Cosmological density perturbations from perturbed couplings

Shinji Tsujikawa
Institute of Cosmology and Gravitation, University of Portsmouth, Mercantile House, Portsmouth PO1 2EG, United Kingdom
(August 19, 2003)

The density perturbations generated when the inflaton decay rate is perturbed by a light scalar field \( \chi \) are studied. By explicitly solving the perturbation equations for the system of two scalar fields and radiation, we show that even in low energy-scale inflation nearly scale-invariant spectra of scalar perturbations with an amplitude set by observations are obtained through the conversion of \( \chi \) fluctuations into adiabatic density perturbations. We demonstrate that the spectra depend on the average decay rate of the inflaton \& on the inflaton fluctuations. We then apply this new mechanism to string cosmologies \& generalized Einstein theories and discuss the conditions under which scale-invariant spectra are possible.

\[ \text{pacs: 98.80.Cq} \]

In inflationary cosmology the seeds of large-scale structure are originated from quantum fluctuations of a light scalar field responsible for inflation—called the inflaton. The adiabatic perturbations generated from inflaton fluctuations exhibit nearly scale-invariant spectra, as required by observations [1]. The energy scale of inflation is determined by the amplitude of large-scale perturbations in the Cosmic Microwave Background (CMB). For example, in the simple model of chaotic inflation with potential \( V = (1/2)m_\phi^2 \phi^2 \), the mass of the inflaton \( \phi \) is constrained to be \( m_\phi \simeq 10^{-6} m_{pl} \) by the Cosmic Background Explorer (COBE) normalization [2,3] (Here \( m_{pl} \simeq 1.22 \times 10^{19} \text{GeV} \) is the Planck mass).

On the other hand, if inflation occurs at a much lower scale, the amplitude of density perturbations is too low to explain the temperature anisotropy in CMB. In this case we need to consider some other mechanism to generate sufficient density perturbations.

Recently a number of authors [4,5] independently proposed an alternative method to produce scalar metric perturbations using the spatial variation of the inflaton decay rate, \( \Gamma \). Since the coupling strength is generally a function of the vacuum expectation value of scalar fields in string theory, it is natural to consider the case where the coupling fluctuates due to scalar field perturbations. When there exists some light field whose mass is smaller than the Hubble rate (such as modulus [5]), its large-scale fluctuations are not exponentially suppressed during inflation. This can provide another source of scalar metric perturbations in addition to the inflaton. In fact the authors in [4] considered the decay of the inflaton by treating it as a matter fluid and showed that density perturbations can be generated by the perturbed decay rate, \( \delta \Gamma \).

In this work we shall extend the analysis of Ref. [4] to the more realistic situation where the inflaton is treated as a scalar field, and completely follow the evolution of metric perturbations during inflation and reheating. This is particularly important when we treat the dynamics of reheating consistently, since the fluid approach can lead to some loss of information (like parametric resonance) by averaging over the scalar field oscillations. We will analyze the case where perturbations of both the inflaton and the decay rate coexist, and estimate the amplitude and the spectral index of metric perturbations after inflaton decay. We shall also apply this new mechanism to string-inspired cosmologies \& generalized Einstein theories.

Let us consider the chaotic inflationary scenario with potential \( V = (1/2)m_q^2 q^2 \). During reheating, the inflaton \( \phi \) decays to standard light particles through the interaction, \( \lambda_0 \phi \sigma \sigma \), whose decay rate, \( \Gamma \), is proportional to \( \lambda_0^2 \) [2]. The inflaton can also decay via non-renormalizable interactions with superfields, e.g., \( \phi (q/M) q q \) (\( M \) is some mass scale), in which case the vacuum expectation value of a scalar component of the \( q \) superfields, \( \langle \chi \rangle \), gives rise to an effective coupling, \( \lambda = \langle \chi \rangle / M \) [4]. Following Ref. [4] we shall analyze the case where the effective coupling \( \lambda(\chi) \) is dependent on the field \( \chi \) as

\[ \lambda(\chi) = \lambda_0 (1 + \chi / M + \cdots) . \]  

When the coupling, \( (1/2)g^2 \phi^2 \chi^2 \), exists during the reheating stage, this can lead to explosive particle production called preheating [6,7]. In order for preheating to occur, we require the large resonance parameter, \( g_{re} \equiv g^2 \phi^2 / (4m_q^2) \gg 1 \), at the end of inflation [7], in which case large-scale perturbations in \( \chi \) are exponentially suppressed during inflation due to the heavy effective mass, \( g|\phi| \), relative to the Hubble rate, \( H \). Then the perturbations of the coupling \( \lambda(\chi) \) are vanishing small, which do not affect the evolution of metric perturbations. Therefore in this work we do not take into account the effect of preheating coming from the interaction, \( (1/2)g^2 \phi^2 \chi^2 \). When this interaction is added to the Born decay term, \( \lambda_0 \phi \chi^2 \), this gives rise to the form of the effective coupling (1) perturbed by the inflaton \( \phi \) instead of \( \chi \). However we are interested in the case where the coupling is perturbed by a light field \( \chi \) other than the inflaton.

Hereafter we will analyze the two-field system of \( \phi \) and \( \chi \) with radiation. We assume that the field \( \chi \) has a small
mass, $m_\chi$, relative to the Hubble rate, $H \equiv \dot{a}/a$, during inflation (here $a$ is a scale factor). Radiation is generated through the decay of the inflaton with the $\chi$-dependent coupling (1). Consider the following perturbed metric for scalar perturbations in the longitudinal gauge [9]:

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)dx^i dx^j.$$ (2)

At linear order one has $\Phi = \Psi$ due to the vanishing anisotropic stress. Then the Fourier transformed, linearized Einstein equations are [10]

$$(k^2/a^2)\Phi + 3H(\Phi + H\Phi) - (4\pi/m_{pl}^2)(\dot{\phi}^2 + \chi^2)\Phi$$

$$= - (4\pi/m_{pl}^2)(\dot{\phi}^2 + m_\phi^2\phi + \chi\dot{\chi} + m_\chi^2\chi\delta\chi + \mu\delta\phi),$$ (3)

$$\delta\dot{\phi} + (3H + \Gamma)\delta\phi + (k^2/a^2 + m_\phi^2)\delta\phi$$

$$= 4\dot{\phi}\Phi - 2m_\phi^2\phi - \Gamma\phi(2\Phi + \delta\Gamma/\Gamma),$$ (4)

$$\delta\dot{\chi} + 3H\delta\chi + (k^2/a^2 + m_\chi^2)\delta\chi = 4\chi\Phi - 2m_\chi^2\chi\Phi,$$ (5)

$$\delta\mu + 4H\delta\mu = 4\mu[\Phi - (k/3a)v] + \Gamma\phi[2\delta\phi + (\delta\Gamma/\Gamma)\phi],$$ (6)

$$\dot{v} + (4H + \dot{\mu}/\mu)v = (k/\mu)[\Phi + (\delta\mu + 3\Gamma\delta\phi)/4\mu].$$ (7)

where $k$ is a comoving wave number (the $k$ subscript is implicit) and $\Gamma$ is the inflaton decay rate that is smaller than the Hubble rate during inflation. $\mu$ is the perturbation of radiation whose energy density is given by $\mu$, and $v$ is the velocity of the radiation fluid.

The decay rate, $\Gamma$, is proportional to $\lambda^2$, and $\lambda$ is dependent on the field $\chi$ through the relation (1). Therefore the fluctuation of the decay rate can be written as

$$\delta\Gamma/\Gamma = \frac{2\delta\lambda}{\lambda} = \frac{2\delta\chi}{\chi + M},$$ (8)

where we dropped the contributions coming from the higher-order terms in Eq. (1). When the mass of the field $\chi$ is smaller than $H$, large-scale $\chi$ fluctuations are not suppressed during inflation, thereby providing source terms for the gravitational potential $\Phi$.

We shall analyze the situation where the background field $\chi$ is not dynamically important relative to the inflaton $\phi$ in order to avoid the second stage of inflation to occur. This means that $\dot{\chi} \ll \dot{\phi}$ in Eq. (3), in which case the correlation between adiabatic and isocurvature perturbations is small [14] due to the negligible angular velocity in field space ($\tan \theta \equiv \dot{\chi}/\dot{\phi} \simeq 0$). In the absence of the $\delta\Gamma$ term the conversion from isocurvature to adiabatic perturbations occurs significantly only when $\dot{\chi}$ and $\dot{\phi}$ can be the same order during inflation [15]. This corresponds to double inflationary scenarios driven by two scalar fields, in which case the strength of the correlation is numerically investigated in Refs. [15,16] for $\delta\Gamma = 0$.

In the presence of the $\delta\Gamma$ fluctuation, the situation is different from the one discussed in Ref. [14]. First of all, even if $\dot{\chi}$ is much smaller than $\dot{\phi}$, the conversion of the $\delta\Gamma$ fluctuation into the gravitational potential occurs as seen from Eqs. (3), (4) and (6). As long as the mass of $\chi$ is light and large-scale $\chi$ fluctuations are not suppressed during inflation, this naturally leads to the conversion to the gravitational potential through the perturbed decay rate, $\delta\Gamma$. The advantage of this scenario is that we do not need to rely on two $\chi$-dependent source terms in Eq. (3) in order to obtain sufficient isocurvature perturbations. In this sense it is also different from metric preheating [17] where the growth of the $\chi$-dependent terms sources the gravitational potential.

In the absence of the $\delta\Gamma$ term, the amplitude of scalar perturbations generated from inflaton fluctuations are estimated as $\delta_H = \sqrt{512\pi/75} V^{3/2}/(m_{pl}^2|V|)|_{k=aH}$ [3]. Substituting the value $\phi \sim 3m_{pl}$ corresponding to the 60 e-foldings before the end of inflation, we get $\delta_H \sim 12m_{\phi}/m_{pl}$. Making use of the COBE result $\delta_H \approx 2 \times 10^{-5}$, the inflaton mass is constrained to be $m_{\phi} \simeq 1.7 \times 10^{-6}m_{pl}$. The spectral index $n$ for the power spectrum, $P_{\Phi} \equiv \frac{k^3}{2\pi^2} \langle |\Phi_k|^2 \rangle$, is estimated as [3]:

$$n = 1 - 6\epsilon + 2\eta \simeq 1 - \frac{4m_{\phi}^2}{3H^2},$$ (9)

where $\epsilon \equiv \frac{m_\phi^2}{16\pi} \left(\frac{\chi'}{\chi}\right)^2 \simeq \frac{m_\chi^2}{4H^2}$ and $\eta \equiv \frac{m_\phi^2}{8\pi} V'/V \simeq \frac{m_\chi^2}{4H^2}$ are the slow-roll parameters. This is a slightly red-tilted spectrum for $m_{\phi} \ll H$.

The above basic picture of the generation of density perturbations can be substantially modified when the $\delta\Gamma$ fluctuations are involved. Let us first estimate the spectral index of the $\delta\chi$ fluctuation in Eq. (5) with mass satisfying $m_\chi \lesssim H$. When the field $\chi$ is subdominant relative to $\phi$ ($\chi \ll \phi$), we can neglect the r.h.s. of Eq. (5) coming from the gravitational potential. Then the spectral index for the super-Hubble $\delta\chi$ fluctuation is estimated as [18]

$$n_{\delta\chi} = 1 - \frac{2}{3H^2}(m_{\phi}^2 - m_{\chi}^2).$$ (10)

In the presence of the $\delta\Gamma$ fluctuation, the power spectra $P_{\Phi}$ exhibit a spectral tilt which is a mixture of Eqs. (9) and (10). When $m_\chi$ is smaller than $m_{\phi}$, one has slightly red-tilted spectra of $\Phi$ which are close to scale-invariant (see Fig. 1). The spectra of $\delta\chi$ are blue-tilted for $m_{\chi} > m_{\phi}$, but this does not necessarily mean that the gravitational potential is also blue-tilted. In fact, even when $m_{\chi} \gtrsim 3m_{\phi}$, super-Hubble $\delta\chi$ fluctuations begin to be exponentially suppressed during inflation, in which case the gravitational potential is dominated by the inflaton fluctuation whose spectrum is given by Eq. (9).
Typically we have red-tilted, nearly scale-invariant spectra of $\Phi$ for wide range of the parameter space.

If the inflaton mass is less than $m_\phi \simeq 10^{-6} m_{pl}$, the amplitude of the power spectra $P_\Phi$ is suppressed compared to the observed value, $P_\Phi \simeq 10^{-9}$ (see Fig. 1). Taking into account the $\delta \Gamma$ fluctuation, the amplitude of density perturbations can be much higher. In fact, as shown in Fig. 1, it is possible to get nearly scale-invariant power spectra whose amplitude are of order $P_\Phi \simeq 10^{-9}$.

$$P_\Phi \simeq 10^{-9},$$

are obtained after inflaton decay. When the $\chi$ mass is light relative to the Hubble rate, one gets the perturbations of order $\delta \chi \sim H$ after Hubble radius crossing. Therefore the fluctuation of the decay rate is approximately estimated as $\delta \Gamma / \Gamma \sim H / M$ for $\chi \ll M$ from Eq. (8). We can have a considerable amount of fluctuations in $\delta \Gamma / \Gamma$ by varying the mass scale, $M$. In Fig. 2 we find that inclusion of the perturbed decay rate allows for the wide possibility to generate the perturbations of order $P_\Phi \simeq 10^{-9}$ even for $m_\phi \ll 10^{-6} m_{pl}$. This figure corresponds to the one where $\chi$ is much smaller than $\phi$ (and $M$). We wish to stress that $\delta \chi$ fluctuations can be sufficiently transferred to adiabatic density perturbations even for $\tan \theta \simeq 0$. When the average decay rate $\Gamma$ gets smaller, we require larger values of $\delta \Gamma / \Gamma$ (corresponding to smaller values of $M$) to lead to sufficient conversion as confirmed by Eqs. (4) and (6). This behavior is actually seen in our numerical simulations of Fig. 2. In the limit of $\Gamma \to 0$ ($\lambda_0 \to 0$), the above conversion does not occur apart from the standard one discussed in Ref. [14].

We can apply the above new mechanism for the more general action

$$S = \int d^4x \sqrt{-g} \left[ \frac{m_{pl}^2}{16\pi} R - \frac{1}{2} (\nabla \varphi)^2 - F(\varphi)(\nabla \chi)^2 - U(\varphi) \right],$$

where $F(\varphi)$ is a function of $\varphi$ and $U$ is a potential of scalar fields including their interactions. The action (11) contains not only various generalized Einstein theories [19,20] but also the tree-level Pre-Big-Bang (PBB) cosmology [21,22] from the dimensional reduction and 4D effective description of ekpyrotic cosmology [23,24].

The effective potential $U$ typically includes an exponential term in generalized Einstein theories, which can lead to inflationary solutions [19,20]. Let us consider the soft inflation model [19] with $U = U_0 \exp(-\sqrt{2/\rho} \varphi)$ and $F = (1/2)e^{\beta \varphi}$ with $\beta$ being a constant (hereafter we set the units such that $8\pi/m_{pl}^2 \equiv 1$). We shall assume that the decay rate of the inflaton is perturbed by the field $\chi$ with a negligible mass. The evolution of the background is given as $a \propto t^\gamma \propto (\tau)^{\rho/(1-p)}$ and $e^{\varphi} \propto \left\{-\left[(p-1)\tau \right]^{\sqrt{2p}/(1-p)}\right\}$ with $\tau \equiv \int a^{-1} dt \left( \tau > 0 \right)$ being a conformal time [25]. Assuming that the background field $\chi$ is not dynamically important, the perturbed equation for $\delta \chi$ is approximately written as

$$\ddot{\delta \chi} + (3H + \beta \dot{\varphi}) \delta \chi + (k^2/a^2) \dot{\delta \chi} = 0.$$

Introducing new variables, $b \equiv e^{\beta \varphi/2} a$ and $\delta \chi \equiv b \delta \chi$, we get

$$\ddot{\delta \chi} + \left( k^2 - b''/b \right) \delta \chi \equiv 0,$$

where a prime denotes the derivative with respect to $\tau$. When the evolution of $b$ is characterized by $b \propto (\tau)^{\gamma}$, the spectral index of the $\delta \chi$ fluctuation is given as $n_{\delta \chi} = 4 - [1 - 2\gamma]$ [26]. In the present case $\gamma$ is given by

$$\gamma = \frac{1}{2}.$$
In the limit of $p \to \infty$, we have $\gamma \to -1$, thereby yielding the scale-invariant spectra, $n_{\delta \chi} = 1$. In the presence of the noncanonical kinetic term ($\beta \neq 0$), we have $n_{\delta \chi} = 1$ for $\beta = -\sqrt{2/p}$ or $\beta = -(3p - 2)/\sqrt{2/p}$ (corresponding to $\gamma = -1$ or $\gamma = 2$). If the decay rate of the inflaton is perturbed by the field $\chi$, it is possible to generate scale-invariant spectra of density perturbations in a similar way as discussed previously.

In PBB and ekpyrotic cosmologies, one has the contracting phase of the universe in the Einstein frame before the bounce [27]. Let us consider the ekpyrotic scenario contracting phase of the universe in the Einstein frame before the bounce [27]. Let us consider the ekpyrotic scenario whose potential is given by $U = -U_0 \exp(-\sqrt{2/p} \varphi)$ with $U_0 < 0$ and $0 < p < 1$. In this case, the contracting phase is characterized by the scale factor, $a \propto (-\tau)^{\beta} \propto (-\tau)^{(1-p)/p}$, with $t < 0$. Since the brane modulus, $\varphi$, evolves as $e^{\varphi} \propto \{(1-p)(-\tau)^{1-p}\}^{\sqrt{2/p}}$, we get the same general formula given by Eq. (14) for the fluctuation $\delta \chi$. In the case of minimal coupling ($\beta = 0$), one has $n_{\delta \chi} = (3-p)/(1-p) \approx 3$ for $p \approx 0$. This is the same blue spectral index as the curvature perturbation, $R$, obtained in the system of the brane-modulus only [28]. We can have $n_{\delta \chi} = 1$ for $p = 2/3$ [29], but the system is found to be unstable in this case [27]. When $\beta = -\sqrt{2/p}$ or $\beta = -(2-3p)/\sqrt{2/p}$, metric perturbations can be scale-invariant provided that $\delta \chi$ fluctuations are sufficiently converted to $\Phi$ through the perturbed decay rate.

The PBB scenario corresponds to the vanishing potential with $p = 1/3$. It is convenient to use the relation $\phi = -\sqrt{2}\varphi$ between the dilaton $\phi$ in the PBB case and the field $\varphi$ in the ekpyrotic case [27]. This is equivalent to replacing $\beta$ in Eq. (11) for new beta $\tilde{\beta}$, as $\beta = -\sqrt{2} \tilde{\beta}$. Then the spectral index for $\delta \chi$ is given by $n_{\delta \chi} = 4 - \sqrt{3}|\tilde{\beta}|$, which allows for scale-invariant spectra for $|\tilde{\beta}| = \sqrt{3}$. Since $|\tilde{\beta}| = 2$ for the standard axionic coupling, we have $n_{\delta \chi} \approx 0.54$ in this case [22]. If we take into account the contribution of the modulus kinetic term, $-n(\nabla \alpha)^2$, in the action (11) (here $n$ is the number of compactified dimensions), it is possible to obtain scale-invariant spectra even for $\tilde{\beta} = 2$ [22]. When the modulus $\alpha$ evolves as $e^{\alpha} \propto (-\tau)^{\alpha}$, we get the spectral index $n_{\delta \chi} = 4 - |\tilde{\beta}| \sqrt{3} - 2n\alpha^2$. The scale-invariant spectra follow for $|\tilde{\beta}| \sqrt{3} - 2n\alpha^2 = 3$.

Of course one must make a numerical analysis in order to check the efficiency of the conversion to adiabatic density perturbations in string-inspired cosmologies with a contracting phase, noncanonical kinetic terms ($\beta \neq 0$) are essentially important to yield scale-invariant spectra in $\Phi$ through the perturbed decay rate coming from $\delta \chi$. The correlated adiabatic and isocurvature perturbations were discussed in Ref. [25] for the same action (11) when $\delta \Gamma = 0$, implying the possibility to generate scale-invariant spectra in entropy field perturbations for $0 < p < 1$. In addition to this, the perturbed decay coupling provides us a new source for isocurvature perturbations, whose conversion to adiabatic perturbations is efficient even when the background field $\chi$ is dynamically negligible.

In PBB and ekpyrotic cosmologies, we have radiation just after the bounce through the decay of scalar fields. The spectra of perturbations can be affected by the details of the background evolution around the bounce as well as the details of the background evolution around the bounce as was recently analyzed in Ref. [30]. It is certainly of interest to investigate the final spectra of $\Phi$ for $\delta \Gamma \neq 0$ by passing through the bounce numerically. Not only in the case of standard inflation, this can also provide us an exciting possibility to generate nearly scale-invariant density perturbations with the amplitude required by observations in the context of string cosmologies.

Acknowledgements – The author is grateful to Bruce Bassett and David Parkinson for useful discussions and comments. He also thanks Anupam Mazumdar for drawing attention to the references [31]. This work is supported from JSPS (No. 049442).