disk} are the size of the bar semi-major axis and disk scale length, respectively. Visual examination of the SCII galaxies yielded 27 strongly barred galaxies (D. Dale 2002; priv. comm.) In both samples, only barred galaxies with M_I ≤ −20.4 could be identified. The Shellflow and SCII sub-samples of barred galaxies are by no means complete, nor are the parent catalogs, and a significant number of bars will be missed especially at low magnitudes and high inclinations where morphological identification becomes problematic.

Figs. 4 & 5 show the distributions of rotational velocities and exponential scale lengths vs I-band absolute magnitudes for Shellflow and SCII galaxies. Different symbols identify the full range of spiral Hubble types; barred galaxies are further emphasized as solid symbols with open circles. Looking at the upper panel of Fig. 4 for Shellflow galaxies, one sees a small offset of barred galaxies from the mean TFR, consistent with these galaxies being systematically brighter for a given mass (line width). The same statistically loose trend for barred galaxies was observed by Sakai et al. (2000). It could be explained if barred galaxies have higher star formation rates. However, Phillips (1996) and Kennicutt (1999)
find that global star formation rates in barred and unbarred galaxies of the same Hubble type are comparable. The TF offset, if real, might also be consistent with maximal disks being brighter than their dark-matter dominated counter-parts at a given mass.

A clearer picture is obtained with the larger SCII sample (Fig. 5) which shows no offset from the mean TFR for SCII barred galaxies. The combined velocity offset for the Shellflow and SCII barred galaxies in the two samples is $\langle \delta \log V \rangle = -0.02 \pm 0.04$, consistent with no deviation of the mean TFR. Note that photometric inclinations are used to deproject velocities in Shellflow and SCII but using kinematic inclinations instead would simply imply a readjustment of the TF zero-point. Provided only one inclination measure is used, the relative distribution of barred and unbarred TF galaxies is not affected by the precise choice of inclination ($\S 2.2$). Recall that all the Shellflow and SCII barred galaxies have $i > 50^\circ$ and are not affected by a putative (kin-phot) inclination offset. Furthermore, if we exclude the few unbarred galaxies that have $i < 50^\circ$ from the Shellflow and SCII samples, the TF distributions remain the same. Thus, we conclude that barred galaxies lie on the same TFR as unbarred galaxies. A similar realisation was also reached by Debattista & Sellwood (2000).

The kinship between barred and unbarred galaxies extends to other properties as well. The lower panels of Figs. 4 & 5 show no statistical differences in the scale lengths of barred and unbarred galaxies (for a given absolute magnitude). Fig. 6 shows the color-magnitude diagram of Shellflow galaxies. Notwithstanding small statistics, barred and unbarred galaxies have similar colors, consistent with their having comparable star formation rates (Kennicutt 1999). MacArthur et al. (2003) find other similarities for structural parameters of barred and unbarred galaxies: their bar/bulge light profiles are close to exponential, and their ratio of bulge effective radius, $r_e$, and disk exponential scale length, $h$, falls in the range $r_e/h = 0.22 \pm 0.09$, expected for late Freeman Type I spirals.

CR99 developed and applied a test for correlated scatter of the TFR. According to this test, pure stellar exponential (maximal) disks should deviate from the mean TF and luminosity-size (LS) relations in such a way that $\partial \log V_{disk} / \partial \log R_{exp} = -0.5$. Thus, strongly correlated TF/LS residuals for the barred spirals would support the suggestion that unbarred spirals have sub-maximal disks (high concentration halos) and that maximal disks are only found, on average, in barred spirals. A new analysis based on the Shellflow and SCII data sets yields residuals that are consistent with $\partial \log V_{disk} / \partial \log R_{exp} = 0.0$ for both barred and unbarred galaxies. CR99 found a similar result for the Courteau-Faber sample. This result further confirms earlier observations about spiral galaxies: barred and unbarred galaxies have similar physical properties and populate the same TF/LS relation and residual space. It also shows that the TFR is fully independent of surface brightness (CR99), a situation which may also result from the fine-tuning of virial parameters. The
analysis of the independence of surface brightness in the TFR, and a revised interpretation of the “Courteau-Rix” test in terms of virial parameter correlations, is presented in Courteau et al. (2003a).

4. Discussion and Conclusion

We have tested the hypothesis that barred and unbarred spiral disks have different structural correlations, such as the Tully-Fisher relation, with barred galaxies possibly having a higher luminous-to-dark matter fraction in their inner parts. New WIYN/SparsePak integral field spectroscopy and deep near-infrared photometry of barred and unbarred spirals allowed us to verify that non-circular motions are not significant at $R_{\text{disk}}$ and that rotation curves from 1-D or 2-D spectroscopy are reliable beyond that radius. Based on this result, and uniform inclination corrections for spiral galaxies with $i > 50^\circ$, we have compared the distribution of barred and unbarred galaxies in the TF plane from extensive redshift-distance surveys of galaxies and found no significant differences.

For a given circular velocity, barred and unbarred galaxies have comparable luminosities, scale lengths, colors, and star formation rates\(^6\). This suggests that barred and unbarred galaxies are close members of the same family and do not originate from different evolutionary trees. Their structural duality may be understood if bars are generated by transient dynamical processes that are likely independent of the initial galaxy formation conditions. Their virial properties would otherwise be different.

Very recent N-body simulations with the highest resolution have relaxed the notion that bars would grow in structures defined by a narrow range of disk/halo parameters. Thus our comparisons cannot be used to ascertain the notion that bars live mostly in spiral disks whose stellar fraction dominates the mass budget within the optical disk. Our results are however consistent with bright barred galaxies having similar dark matter fractions as do their unbarred cousins (Debattista & Sellwood 2000; Courteau et al. 2003a). Stellar velocity dispersions, which provide robust disk $M/L$ ratios, hold the promise of breaking the disk/halo degeneracy in mass modeling of barred and unbarred galaxies.

If the presence of bars in rotating disks is not directly related to their virial structure but rather to their local dynamical state, it can surely be used as a signpost of galaxy evolution.

\(^6\)A comparative study Sheth et al. (2002) of the molecular gas properties of barred and unbarred galaxies in the BIMA Survey of Nearby Galaxies shows striking differences. However, their data (see their Fig.2) show less striking differences for the star formation rates between barred and unbarred galaxies but based on scanty information. More data are clearly needed to elucidate these questions!
Given that bars may be just as important as mergers in shaping field disk galaxies, significant efforts should be invested in programs to probe differences between barred and unbarred galaxies. Bars, which can be triggered spontaneously by the global dynamical instability of a rotationally supported disk, can also be induced by interactions with a satellite. One might thus expect an increase of the fraction of barred disks at higher redshift, unless these younger disks are too dynamically hot to sustain bar unstable modes. Van den Bergh et al. (2002) studied the visibility of bars in the northern Hubble Deep Field (HDF-N) and reported a dearth of bars at $z > 0.7$ in the rest-frame V-band. Taken at face value, this could indicate a dependence of bar strength on the local galaxy density which grows with time. However, a similar study by Sheth et al. (2003) based on the NICMOS Deep Field reveals numerous strongly barred galaxies up to $z = 1.1$. Extinction effects in the bluer band explored by van den Bergh et al. (2002) thus thwarted their ability to detect dust enshrouded bars. Given the detection of stable disks beyond $z \sim 1.3$ (van Dokkum & Stanford 2001; Genzel et al. 2003), it is thus reasonable to posit the existence of bars at comparably high redshifts. The cosmological volumes sampled in two HDF studies above are very small and robust statistics on the barredness of galaxies with look-back time awaits wider coverage and more extensive sky surveys, especially with telescopes like ALMA within the next decade.

Closer to home and on shorter time scales, our comparison of a few dozen barred galaxies with TF samples of unbarred disks should soon be superceded, it is wished, by systematic studies of structural and environmental properties of thousands of barred and unbarred galaxies in the SLOAN and 2MASS galaxy catalogs. Only with such large-scale, systematic local investigations can we make significant progress in mapping galaxy evolution at high-redshift and linking the near and far-field Universe.

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A. Rotation Curves, Velocity Fields and I-band images for the WIYN02 Sample

This section shows long-slit (1-D) and SparsePak (2-D) velocity fields for all galaxies observed at WIYN in March 2002. See §2 for details about the sample and data analysis. Shown for each galaxy are the position-velocity contours (“spider” diagrams) in the upper window, superimposed on the galaxy I-band image, and in the lower window, the SparsePak velocities (in the plane of the sky, i.e. not corrected for projection effects). The velocity data were extracted according to various techniques described in the text and, whenever available, matching rotation curves are shown from the Shellflow collection of long-slit spectra.

Smoothed versions of the observed velocity field were produced using the patch routine within the GIPSY analysis package (van der Hulst et al. 1992; Vogelaar & Terlouw 2001). The SparsePak velocity field shown in the lower window is extracted from a model which includes inclination, PA, disk center, rotation velocity, scale length, and systemic velocity. The parameterization of the velocity field is given by Eq. (1).

SparsePak Hα position-velocity diagrams are constructed using 2 representations of the 2-D velocity field: The first includes all measurements with a simulated 6′′ “slit” for the best-fit kinematic position angle (filled triangles). The second SparsePak rotation curve uses all measured velocities within ±60 ° of the kinematic major axis in the inclined plane of the galaxy (open squares). Using the modeled kinematic inclination and position angle, we can project each measured rotation velocity onto the major axis. This second, ”wedge,” approach is relatively insensitive to inclination-induced beam smearing which affects the simulated slit measurements. However, the wedge does not spatially sample the inner 10′′ as well as the slit. Our best fit model (solid line) is adjusted for beam smearing induced by the ~5″ fibers of SparsePak. When comparing this model to the data, remember that the simulated slit data (filled triangles) have not been projected onto the major axis; the magnitude of these velocities serve only as a lower limit. Thus, a black triangle in the center of the RC that does not have a corresponding open box at the same radius implies that the center of that fiber lies more than 60° from the major axis and its azimuth correction is large (greater than 2). The velocity models based on Eq. (1) trace the open boxes only. Further details about velocity field modeling are given in §2.2.
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Fig. 1.— SparsePak fiber footprint for one pointing overlayed on our CCD I-band image for the SBbc galaxy UGC 5141.
Fig. 2.— Differences in measurements of kinematic and photometric position angles and inclinations for galaxies with available 2-D velocity fields and I-band imaging. Inclinations shown next to the galaxy names correspond to the kinematic and photometric estimates, respectively. Inclination differences are larger for progressively face-on orientations.
Fig. 3.— Difference between kinematic and photometric inclinations vs kinematic inclination for four galaxy samples. The point types are squares (this study); circles (Andersen 2001); triangles (Peletier & Willner 1991); and pentagons (Sakai et al. 2000). Open, gray-filled, and black-filled symbols represent unbarred, weakly-barred, and strongly barred galaxies, respectively. The top panel shows simple regressions to the Courteau et al. (this study), Andersen, and Peletier & Willner samples (independently). We exclude the Peletier & Willner sample in the bottom sample.
Fig. 4.— Line width-luminosity (top) and size-luminosity (bottom) diagrams for Shellflow galaxies. Line widths are measured at 2.2 disk scale lengths and disk scale lengths are obtained from B/D decompositions of the surface brightness profile. Barred galaxies have filled symbols consistent with their Hubble type and are further emphasized with an open circle. Barred galaxies lie below the mean TFR, appearing to be systematically brighter for their rotational velocity. As in Sakai et al. (2000), this is a small number artifact. The solid line is a fit from our data—model minimization technique (Courteau et al. 2003a).
Fig. 5.— Line width-luminosity (top) and size-luminosity (bottom) diagrams for SCII galaxies. Line widths are measured from Hα rotation curves and H I line widths and disk scale lengths are measured using the “marking the disk” technique (see text). Symbols are as in Fig. 4. The TFR is the same for barred and unbarred galaxies. The solid and dashed lines show data—model minimization fits from Courteau et al. (2003a) and Dale et al. (1999), respectively.
Fig. 6.— Color-magnitude diagram for Shellflow galaxies. Barred galaxies have mean colors consistent with the general spiral population. Symbols are as in Fig. 4.
Fig. 7.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for IC 784.
Fig. 8.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for IC 2104. This galaxy has a pathological velocity field with significant non-circular motions, a continuously rising rotation curve, and a small inner velocity bump representative of a strong bar and/or bulge (that is poorly matched by the velocity model).

Fig. 9.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for NGC 2540.

Fig. 10.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for the unbarred galaxy NGC 3029.

Fig. 11.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for NGC 3128.

Fig. 12.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for NGC 3469.

Fig. 13.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for NGC 3832.

Fig. 14.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for NGC 4999.

Fig. 15.— Velocity contours and V-band image (top) and rotation curve data with velocity model (bottom) for NGC 5504. The I-band image was not available.

Fig. 16.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for UGC 4416. The vertical trace in the upper image is due to an internal image reflection.

Fig. 17.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for UGC 5141.

Fig. 18.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for the weakly barred galaxy UGC 6895.
Fig. 19.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for UGC 7173.

Fig. 20.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for UGC 8229.

Fig. 21.— Velocity contours and I-band image (top) and rotation curve data with velocity model (bottom) for UGC 8241.