Particle Production on Half S-brane

J. Klusoň *

Institute of Theoretical Physics, University of Stockholm, SCFAB
SE- 106 91 Stockholm, Sweden
and
Institutionen för teoretisk fysik
BOX 803, SE- 751 08 Uppsala, Sweden
E-mail: josef.kluson@teorfys.uu.se

ABSTRACT: In this paper we will study quantum field theory of fluctuation modes around the rolling tachyon solution on non-BPS D-brane effective action. The goal of this paper is to study particle production during the decay of non-BPS D-brane and explore possible relation with minisuperspace calculation. We find that the number of particles produced on half S-brane exponentially grows for large time which suggests that linearised approximation breaks down and also that backreaction of fluctuation field on classical solution should be taken into account.

*On leave from Masaryk University, Brno
1. Introduction

A spacelike brane, or S-brane is almost the same as ordinary D-brane except that one of its transverse dimensions includes time. S-brane can be also seen as time-dependent, soliton-like configurations in a variety of field theories. In string theory, the potential for the open string tachyon field on the world-volume of unstable D-brane leads to S-branes [1] in a time-dependent version of the construction of D-branes as solitons of the open string tachyon. These S-branes can be thought as the creation and subsequent decay of an unstable brane. This process has recently attracted much attention [1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 21, 22, 23, 24, 21, 27, 28, 11, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43] \(^1\). S-branes have been also extensively studied in supergravity approach with potentially interesting cosmological applications [44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66].

Very nice world-sheet construction of S-branes was given in the classical \(g_s = 0\) limit by A. Sen [2, 3, 4] where he introduced class of models in bosonic string theory obtained by perturbing the flat space \(c = 26\) CFT with the exactly marginal deformation

\[
S_{boud} = \lambda \int d\tau \cosh X^0(\tau) ,
\]

(1.1)

where \(X^0\) is time coordinate, \(t\) is a coordinate on the world-sheet boundary and \(\lambda\) is a free parameter in the range \(0 \leq \lambda \leq \frac{1}{2}\) \(^2\). This is family of exact solutions of classical open string theory whose space-time interpretation is that of an unstable brane being created at a time \(X^0 \sim -\tau\) and decaying at a time \(X^0 \sim \tau\) with \(\tau = -\log(\sin(\pi \lambda))\).

As was shown in [5, 6, 7] this time-dependent process of tachyon condensation has many intriguing properties. In particular, it is known that in time-dependent backgrounds there is in general no preferred vacuum and particle production is unavoidable. In [7] the open string vacua on S-brane were studied and it was shown that they have very mysterious properties. In particular, for (1.1) there is open string pair production with a strength characterized by the Hagedorn temperature \(T_H = \frac{1}{4\pi} [5, 6]\). As was argued [5, 6, 7] this temperature arises from the periodicity of the boundary interaction (1.1) in imaginary time.

\(^1\)For the most recent discussion of S-branes and other time-dependent processes in string theory, see [84, 82, 81, 83, 77, 79, 78, 80, 85].

\(^2\)We work in units \(\alpha' = 1\).
Recent intensive work on the dynamics of unstable D-branes in string theory has led to an effective action for the open string tachyon $T$ and massless open string modes $A_\mu$ (the gauge field on the D-brane) and $Y^I$ (the scalar fields describing the location of the D-brane in the transverse directions) [2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21]. As it was shown in [12, 21] the effective action for non-BPS D-brane (3.1), (3.2) reproduces several non-trivial aspects of open string dynamics, for very nice recent discussion, see [8, 12, 13, 14, 11, 15]. These successes lead one to believe that the action (3.1) captures some class of phenomena in the full classical open string theory. As was argued in [12] this action should be thought of as a generalization of the DBI action describing the gauge field $A_\mu$ and scalars $Y^I$ on the brane. On the other hand we should be more careful when we speak about the notion “low energy effective action” in case of unstable D-brane. The fact that the tachyon has negative $m^2$ of the order of the string scale, the notion of an effective action which is normally considered as a result of integration out the heavy modes for describing the dynamics of light modes, is somewhat unclear here. As was recently argued in [15], the situation is even worse by the fact that there are no physical states around the tachyon vacuum and hence the usual method of deriving an effective action-by comparing the $S$-matrix elements computed from string theory with those computed from the effective action-does not work. The question of usefulness of the effective action for non-BPS D-brane was extensively discussed in [12] where it was argued that its usefulness can also be judged by comparing the classical solution of the equation of motion derived from the effective action with the classical solutions in open string theory which are described by boundary conformal field theory. In this respect the effective action constructed in [8, 21, 12] has had some remarkable success.

Motivated by this success of effective action description of the open string phenomena on unstable D-brane and also very nice analysis performed in [5, 6, 7] we asked the question whether there should be some connection between effective action description of fluctuation fields around S-brane, or half S-brane, and minisuperspace description of open strings on the same background. In other words, the goal of this paper is to construct quadratic action for fluctuation field around half S-brane solution on non-BPS D-brane. Since the classical solution describing half S-brane explicitly depends on time we can expect that metric and mass term in the action for fluctuation field will be function of time as well. As is well known from the study of quantum field theory in curved space-time time-dependent background generally leads to the creation of particles during the time evolution. Then we could expect that particles will be produced during unstable D-brane decay as well. In order to study this process we will follow [72, 73, 74, 67, 68].

Let us sketch the main idea of particle creation on half S-brane. We start with non-BPS D-brane in unstable minimum of the tachyon potential in the asymptotic past $t \rightarrow$
This corresponds to the situation when the vacuum value of the tachyon is zero and small fluctuation modes oscillate around it. Half S-brane solution then describes process when the homogeneous tachyon field starts to roll from unstable minimum of the potential \( T = 0 \) at the time \( t = -\infty \) to stable one \( T = \infty \) at asymptotic future \( t = \infty \). This rolling tachyon solution forms background for the fluctuation modes which is manifest-ally time-dependent. It is important to know whether these fluctuations, that were small at the beginning of the rolling tachyon process, remain small during time evolution or if they become large and hence they could affect the classical evolution. Since large value of fluctuation field can be interpreted as creation of large number of particles we will be mainly interested in the time evolution of this quantity. We find that number of particles created during D-brane decay exponentially grows at asymptotic future. The similar behavior of the fluctuation modes was observed in [38, 39, 40], where it was argued that the growing of fluctuation modes quickly destabilize the linearised analysis and hence backreaction of fluctuation field on homogeneous solution should be taken into account. The result that we get in this paper also seems to indicate that the linearised analysis of fluctuation modes is only suitable for the study of fluctuation at the beginning of the rolling tachyon process. This is consistent with the fact that only at the asymptotic past where the tachyon is in its unstable minimum open string perturbative states exist and one can define open string S-matrix. It is also important to stress that the analysis given in this paper is restricted to the pure classical approximation where we consider the string coupling \( g_s \) equal to zero. However it was shown recently in [21, 22, 24, 77, 79] that the coupling between closed and open strings in the rolling tachyon process plays fundamental role which suggests that time-dependent D-brane decay is very complex problem and its study could answer some fundamental questions considering nature of the string theory.

This paper is organized as follows. In the next section (2) we briefly review minisuperspace analysis of half S-brane to show that particle production during D-brane decay is natural process from the point of view of quantum field theory on D-brane. In section (3) we introduce a non-BPS Dp-brane effective action proposed in [21, 12]. Section (4) is devoted to study of fluctuation modes above half S-brane solution on non-BPS D-brane effective action. In conclusion (5) we outline the results obtained in this paper and give their possible interpretation.

### 2. Review of the minisuperspace approximation

In this section we present a short review of minisuperspace approach to the study of S-brane dynamics, following seminal papers [5, 6, 7].

We wish to understand the dynamics of the open string world-sheet theory with a time-dependent tachyon

\[
S = -\frac{1}{4\pi} \int_{\Sigma_2} d^2\sigma \partial^a X^\mu \partial_a X_\mu + \int_{\partial\Sigma_2} d\tau \, m^2 (X^0) .
\]  

\[ \text{(2.1)} \]
For the open bosonic string $m^2 = T$ where $T$ is the space-time tachyon, while for the open superstring $m^2 \sim T^2$ after integrating out world-sheet fermions. We use the symbol $m^2$ to denote the interaction because the coupling (among other effects) impacts a mass to the open string states. We consider three interesting classes described by the marginal interactions

\begin{align*}
m_+^2(X^0) &= \frac{\lambda}{2} e^{X^0} \quad (2.2) \\
m_-^2(X^0) &= \frac{\lambda}{2} e^{-X^0} \quad (2.3) \\
m_S^2(X^0) &= \lambda \cosh X^0. \quad (2.4)
\end{align*}

The first case $m_+^2$ describes the process of brane decay, in which an unstable brane decays via tachyon condensation. The second case describes the time-reverse process of brane creation, in which an unstable brane emerges from the vacuum. The final case describes an S-brane, which is the process of brane creation followed by brane decay. Brane decay can be thought of as the future (past) half of an S-brane, i.e. as the limiting case where the middle of the S-brane is pushed into the infinite past (future).

In [5, 6, 7] this problem has been studied using the minisuperspace analysis in which the effect of the interaction is simply to give a time-dependent shift to the masses of all the open string states. In the minisuperspace approximation only the zero-mode dependence of the interaction $m^2(X^0)$ is considered. In this case we can plug in the usual mode solution for the free open string with oscillator number $N$ to get an effective action for the zero modes

$$S = \int d\tau \left( -\frac{1}{4} \dot{X}^{\mu} \dot{X}_\mu + (N - 1) + 2m^2(x^0) \right). \quad (2.5)$$

This is the action of a point particle with a time-dependent mass. Here $x^\mu(\tau)$ is the zero mode part of $X^\mu(\sigma, \tau)$, and the second term in is an effective contribution from the oscillators, including the usual normal ordering constant. From upper action we can write down the Klein-Gordon equation for the open string wave function $\phi(t, x)$,

$$\left( \partial^\mu \partial_\mu - 2m^2(t) - (N - 1) \right) \phi(t, x) = 0, \quad (2.6)$$

where $(t, x)$ are the space-time coordinates corresponding to the world-sheet fields $(X^0, X^i)$. This is the equation of motion for a scalar field with time-dependent mass. Quantum field theory of the scalar field with time-dependent mass is similar with with the QFT in time-dependent background [69, 70, 71] hence we we should make few remarks about such field theories. Time translation invariance has been broken, so energy is not conserved and there is no preferred set of positive frequency modes. As a consequence this leads in generally to particle creation. The probability current $j_\mu = i(\phi^* \partial_\mu \phi - \partial_\mu \phi^* \phi)$ is still conserved, allowing us to define the Klein-Gordon inner product

$$\langle f | g \rangle = i \int d\Sigma(\Sigma)^\mu (f^* \partial_\mu g - \partial_\mu f^* g), \quad (2.7)$$
where Σ is a spacelike slice. This norm does not depend on the choice of Σ if \( f \) and \( g \) solve the wave equation. Normalized positive frequency modes are chosen to have \( \langle f|f \rangle = 1 \). Negative frequency modes are complex conjugates of positive frequency modes, with \( \langle f^*|f^* \rangle = -1 \). There is a set raising and lowering operators associated to each choice of mode decomposition — these operators obey the usual oscillator algebra if the corresponding modes are normalized with respect to (2.7). We also define a vacuum state associated to each mode decomposition — it is the state annihilated by the corresponding lowering operators.

To illustrate this idea we give an example of the half S-brane corresponding to the decay of unstable D-brane so that (2.6) contains following time-dependent mass term

\[
m^2(t) = \frac{\lambda}{2} e^t. \tag{2.8}
\]

Since in the past \( t \to -\infty \) we have ordinary D-brane with perturbative open string spectrum it is natural to consider \( |\text{in} \rangle \) vacuum as a vacuum with no particle present. Expanding field in plane waves

\[
\phi(t, x) = e^{ipx} u(t) \tag{2.9}
\]

the wave equation becomes

\[
(\partial^2_t + \lambda e^t + \omega^2)u = 0, \omega^2 = p^2 + N - 1 \tag{2.10}
\]

This is a form of Bessel’s equation. It has normalized, positive frequency solutions

\[
u^{\text{in}} = \lambda \frac{\Gamma(1 - 2i\omega)}{\sqrt{2\omega}} J_{-2i\omega}(2\sqrt{\lambda} e^{t/2}) \tag{2.11}
\]

where superscript \( \text{in} \) and \( \text{out} \) on a wave function denotes solutions that are purely positive frequency when \( t \to -\infty \) or \( t \to \infty \). Using properties of Bessel’s functions we can easily show that \( u^{\text{in}} \) approaches flat positive frequency plane waves in the far past \( t \to -\infty \)

\[
u^{\text{in}} \sim \frac{1}{\sqrt{2\omega}} e^{-i\omega t}. \tag{2.12}
\]

We can also consider the wave functions

\[
u^{\text{out}} = \sqrt{\frac{\pi}{2}} (ie^{2\pi\omega})^{-1/2} H_{-2i\omega}^{(2)}(2\sqrt{\lambda} e^{t/2}), \tag{2.13}
\]

that are purely positive frequency in the far future \( t \to \infty \)

\[
u^{\text{out}} \sim \frac{\lambda^{-1/4}}{\sqrt{2}} \exp \left\{ -t/4 - 2i\sqrt{\lambda} e^{t/2} \right\}. \tag{2.14}
\]

Generally \( u^{\text{out}} \) and \( u^{\text{in}} \) are related by celebrated Bogolubov transformations

\[
u^{\text{out}} = Au^{\text{in}} + Bu^{\text{in}} \tag{2.15}
\]
where coefficients $A, B$ can be determined from known solutions $u^\text{in}, u^\text{out}$. In particular, for modes given above it can be shown that
\begin{equation}
A = e^{2\pi \omega + \pi i/2} B^* = \sqrt{\omega} e^{\pi \omega - \pi i/4} \left( \frac{\lambda^{-i\omega}}{\sinh 2\pi \omega \Gamma(1 - 2i\omega)} \right) .
\end{equation}
These coefficients obey unitarity relation $|A|^2 - |B|^2 = 1$. The relation (2.15) between in and out modes implies the relation between in and out creation and annihilation operators
\begin{equation}
a_{\text{in}} = A a_{\text{out}} + B^* a_{\text{out}}^\dagger .
\end{equation}
From this relation we see that the condition $a_{\text{in}} |\text{in}\rangle = 0$ implies that $|\text{in}\rangle$ is squeezed state [7]. Physically, this is the statement that particles are produced during brane decay: if we start in a state with no particles at $t \to -\infty$, there will be many particles at time $t \to \infty$. The density of particles of momentum $p$ is given
\begin{equation}
n_p = |\gamma|^2 = e^{-4\pi \omega} .
\end{equation}
We see that despite the fact that $|\text{in}\rangle$ is pure state this is precisely the Boltzmann density of states at temperature $T_H = 1/4\pi$. In string units, $T_H$ is Hagedorn temperature. As was argued in [5, 6, 7] the fact that this temperature is so high means that $g_s$ corrections are likely important even for $g_s \to 0$.

The main conclusion which we can get from this brief review of minisuperspace analysis of S-branes is that in the limit $g_s \to 0$ there is a particle production during D-brane decay. We would like to see the similar behavior from the effective action point of view. We will discuss this problem in next sections.

3. Non-BPS D-brane effective action

The dynamics of the tachyon field $T$ on non-BPS D-brane can be described by effective field theory where the value of $T$ and its derivatives satisfy some conditions [11, 12, 13]. The effective action for real tachyon $T$ on non-BPS Dp-brane in Type II string theory is expected in the form [16, 17, 18, 19]
\begin{equation}
S = \int d^{p+1}x L = -M_p \int d^{p+1}x \sqrt{1 + \eta^{\mu\nu} \partial_\mu T \partial_\nu T} ,
\end{equation}
where $M_p$ is tension of non-BPS Dp-brane \footnote{We are using the convention where $\eta_{\mu\nu} = \text{diag}(-1,1,\ldots,1)$, $\mu, \nu = 0,\ldots,p$, $\alpha, \beta = 1,\ldots,p$ and the fundamental string tension has been set equal to $(2\pi)^{-1}$ in the calculation of the one-loop effective action.}. Another form of the effective action for non-BPS D-brane has been obtained recently in [12]
\begin{equation}
S = \int d^{p+1}x L = -M_p \int d^{p+1}x V(T) \sqrt{1 + \frac{T^2}{2} + \eta^{\mu\nu} \partial_\mu T \partial_\nu T} , V^2(T) = \frac{1}{1 + \frac{1}{2} T^2} ,
\end{equation}
(3.2)
where the tachyon field $T$ given in (3.2) is related to $\mathcal{T}$ in (3.1) by the field redefinition

$$\frac{T}{\sqrt{2}} = \sinh \frac{\mathcal{T}}{\sqrt{2}}. \quad (3.3)$$

We will concentrate in this paper on the action (3.2) since it has the tachyon solution of equation of motion which has the same form as classical solution in open string theory that in turn is described by boundary conformal field theory (BCFT). For that reason we believe that field theory for fluctuation modes in the effective action (3.2) could have some relation to the quantum field theory reviewed in the previous section that arises as minisuperspace approximation of exact BCFT.

We would like also study the behavior of massless modes on non-BPS D-brane effective action during D-brane decay. For that reason we should include them in (3.2). In order to do this we use an equivalence between (3.2) and (3.1) where the action (3.1) containing massless modes is well known [16, 17, 18, 19]

$$S = -M_p \int d^{p+1}x \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu Y^I \partial_\nu Y^J \delta_{IJ} + F_{\mu\nu} + \partial_\mu \mathcal{T} \partial_\nu \mathcal{T})},$$

(3.4)

where $A_\mu$ and $Y^I$ for $\mu, \nu = 0, \ldots, p, I = p + 1, \ldots, 9$ are the gauge and the transverse scalar fields on the world-volume of the non-BPS D-brane. If we expect that the field redefinition (3.3) remains the same even with nonzero massless fields and also that these massless modes are the same in both forms of the action we can write

$$T = \sqrt{2} \sinh \frac{\mathcal{T}}{\sqrt{2}}, \quad V(\mathcal{T}) = \frac{1}{\cosh \frac{\mathcal{T}}{\sqrt{2}}} = \frac{1}{\sqrt{1 + \frac{T^2}{2}}}, \quad \partial_\mu \mathcal{T} = \frac{\partial_\mu T}{\sqrt{1 + \frac{T^2}{2}}},$$

(3.5)

and hence we obtain generalized form of (3.2)

$$S = -M_p \int d^{p+1}x V(T) \sqrt{-\det A},$$

(3.6)

where

$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu Y^I \partial_\nu Y^J \delta_{IJ} + F_{\mu\nu} + V^2(T) \partial_\nu \mathcal{T} \partial_\mu \mathcal{T}.$$

$$V(T) = \frac{1}{\sqrt{1 + \frac{T^2}{2}}}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

(3.7)

Action (3.6) will be useful when we will discuss the massless mode on half S-brane. For classical solutions that depend on $T$ only the action (3.2) is sufficient so that the equation of motion that arises from it is

$$-\frac{T}{(1 + \frac{T^2}{2})^2} \sqrt{B} + \frac{T}{2(1 + \frac{T^2}{2}) \sqrt{B}} - \partial_\mu \left( \frac{\eta^{\mu\nu} \partial_\nu T}{(1 + \frac{T^2}{2}) \sqrt{B}} \right) = 0,$$

(3.8)

where

$$B = 1 + \frac{T^2}{2} + \eta^{\mu\nu} \partial_\mu T \partial_\nu T.$$

(3.9)
Solution of (3.8) that is interpreted as half S-brane is given by

\[ T_c = a \sqrt{2} e^{\sqrt{2} t}, \]  

(3.10)

where \( a \) is some constant which can be set to 1 by time translation[21].

In the next section we present an analysis of the fluctuation modes above classical solution (3.10).

4. Analysis of fluctuations

In this section we will analyse the fluctuation field around the classical half S-brane solution (3.10). To do this we write the tachyon field \( T \) that appears in (3.2) as

\[ T(t, x) = T_c(t) + \phi(t, x), \]  

(4.1)

where \( T_c \) is given in (3.10) and \( \phi(t, x) \) is fluctuation field around \( T_c \) where we presume that it is small with respect to the classical solution.

We insert (4.1) into (3.2) and perform an expansion with respect to \( \phi \) up to second order. As a result we obtain a quadratic action for free massive scalar field where metric and mass term are functions of classical solution \( T_c \). More precisely, let us write

\[ S = - \int dx L(T_c + \phi) = - \int dx L(T_c) - \int dx \left( \frac{\delta L(T_c)}{\delta T} \phi + \frac{\delta L(T_c)}{\delta \partial \mu T} \partial \mu \phi \right) - \frac{1}{2} \int dx \left( \frac{\delta^2 L(T_c)}{\delta T \delta T} \phi^2 + 2 \frac{\delta^2 L(T_c)}{\delta T \delta \partial \mu T} \partial \mu \phi + \frac{\delta^2 L(T_c)}{\delta \partial \mu T \delta \partial \nu T} \partial \mu \partial \nu \phi \right) + \ldots, \]  

(4.2)

where dots mean terms of higher order in \( \phi \). Using equation of motion and integration by parts we can easily show that the expression on the second line in (4.2) is equal to zero. The quadratic effective action for \( \phi \) is then given on the third line in (4.2). We can rewrite it in more familiar form using

\[ \int dx \frac{\delta^2 L(T_c)}{\delta T \delta \partial \mu T} \phi \partial \mu \phi = \int dx \partial \mu \left[ \frac{\delta^2 L(T_c)}{\delta T \delta \partial \mu T} \phi^2 \right] - \int dx \partial \mu \left[ \frac{\delta^2 L(T_c)}{\delta T \delta \partial \mu T} \right] \phi^2 - \frac{1}{2} \int dx \partial \mu \left[ \frac{\delta^2 L(T_c)}{\delta T \delta \partial \mu T} \right] \phi^2 \Rightarrow \int dx \frac{\delta^2 L(T_c)}{\delta T \delta \partial \mu T} \phi \partial \mu \phi = - \frac{1}{2} \int dx \partial \mu \left[ \frac{\delta^2 L(T_c)}{\delta T \delta \partial \mu T} \right] \phi^2, \]  

(4.3)

where as usual we do not carry about boundary terms. Then we obtain the action for fluctuation modes up to second order in \( \phi \)

\[ S = - \frac{1}{2} \int dt dx \left[ G^{\mu \nu} (T_c) \partial \mu \phi \partial \nu \phi + M^2 (T_c) \phi^2 \right], \]
\[ G^{\mu\nu}(T_c) = \frac{\delta^2 L(T_c)}{\delta \partial_\mu T \delta \partial_\nu T}, \quad M^2(T_c) = \frac{\delta^2 L(T_c)}{\delta T \delta T} - \partial_\mu \left[ \frac{\delta^2 L(T_c)}{\delta T \partial_\mu T} \right]. \] (4.4)

For half S-brane solution (3.10) we obtain from (3.2) following metric and mass term that appear in (4.4)

\[
G^{00} = -\frac{M_p V^2}{\sqrt{B}} \frac{\dot{T}^2}{(B)^{3/2}} = -M_p,
\]
\[
G^{ab} = \frac{M_p V^2 \delta^{ab}}{\sqrt{B}} = \frac{M_p \delta^{ab}}{1 + e^{\sqrt{2} t}} \equiv M_p G(t) \delta^{ab},
\]
\[
M^2 = -\frac{M_p^2}{2},
\] (4.5)

where for (3.10) we have \( B = 1 + \frac{T^2}{2} - \dot{T}^2 = 1 \). According to (4.5) the action for fluctuation modes around half S-brane solution has the form

\[
S = -\frac{M_p}{2} \int dt dx \left( -\partial_t \phi \partial_t \phi + \frac{1}{1 + e^{\sqrt{2} t}} \partial_i \phi \partial_i \phi - \frac{1}{2} \phi^2 \right). \] (4.6)

This form of the action will be starting point for the analysis of particle production on half S-brane. We will show that (4.6) has similar form as the action describing scalar field with time-dependent mass. Such quantum field theories are well known, for very nice and detailed discussion, see in particular [72, 73, 68, 67].

To begin with, we should determine the conjugate momentum to \( \phi \) and the Hamiltonian that arise from (4.6)

\[
\Pi(t, x) = \frac{\delta L}{\delta \dot{\phi}(t, x)} = M_p \dot{\phi}(t, x),
\]
\[
\mathcal{H}(t, x) = \Pi(t, x) \dot{\phi}(t, x) - L = \frac{1}{2} \left( \frac{\Pi^2}{M_p} + G^{ab}(t) \partial_a \phi \partial_b \phi + M^2 \phi^2 \right),
\]
\[
H(t) = \int dxdt \mathcal{H}(t, x). \] (4.7)

Since the effective metric and mass are functions of time only it is natural to write operator \( \phi(t, x) \) as [73]

\[
\phi(t, x) = \int dke^{i k \cdot x}, \quad \Pi(t, x) = M_p \int dke^{i k \cdot x}, \] (4.8)

where we have included the normalization factor \( \frac{1}{(2\pi)^p} \) into the definition of \( dk \). When we insert (4.8) into the Hamiltonian we get

\[
H(t) = \frac{M_p}{2} \int dk \left( P_k P_{-k} + \left[ G(t) k^2 + \frac{M^2}{M_p} \right] Q_k Q_{-k} \right). \] (4.9)
Condition \(\phi(t, x) = \phi^\dagger(t, x)\) implies \(Q^\dagger_k = Q_{-k}\) and consequently the Hamiltonian for the scalar field \(\phi\) can be seen as collection of Hamiltonians \(H_k\) of quantum oscillators with complex coordinates \(Q_k\)

\[
H = M_p \int d\mathbf{k} H_k , H_k = \frac{1}{2} \left( P_k P^\dagger_k + \Omega_k^2(t) Q_k Q^\dagger_k \right)
\]  

(4.10)

and with the time-dependent frequencies

\[
\Omega^2_k(t) = G(t) k^2 + \frac{M^2}{M_p} , \Omega_k = \Omega_{-k} , k^2 = \delta_{ab} k_a k_b .
\]  

(4.11)

Canonical equal time commutation relations for scalar field

\[
\begin{align*}
[\phi(t, x), \Pi(t, y)] &= i \delta(x - y) , \\
[\phi(t, x), \phi(t, y)] &= 0 , \\
[\Pi(t, x), \Pi(t, y)] &= 0 ,
\end{align*}
\]

(4.12)

imply following commutation relations between \(Q_k, P_k\)

\[
[Q_k, P_{k'}] = i \delta(k + k') , [Q_k, Q_{k'}] = [P_k, P_{k'}] = 0 .
\]  

(4.13)

In the following we will review the basic properties of quantum oscillators with time-dependent frequencies. We will mainly follow [73].

We would like to go to a time-dependent frame in Hilbert space in which the Hamiltonian \(H_k\) is diagonal at every moment of time. For that reason we define time-dependent operators \(a_k(t)\) and \(a^\dagger_k(t)\) by

\[
a_k = \frac{e^{i \int \Omega_k dt}}{\sqrt{2 \Omega_k}} \left( Q_k \Omega_k + i P_k \right) , \\
a^\dagger_k = \frac{e^{-i \int \Omega_k dt}}{\sqrt{2 \Omega_k}} \left( Q^\dagger_k \Omega_k - i P^\dagger_k \right) .
\]  

(4.14)

These operators are Hermitian conjugate and have the standard commutation relation of creation-annihilation operators

\[
[a_k, a^\dagger_{k'}] = \delta(k - k') .
\]  

(4.15)

Using these operators one can write

\[
\begin{align*}
Q_k &= \frac{1}{\sqrt{2 \Omega_k}} \left( e^{-i \int \Omega_k dt} a_k + e^{i \int \Omega_k dt} a^\dagger_{-k} \right) , \\
P_k &= \frac{\sqrt{2 \Omega_k}}{2i} \left( e^{-i \int \Omega_k dt} a_k - e^{i \int \Omega_k dt} a^\dagger_{-k} \right) .
\end{align*}
\]

(4.16)
so that the Hamiltonian $H_k$ has the form

$$H_k = \frac{\Omega_k}{2} \left(1 + a_k a_k^\dagger + a_{-k}^\dagger a_{-k}\right). \quad (4.17)$$

The orthonormal frame in Hilbert space which diagonalizes the Hamiltonian $H_k$ is given by acting creation operators $a_k^\dagger$ on time dependent vacuum state $|0_i\rangle$ which is annihilated by the operator $a_k(t)$. The operators $a_k(t), a_k^\dagger(t)$ obey following equation of motion

$$\dot{a}_k = \frac{\dot{\Omega}_k}{2\Omega_k} e^{2i\int \Omega_k dt} a_{-k}^\dagger,$$

$$\dot{a}_k^\dagger = \frac{\dot{\Omega}_k}{2\Omega_k} e^{-2i\int \Omega_k dt} a_{-k}. \quad (4.18)$$

The solution of (4.18) can be written in terms of constant creation and annihilation operators $a_{0,k}^\dagger, a_{0,k}$ as follows

$$a_k(t) = \alpha_k(t)a_{0,k} + \beta_k^*(t)a_{0,-k}^\dagger,$$

$$a_{-k}^\dagger(t) = \beta_k(t)a_{0,k} + \alpha_k^*(t)a_{0,-k} \quad (4.19)$$

If we insert (4.19) to (4.18) we easily get

$$\dot{\alpha}_k = \frac{\dot{\Omega}_k}{2\Omega_k} e^{2i\int \Omega_k dt} \beta_k(t), \quad \dot{\beta}_k = \frac{\dot{\Omega}_k}{2\Omega_k} e^{-2i\int \Omega_k dt} \alpha_k(t). \quad (4.20)$$

System (4.19) represents so named Bogolubov transformation between two pairs of creation-annihilation operators. If the oscillator initially ($t = -\infty$) is in the vacuum state then its state $|0_i\rangle$ is annihilated by the operator $a_{0,k}$ and the initial condition for functions $\alpha_k(t), \beta_k(t)$ are

$$|\alpha_k(-\infty)| = 1, \quad |\beta_k(-\infty)| = 0. \quad (4.21)$$

At the moment $t$ it will not be in the vacuum state $|0_i\rangle$ annihilated by the operator $a_k(t)$. Recall that we are working in Heisenberg representation where the state does not evolve. There is relation between the states considered

$$|0_0\rangle = \prod_k \frac{1}{\sqrt{|\alpha_k(t)|}} \exp \left(\frac{\beta_k^*(t)}{2\alpha_k(t)} a_k^\dagger(t)a_{-k}^\dagger(t)\right) |0_i\rangle. \quad (4.22)$$

At the moment $t$ the average number of particles produced in the $k$-th mode is

$$N_k(t) = \langle 0_0 |a_k^\dagger(t)a_k(t)|0_0\rangle = |\beta_k(t)|^2. \quad (4.23)$$

In Heisenberg representation the equations of motion for $Q_k, P_k$ are

$$\dot{Q}_k = i[H_k, Q_k] = P_k, \quad \dot{P}_k = -\Omega_k^2(t)Q_k. \quad (4.24)$$
Their solutions can be written using operators $a_{0,k}$, $a_{0,-k}^\dagger$ as follow

$$Q(t) = Q^{(-)}_k(t)a_{0,k} + Q^{(+)}_k(t)a_{0,-k}^\dagger,$$  \hspace{1cm} (4.25)

where modes $Q^{(\pm)}_k$, $Q^{(-)}_k(t) = Q^{(+)}_{-k}$ obey differential equation

$$\ddot{Q}^{(\pm)}_k + \Omega^2_k(t)Q^{(\pm)}_k = 0.$$  \hspace{1cm} (4.26)

We can easily express $\beta_k(t)$ in terms of these modes

$$\beta_k(t) = \frac{1}{\sqrt{2W_k}} e^{-i \int \Omega_k dt} \left( \Omega_k Q^{(-)}_k - i \dot{Q}^{(-)}_k \right)$$ \hspace{1cm} (4.27)

so that the number $N_k(t)$ of particle produced is equal to

$$N_k(t) = \frac{\Omega_k}{2} \left( \frac{\dot{Q}^{(-)}_k \dot{Q}^{(+)}_k}{\Omega^2_k} + Q^{(-)}_k Q^{(+)}_k \right) - \frac{1}{2}.$$ \hspace{1cm} (4.28)

Now we apply the formalism reviewed above to the case of fluctuation field around half S-brane solution (4.6). Since the number $N_k(t)$ of particle produced (4.28) is function of $Q^{(\pm)}_k$ obeying (4.26), we must solve this equation. In case when we do not need to know an exact solution the most efficient method to solve (4.26) is to use WKB approximation (For review of the application of this method in QFT in curved space-time, see [69], for recent discussion, see [74].) In fact, we are mainly interested in estimation of $N_k(t)$ at asymptotic future so that the WKB approximation can be applied. Setting

$$Q^{(-)}_k = \frac{1}{\sqrt{2W_k}} \exp(-i \int dt W_k)$$ \hspace{1cm} (4.29)

one can see that the solution of equation (4.26) is equivalent to the condition

$$W^2_k = \Omega^2_k - \frac{1}{2} \left( \frac{\dot{W}_k^2}{W_k} - \frac{3}{2} \frac{\dot{W}_k^2}{W_k^2} \right).$$ \hspace{1cm} (4.30)

If following condition holds

$$\left| \frac{Q_k}{\Omega_k^2} \right| \ll 1, \quad Q_k \equiv \frac{1}{2} \left( \frac{\dot{\Omega}_k}{\Omega_k} - \frac{3 \dot{\Omega}_k^2}{2 \Omega_k^2} \right)$$ \hspace{1cm} (4.31)

we can write

$$W_k(t) = \Omega_k(t).$$ \hspace{1cm} (4.32)

Let us apply WKB approximation for equation (4.26), following very nice analysis given in [74]. Firstly we observe that there is a time $t_*$ when $\Omega_k^2$ vanishes

$$\Omega_k^2(t_*) = 0 \Rightarrow e^{\sqrt{\Omega_*}} = 2\omega^2_0, \omega^2_0 = k^2 - \frac{1}{2},$$ \hspace{1cm} (4.33)

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i.e. \( t_\ast \) is classical turning point. According to [74] we define region I as the region such that \( \Omega^2_k > 0 \) and region II the region where \( \Omega^2_k < 0 \). At some time \( t \) deep in region I the WKB approximation of \( Q_k^{(-)} \) can be written as

\[
Q_k^{(-)} = \frac{A}{\sqrt{2\Omega_k}} \exp(-i \int_{t_i}^{t} dt \Omega_k) ,
\]

where \( t_i \) is the initial time at which the normalization is performed. We can also easily see that deep in region I the condition (4.31) is valid. Since for \( t \to -\infty \) the half S-brane solution (3.10) tends to zero corresponding to the original non-BPS D-brane it is natural to demand that for \( t \to -\infty \) \( Q_k^{(-)} \) approaches the plane wave mode

\[
Q_k^{(-)} = \frac{1}{\sqrt{2\omega_{0,k}}} \exp(-i\omega_{0,k}(t - t_i)) .
\]

In fact, deep in the region I we have

\[
\Omega^2_k = \omega^2_{0,k} - k^2 e^{\sqrt{2t}}
\]

and hence \( Q_k^{(-)} \) is equal to

\[
Q_k^{(-)} = \frac{A}{\sqrt{2\omega_{0,k}}} e^{-i\omega_{0,k}(t-t_i)+i\frac{k^2}{2\sqrt{2\omega_{0,k}}}(e^{\sqrt{2t}}-e^{\sqrt{2t_i}})} .
\]

Comparing with (4.34) we immediately obtain the value of normalization constant \( A \) equal to 1. We must also stress that we are considered the scalar modes with the initial frequency \( \omega^2_{0,k} = k^2 - \frac{1}{2} > 0 \) since these correspond to the standard fluctuation modes on non-BPS D-brane in the past infinity. For modes that obey \( k^2 - \frac{1}{2} < 0 \) the analysis is much more complicated, as was shown in [75, 76].

In the region II the situation is different. In this case \( \Omega_k \) becomes complex. However this does not prevent to use WKB approximation [74]. If we write \( \Omega_k(t) \) as \( \Omega_k(t) = i|\Omega_k(t)| \) then the solution in region II in WKB approximation is

\[
Q_k^{(-)}(t) = \frac{C_+}{|\Omega_k(t)|^{1/2}} \exp \left[ \int^t dt' |\Omega_k(t')| \right] + \frac{C_-}{|\Omega_k(t)|^{1/2}} \exp \left[ - \int^t dt' |\Omega_k(t')| \right] .
\]

This solution should be matched on the solution in the region I. At the turning point \( t = t_\ast \) the WKB approximation breaks down and we must use the usual WKB procedure [74] and approximate the potential around the point \( t_\ast \) by a straight line such that

\[
\Omega_k^2(t) \approx -\alpha(t - t_\ast) ,
\]

where

\[
\alpha = -\frac{d\Omega_k^2(t_\ast)}{dt} = 2\sqrt{2\omega_{0,k}^2(\omega_{0,k}^2 + \frac{1}{2})} > 0 .
\]

The solutions of the equation of motion with such a potential are given in terms of Airy functions of first and second kinds

\[
Q_k^{(-)} = B_1Ai(s) + B_2Bi(s) , s \equiv \alpha^{1/3}(t - t_\ast) .
\]
Now we use the asymptotic behavior of the Airy functions to calculate the relation between $B_1$, $B_2$ with $A$ on one hand and $C_\pm$ on the other. In region I, for a value of $t$ not too far from $t_*$, (4.41) can be written as

\[ Q_{k,1}^{(-)}(t) \approx \frac{\alpha^{-\frac{1}{6}}}{2\sqrt{\pi}}|t - t_*|^{-\frac{1}{4}} \left[ (B_1 - iB_2) \exp \left( \frac{2i}{3} \alpha^\frac{1}{2} |t - t_*|^\frac{3}{2} + \frac{\pi i}{4} \right) + 
\right.
\]
\[ \left. + (B_2 + iB_1) \exp \left( -\frac{2i}{3} \alpha^\frac{1}{2} |t - t_*|^\frac{3}{2} - \frac{\pi i}{4} \right) \right] , \] 

(4.42)

while in region II, under the same conditions, the function $Q_{k,II}^{(-)}(t)$ can be expressed as

\[ Q_{k,II}^{(-)}(t) \approx \frac{\alpha^{-\frac{1}{6}}}{2\sqrt{2}}(t - t_*)^{-\frac{1}{4}} \left\{ \frac{B_1}{2} \exp \left[ -\frac{2}{3} \alpha^\frac{1}{2} (t - t_*)^\frac{3}{2} \right] + B_2 \exp \left[ \frac{2}{3} \alpha^\frac{1}{2} (t - t_*)^\frac{3}{2} \right] \right\} . \] 

(4.43)

Now we evaluate in region I the solution (4.34) for the potential (4.39). The integral in the exponent can be written as

\[ \int_{t_1}^{t} dt' \Omega_k(t') = \int_{t_1}^{t_*} dt' \Omega_k(t') + \int_{t_*}^{t} dt' \Omega_k(t') \equiv \Phi_k + \int_{t_*}^{t} dt' \Omega_k(t') . \] 

(4.44)

The frequency (4.39) is used in the second integral only, assuming that $t$ is not too far away from $t_*$. The quantity $\Phi_k$ is just a number and its calculation would require the knowledge of $\Omega_k$ in the whole region I. However as we will see it does not enter in final result and therefore we are not interested in its value. Then we find

\[ Q_{k,1}^{(-)}(t) = \alpha^{-\frac{1}{4}}|t - t_*|^{-\frac{1}{4}} \exp \left( \frac{2i}{3} \alpha^\frac{1}{2} |t - t_*|^\frac{3}{2} + i\Phi \right) \] 

(4.45)

so that comparing with (4.42) we find

\[ B_1 = iB_2 , B_2 = \sqrt{\pi} \alpha^{-\frac{1}{6}} e^{\varphi_k - \frac{\pi i}{4}} . \] 

(4.46)

In the same way we can establish the link between $C_+, C_-$ and $B_1, B_2$. This can be done in the same way as in the region I. If we define $\Psi_k$ as

\[ \Psi_k \equiv \int_{t_*}^{t_f} |\Omega_k(t')|dt' \] 

(4.47)

then (4.38) reduces to the expression

\[ Q_{k,II}^{(-)}(t) = \alpha^{-\frac{1}{4}}(t - t_*)^{-\frac{1}{4}} \left\{ C_+ e^{\Psi_k} \exp \left[ -\frac{2}{3} \alpha^\frac{1}{2} (t - t_*)^\frac{3}{2} \right] + 
\right. \]
\[ \left. + C_- e^{-\Psi_k} \exp \left[ \frac{2}{3} \alpha^\frac{1}{2} (t - t_*)^\frac{3}{2} \right] \right\} . \] 

(4.48)

By comparison with (4.43) we get

\[ C_+ = \frac{B_2}{2\sqrt{\pi}} \alpha^{1/6} e^{-\psi_k + i\pi/2} , C_- = \frac{B_2}{\sqrt{\pi}} \alpha^{\frac{1}{6}} e^{\psi_k} . \] 

(4.49)
The upshot of this analysis is that generally both coefficients $C_+, C_-$ are nonzero when we match the solution in region II to the solution in region I that corresponds to the plane wave at asymptotic past $t \to -\infty$. For $t \gg t_*$ the leading term in $Q_k^{(-)}$ is proportional to $C_-$ so that we have

$$Q_k^{(-)}(t) = \frac{B_2}{\sqrt{\pi}|2\Omega_k|^2} \alpha^\frac{1}{2} e^{\Psi_k} e^{-\frac{1}{2} \int_{t}^{t'} dt' |\Omega_k(t')|}$$

and consequently the number $N_k(t)$ of particle produced for large $t$ is

$$N_k(t) = e^{2\Psi_k} \frac{1}{2} e^{-2 \int_{t}^{t'} dt' |\Omega_k(t')|} - \frac{1}{2}.$$ (4.51)

In asymptotic future $t \to t_f$ and we get

$$N_k(t) \sim e^{2\Psi_k} \sim e^{\sqrt{2}t_f},$$ (4.52)

where we have used the fact that for $t \to \infty |\Omega_k| \to \frac{1}{\sqrt{2}}$. This result implies that during D-brane decay infinite number of particles is produced. The result is in agreement with [38, 39, 40], where similar exponential instability of fluctuation modes was found in case of different non-BPS effective actions. It was then argued there that this result implies that the approximation, where we consider fluctuation modes as small with respect to classical condensate and hence we can neglect their backreaction, breaks down very early from the beginning of the time evolution. We observe the same behavior in our approach as well. More precisely, we can easily see that when we insert asymptotic form of fluctuation $\phi \sim e^{t\sqrt{2}}$ to the action (4.6) that this action dominates over the action (3.2) evaluated on half S-brane solution

$$S(T_c) = -M_p \int dt \frac{1}{1 + e^{\sqrt{2}t}}.$$ (4.53)

Then it follows that the approximation when we regard $\phi$ as small fluctuation modes without any backreaction on classical solution breaks down at late times. It would be nice to perform analysis of the fluctuation modes around half S-brane solution when we take this backreaction into account. We hope to return to this problem in future.

4.1. Massless fluctuation on half S-brane

In this section we extend our analysis to the case of the massless fluctuations on half S-brane. We start with the action

$$S = -M_p \int d^{p+1}x V(T) \sqrt{-\det (\eta_{\mu\nu} + \partial_\mu Y^I \partial_\nu Y^J \delta_{IJ} + F_{\mu\nu} + V^2(T) \partial_\mu T \partial_\nu T)}.$$ (4.54)

We will focus on scalar modes $Y^I$ only since for gauge field $A_\mu$ the situation is completely the same. When we perform an expansion of (4.54) around classical solution $T_c$ up to
second order in \( Y^I \) we get

\[
S = -M_p \int d^{p+1}x V(T_c) \sqrt{-\det (\eta_{\mu\nu} + V^2(T_c) \partial_\mu T_c \partial_\nu T_c + \partial_\mu Y^I \partial_\nu Y^J \delta_{IJ})} =
\]

\[
= -M_p \int d^{p+1}x V(T_c) \sqrt{-\det G(T_c) \sqrt{\det (\delta_{\mu\nu} + G^{\mu\nu}(T_c) \partial_\mu Y^I \partial_\nu Y^J \delta_{IJ})}} =
\]

\[
= -M_p \int d^{p+1}x V(T_c) \sqrt{-\det G(T_c) - \frac{M_p}{2} \int d^{p+1}x V(T_c) \sqrt{-\det G(T_c) G^{\mu\nu}(T_c) \partial_\mu Y^I \partial_\nu Y^J \delta_{IJ}}} ,
\]

(4.55)

where

\[
G(T_c)_{\mu\nu} = \eta_{\mu\nu} + V^2(T_c) \partial_\mu T_c \partial_\nu T_c
\]

(4.56)

which for half S-brane solution (3.10) is equal to

\[
G(T_c)_{\mu\nu} = \text{diag} \left( \frac{1 + \frac{T_c^2}{2} - \frac{\dot{T}_c^2}{2}}{1 + \frac{T_c^2}{2}} , 1, \ldots, 1 \right).
\]

(4.57)

Consequently the quadratic effective action for scalar fluctuation modes \( Y^I \) around half S-brane solution has the form

\[
S = -\frac{M_p}{2} \int d^{p+1}x V(T_c) \sqrt{-\det G(T_c) G^{\mu\nu}(T_c) \partial_\mu Y^I \partial_\nu Y^J \delta_{IJ}} =
\]

\[
= -\frac{M_p}{2} \int dx dt \left( -\partial_t Y^I \partial_t Y^I + \frac{\delta_{ab}}{1 + e^{-\sqrt{2} t}} \partial_a Y^I \partial_b Y^I \right)
\]

(4.58)

Let us consider one scalar mode, say \( Y^{p+1} \equiv Y \). From (4.58) we get conjugate momentum to \( Y \)

\[
\Pi(t, x) = \frac{\delta L}{\delta \dot{Y}(t, x)} = M_p \dot{Y}(t, x)
\]

(4.59)

so that the Hamiltonian is

\[
H = \frac{1}{2} \int dx \left( \Pi \dot{Y} - L \right) = \frac{1}{2} \int dx \left( \frac{\Pi^2}{M_p} + M_p G(t)^{ab} \partial_a Y \partial_b Y \right)
\]

(4.60)

If we take \( Y = \int dk Y_k(t) e^{ikx} \) the Hamiltonian reduces to the collection of Hamiltonians \( H_k \) with frequency \( \Omega_k^2(t) \)

\[
H = \int dk H_k , H_k = \frac{1}{2} \left( \frac{\Pi_k \Pi_k^\dagger}{M_p} + \Omega_k^2(t) Y_k Y_k^\dagger \right)
\]

(4.61)

where

\[
\Omega_k^2(t) = \frac{M_p k^2}{1 + e^{\sqrt{2} t}}.
\]

(4.62)
As we know from the previous section in order to determine number \( N_k(t) \) of particle produced at time \( t \) we should solve following equation

\[
\ddot{Q}_k^{(-)}(t) + \Omega_k^2(t)Q_k^{(-)} = 0 ,
\] (4.63)

where \( \Omega_k \) is given in (4.62). As opposite to the case of tachyon fluctuation mode studied in previous section from \( \Omega_k \) given in (4.62) we get

\[
\lim_{t \to \infty} \frac{\dot{\Omega}_k}{\Omega_k} = -\frac{1}{\sqrt{2}}, \quad \lim_{t \to \infty} \Omega_k(t) = 0
\] (4.64)

and hence WKB approximation cannot be used. However in order to estimate number of particles at asymptotic future we does not need to know an exact solution in the whole region, rather we could try to solve (4.63) in asymptotic future. For large \( t \) we can write

\[
\Omega_k^2(t) = k^2 e^{-\sqrt{2}t}
\] (4.65)

so that (4.63) is

\[
\ddot{Q}_k^{(-)} + k^2 e^{-\sqrt{2}t} Q_k^{(-)} = 0 .
\] (4.66)

After substitution \( m = \sqrt{2}k e^{-\frac{\sqrt{2}t}{2}} \) we obtain following equation

\[
m^2 \frac{d^2 Q_k^{(-)}}{dt^2} + m \frac{dQ_k^{(-)}}{dm} + m^2 Q_k^{(-)} = 0 .
\] (4.67)

This is Bessel’s equation of the first kind with the general solution

\[
Q_k^{(-)}(m) = A_1 J_0(m) + A_2 Y_0(m).
\] (4.68)

For \( m \to 0 \) we have

\[
J_0(m) \sim 1 - \frac{m^2}{4}, \quad Y_0(m) \sim \ln m
\] (4.69)

and hence

\[
Q_k^{(-)}(t) \sim A_1(1 - \frac{k^2}{2} e^{-\sqrt{2}t}) + A_2 t .
\] (4.70)

Let us consider for a moment the situation when \( A_2 = 0 \). Then the number \( N_k \) of particle produced for large \( t \) is equal to

\[
N_k \sim \Omega_k \left( \frac{A_1^2 k^4 e^{-2\sqrt{2}t}}{2 \Omega_k^2} + A_1^2 \right) - \frac{1}{2} \sim A_1^2 \frac{e^{-2\sqrt{2}t}}{e^{-\sqrt{2}}} - \frac{1}{2} \sim -\frac{1}{2}
\] (4.71)

which is clearly nonphysical result since the number of particles cannot be negative. For that reason any physical solution must contain term proportional to \( A_2 \). However this term dominates at large time so we can write

\[
Q_k^{(-)} \sim A_2 t .
\] (4.72)
For such a solution the number \( N_k(t) \) of massless particles produced during D-brane decay is equal to

\[
N_k \sim \Omega_k \left( \frac{A_2^2}{\Omega_k^2} + A_2 t^2 \right) - \frac{1}{2} \sim \frac{A_2}{\Omega_k} \sim e^{\sqrt{2} t}. \tag{4.73}
\]

We see that the number of particles that are created during the D-brane decay exponentially grows. This is consequence of the fact that for \( t \to \infty \) the effective frequency \( \Omega_k \) goes to zero. Following [38, 39, 40] we can interpret this exponential growth of particles as breaking of linearised approximation and their backreaction on classical solution should be taken into account in the process of D-brane decay.

### 5. Conclusion

In this paper we have studied the particle creation during the time-dependent process of non-BPS D-brane decay (half S-brane) in the effective field theory approximation. We hoped that the behavior of fluctuation modes around rolling tachyon solution on non-BPS D-brane could reflect the behavior of open string modes as was described using minisuperspace approach [5, 6, 7]. In particular, we have been mainly interested in the question whether particle production occurs during D-brane decay and whether vacuum states of fluctuation modes in asymptotic past and future have similar properties as vacuum states introduced in [5, 6, 7]. While the particle production was unavoidable in our effective action description as well since the action for fluctuation field around half S-brane solution corresponds to the quantum field theory action in time-dependent background, we have find that linearised approximation is not completely sufficient to reproduce results given in [5, 6, 7]. In particular we have shown that number of fluctuation modes exponentially grows for large time which implies that the linearised approximation breaks down. Comparing the effective action for fluctuation field given in this paper and the quantum field theory action given in [5, 6, 7] we immediately see reason why our results are so different. In [5, 6, 7] effective frequency for given mode grows to infinity in the asymptotic future and hence given mode exponentially decays. On the other hand, in the quadratic action for fluctuation field given in previous sections the time-dependent frequency either approach constant value, as in the case of tachyon fluctuation, or approaches zero for the case of massless fluctuations. This behavior of effective frequency results into the exponential growth of number of particles produced during D-brane decay. Similar exponential instability of fluctuations has been observed in [38, 39, 40], where this huge growth of number of particles produced has been interpreted as breaking of the linearised approximation. To conclude we mean that the linearised description of fluctuation modes around half S-brane is only efficient at the beginning of the rolling tachyon process. After some time it breaks down and we should take into account backreaction of fluctuation field on the classical solution.

We must also stress that in this paper, following [5, 6, 7], we restrict to the case \( g_s = 0 \). However recent results given in very interesting papers [21, 22, 24, 77, 79]
show that in order to correctly describe D-brane decay we should take into account interaction between closed and open strings and hence to go beyond the free case $g_s = 0$.

To conclude, we hope that our modest results given in this paper could be helpful for other study of non-BPS D-brane effective action and time-dependent process of D-brane decay.

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