Neutrino Magnetic Moments and Minimal Supersymmetric SO(10) Model

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Abstract

We examine supersymmetric contributions to transition magnetic moments of Majorana neutrinos. We first give the general formula for it. In concrete evaluations, informations of neutrino mass matrix elements including CP phases are necessary. Using unambiguously determined neutrino mass matrices in recently proposed minimal supersymmetric SO(10) model, the transition magnetic moments are calculated. The resultant neutrino magnetic moments with the input soft supersymmetry breaking masses being of order 1 TeV are found to be roughly an order of magnitude larger than those calculated in the standard model extended to incorporate the see-saw mechanism.

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The stellar objects like the Sun, White Dwarfs, Supernovae, and Magnetors have magnetic field roughly from $10^3$ to $10^{15}$ Gauss (G). The motions of neutrinos in the medium of these objects are affected by these magnetic fields if neutrinos have the sufficient magnitude of magnetic moments. Experiments show the following limits on the magnetic moments \[1\],

\[
\begin{align*}
\mu_{\nu_e} &\leq 1.5 \times 10^{-10} \mu_B, \\
\mu_{\nu_\mu} &\leq 6.8 \times 10^{-10} \mu_B, \\
\mu_{\nu_\tau} &\leq 3.9 \times 10^{-7} \mu_B
\end{align*}
\]  

in units of the Bohr magneton, $\mu_B \equiv \frac{e\hbar}{2m_e}$. Astrophysical observations provide severer constraints: from anomalous stellar cooling due to the plasmon decay $\gamma \rightarrow \nu \bar{\nu}$ \[2\],

\[
(\sum_{i,j} |\mu_{ij}|^2)^{1/2} \leq 3 \times 10^{-12} \mu_B,
\]  

$\nu_L \rightarrow \nu_R$ conversion process in SN1987A \[3\] gives

\[
\mu_{\nu_e} < (1 - 4) \times 10^{-12} \mu_B,
\]  

and

\[
\mu_{\nu_e} \leq 3.9 \times 10^{-12} \mu_B
\]  

from solar neutrino observations \[4\] for the moderate solar magnetic field $B = 50kG$ etc. Eq. \[2\] is especially important constraint since it includes all the off-diagonal entries of neutrino magnetic moments. If $\mu_{ij}$ is large enough (but is still satisfied by current upper bound),

\[
\mu_{\nu} \geq 10^{-14} \mu_B,
\]  

the neutrino spin flavor precession (SFP) can occur in the solar \[5\] or in the supernovae \[6\] and we can expect the observable effects. The comprehensive arguments under the assumption of large neutrino magnetic moment are given, for instance, in the text by Raffelt \[7\].

In theoretical point of view, it is interesting to examine how large neutrino magnetic moments are obtained in massive neutrino models. For the Dirac neutrinos in the standard model extended to include right-handed neutrinos, the magnetic moments are found to be \[8\]

\[
\mu_\nu = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} \sim 3.20 \times 10^{-19} \left( \frac{m_\nu}{1\text{eV}} \right) \mu_B,
\]  

which is strongly suppressed owing to the chiral symmetry and the GIM cancellations. If neutrinos are Majorana particles, they can only have transition magnetic moments
(TMMs) because of CPT invariance of the theory. The transition matrix corresponding to TMM such as \( \nu_i(p) \rightarrow \nu_j(p-q) + \gamma(q) \) \((i \neq j)\) is described as

\[
T_{ji} = -ie^\mu \tilde{\nu}_j(p-q) \left[ 2i\Im F_2(q^2)_{ji} \right] \sigma_{\mu
u} q^\nu \nu_i(p) ,
\]  

where \( F_2(q^2)_{ji} \) is the photo-penguin coupling. TMM is defined as \( \mu_{ij} = 2i\Im F_2(q^2)_{ji} \).

Note that both of lepton flavor and CP violations are necessary for non-zero TMM to be induced (except the CP invariant case with relative CP-phase \( \pi \)). For Majorana neutrinos in the standard model extended to incorporate the see-saw mechanism \[9\], TMM is found to be \[10\]

\[
\mu_{ij} = \frac{3eG_F}{16\pi^2\sqrt{2}} (m_i + m_j) \sum_{\alpha=e,\mu,\tau} \Im \left[ (U^\dagger_{MNS})_j\alpha \left( \frac{m_\alpha}{M_W} \right)^2 (U_{MNS})_{\alpha i} \right] ,
\]

where \( m_i \) is the mass eigenvalue of the \( i \)-th generation neutrino, \( U_{MNS} \) is the Maki-Nakagawa-Sakata mixing matrix \[11\], and \( M_W \) is the weak boson mass. Note that in order to evaluate the TMM we need informations of the absolute values of neutrino masses and all the elements in \( U_{MNS} \) including three CP-phases.

If the model is supersymmetric (SUSY) one, there are additional contributions to neutrino magnetic moments through radiative corrections with sparticles running in a loop. It is worth investigating how large SUSY contributions can be. In minimal supersymmetric standard model (MSSM) with Dirac neutrinos, NMMs have been calculated and found to be the same order of magnitude as those in the standard model with Dirac neutrinos \[12\].

In this article, we discuss the case of the MSSM with Majorana neutrinos through the see-saw mechanism. As in the case of the standard model with Majorana neutrinos discussed above, informations of neutrino masses and all the elements of MNS mixing matrix are necessary for the evaluation of neutrino TMM. In addition, we need further informations of neutrino Dirac Yukawa couplings and the heavy right-handed neutrino masses, since soft SUSY breaking parameters at the weak scale depend on them through renormalization group equation (RGE) evolutions. In the following, we evaluate the TMM by using unambiguously determined neutrino mass matrices in recently proposed minimal SUSY SO(10) model \[13\].

Let us first give the general formula of SUSY contributions to TMM\[1\]. The basic theory which we analyze is the MSSM with the right-handed Majorana neutrinos. The superpotential in the leptonic sector is given by

\[
W_Y = Y_{\nu}^{ij}(\nu_R^i)_{i} \ell_j H_u + Y_e^{ij}(e_R^i)_{i} \ell_j H_d + \frac{1}{2} M_{R_{ij}}(\nu_R^i)_{i}(\nu_R^j)_{j} + \mu H_d H_u ,
\]

where the indeces \( i, j \) run over three generations, \( H_u \) and \( H_d \) denote the up-type and down-type MSSM Higgs doublets, respectively, and \( M_R \) is the heavy right-handed Majorana neutrino mass matrix. \( Y_\nu \) and \( Y_e \) are the neutrino Dirac Yukawa matrix and

\[1\] The formulas with the mass insertion approximation have been given in Ref. \[14\].
the charged-lepton Yukawa matrix, respectively. In the following, we work in the basis where $Y_e$ and $M_R$ are real-positive and diagonal matrices: $Y_{ij} = Y_e \delta_{ij}$ and $M_{Rij} = \text{diag}(M_{R1}, M_{R2}, M_{R3})$. The soft SUSY breaking terms in the leptonic sector is described as

$$\mathcal{L}_{\text{soft}} = \bar{\ell}_i (m_{\tilde{\ell}}^2)_{ij} \tilde{\ell}_j + \bar{\nu}_{Ri} (m_{\tilde{\nu}}^2)_{ij} \tilde{\nu}_{Rj} + \bar{e}_{Ri} (m_{\tilde{e}}^2)_{ij} \tilde{e}_{Rj}$$

$$+ \frac{1}{2} B_{\nu ij} \bar{\nu}_{Ri} \tilde{\nu}_{Rj} + h.c.$$ (10)

At energies lower than $M_{R1}$, the right-handed neutrinos are decoupled, and light Majorana neutrinos via the see-saw mechanism are only neutrinos relevant for the low energy effective action. The effective interaction Lagrangian among the mass eigenstates of light Majorana neutrino $\nu_i$, chargino $\chi^-_A$ and charged-slepton $\tilde{e}_X$ is given by [15] [16]

$$\mathcal{L}_{\text{int}} = \bar{\nu}_i (C_{iAX}^R P_R) \tilde{\chi}^+_A \tilde{e}_X + h.c.$$ (11)

where $P_R = (1 + \gamma_5)/2$ is the right-handed chirality projection operator, and

$$C_{iAX}^R = g_2 (U_{MNS}^*)_{ai} \left( - (O^*_L) A_1 (U^*_\tilde{e}) x_\alpha + \frac{m_\alpha}{\sqrt{2} M_W \cos \beta} (O^*_L) A_2 (U^*_\tilde{e}) x_{\alpha + 3} \right)$$ (12)

with $m_\alpha = m_e, m_\mu, m_\tau$. Chargino mass matrix $M_C$ and charged-slepton mass-squared matrix $M_{\tilde{e}}$ are diagonalized by the unitary matrices $O_L$, $O_R$ and $U_{\tilde{e}}$, respectively, such that

$$O_R M_{\tilde{e}} O_L^\dagger = \text{diag}(m_{\tilde{\chi}^-_1}, m_{\tilde{\chi}^-_2})$$

$$U_{\tilde{e}} M_{\tilde{e}} U_{\tilde{e}}^\dagger = \text{diag}(m_{\tilde{e}_1}^2, \ldots, m_{\tilde{e}_6}^2).$$ (13)

Note that the left-handed chirality part is absent in Eq. (11) because of the decoupling of the right-handed Majorana neutrinos.

One-loop corrections with chargino and charged-slepton running in the loop give dominant contributions to the TMM. Corresponding Feynman diagrams are depicted in Fig. 1. Note that two diagrams are summed because of Majorana property of the external line neutrinos. Considering all the diagrams, we obtain the TMM normalized by the Bohr magneton such as

$$\mu_{ij} = - \frac{1}{16\pi^2} \sum_{A=1}^{2} \sum_{X=1}^{6} \frac{m_e}{m_{\tilde{\chi}^-_A}} \left[ \frac{m_i + m_j}{m_{\tilde{\chi}^-_A}} \left( 3 |C_{iAX}^R C_{jAX}^{R*}| \right) f(x_{AX}) \right] \mu_B.$$ (14)
where $m_i$ is the neutrino mass of the $i$-th generation, $x_{AX} = m_{\chi A}^2/m_{\tilde{e}X}^2$, and $f(x)$ is the loop function defined as

$$f(x) = \frac{x}{2(1-x)^2} \left[ \frac{2x}{1-x} \log x + 1 + x \right],$$

which is a monotonically increasing function of $x$ varying from $f(x \rightarrow +0) = 0$ to $f(x \rightarrow \infty) = \frac{1}{2}$. We can easily check that $\mu_{ij} = -\mu_{ji}$, and hence $\mu_{ii} = 0$.

As can be seen in the general formula of Eq. (14), we need knowledge of neutrino mass eigenvalues, all the components of the MNS mixing matrix and the soft SUSY breaking terms at the weak scale, for the evaluation of concrete TMM values. There are little models which can determine all the fermion mass matrices including CP phases. In this article, we refer the results in the recently proposed minimal SUSY SO(10) model [13], where all the fermion mass matrices are unambiguously determined. Using the results in [13], SUSY contributions to the lepton flavor violating processes and muon $g - 2$ has been calculated based on the minimal supergravity (mSUGRA) boundary conditions [17]. In the following calculations, we apply the same strategy and input parameters as in [17].

First we list the results in [13] for the case of $\tan \beta = 45$ which are used in the following analysis. The light Majorana neutrino mass eigenvalues are fixed as (in GeV) $m_1 = 2.45 \times 10^{-12}$, $m_2 = 1.95 \times 10^{-11}$ and $m_3 = 4.88 \times 10^{-11}$, while the right-handed heavy Majorana neutrino mass eigenvalues are found to be (in GeV) $M_{R1} = 1.64 \times 10^{11}$, $M_{R2} = 2.50 \times 10^{12}$ and $M_{R3} = 8.22 \times 10^{12}$, when a model parameter is fixed so as to provide $\Delta m^2_{\odot} = 2 \times 10^{-3}$eV$^2$ for the atmospheric neutrino oscillation data. The MNS mixing matrix has been found to be

$$U_{MNS} = \begin{pmatrix} 0.168 + 0.838i & -0.467 + 0.0940i & -0.00508 + 0.207i \\ 0.0519 + 0.498i & 0.651 - 0.0473i & 0.0189 - 0.569i \\ 0.0745 + 0.116i & 0.450 - 0.381i & 0.431 + 0.669i \end{pmatrix}. \quad (16)$$

In the basis where both of the charged-lepton and right-handed Majorana neutrino mass matrices are diagonal with real and positive eigenvalues, the neutrino Dirac Yukawa matrix at the grand unification (GUT) scale is found to be

$$Y_{\nu} = \begin{pmatrix} -0.000135 - 0.00273i & 0.00113 + 0.0136i & 0.0339 + 0.0580i \\ 0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\ -0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i \end{pmatrix}. \quad (17)$$

The soft SUSY breaking parameters at the weak scale are evaluated through the RGE evolutions by imposing mSUGRA universal boundary conditions at the GUT scale, $M_{GUT} \sim 2 \times 10^{16}$GeV, such that

$$\left( m^2_{\tilde{H}} \right)_{ij} = \left( m^2_{\tilde{e}} \right)_{ij} = \left( m^2_{\tilde{\chi}} \right)_{ij} = m_0^2 \delta_{ij},$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2,$$

$$A^i_{\nu} = A_0 Y^i_{\nu}, \quad A^i_{\tilde{e}} = A_0 Y^i_{\tilde{e}},$$

$$M_1 = M_2 = M_3 = M_{1/2}. \quad (18)$$
The neutrino Dirac Yukawa matrix contributes the RGE evolutions of the soft SUSY breaking parameters at the energy scales above $M_{R_i}$. For example, RGE for the left-handed slepton mass-squared matrix is given by

$$\mu \frac{d}{d\mu} \left( m_{\tilde{\ell}}^2 \right)_{ij} = \mu \frac{d}{d\mu} \left( m_{\tilde{\ell}}^2 \right)_{ij} |^{\text{MSSM}} + \frac{1}{16\pi^2} \left( m_{\tilde{\ell}}^2 Y^\dagger \nu Y_{\nu} + 2Y^\dagger_{\nu} m^2_{\tilde{\nu}} Y_{\nu} + 2m^2_{H_u} Y^\dagger_{\nu} Y_{\nu} + 2A^i_\nu A_{\nu} \right)_{ij},$$

(19)

where the first term in the right hand side denotes the normal MSSM term. In the leading-logarithmic approximation, additional contribution due to the neutrino Dirac Yukawa couplings are estimated as

$$\left( \Delta m^2_{\tilde{\ell}} \right)_{ij} \sim -\frac{3m^2_{0} + A^2_{\nu}}{8\pi^2} \left( Y^\dagger_{\nu} L Y_{\nu} \right)_{ij},$$

(20)

where the distinct thresholds of the right-handed Majorana neutrinos are taken into account by the matrix $L_{ij} = \log[M_G/M_{R_i}]\delta_{ij}$.

Now let us present our numerical results. The resultant $|\mu_{ij}|$ as a function of the universal scalar mass $m_0$ is depicted in Fig. 2 for fixed $M_{1/2} = 600\,\text{GeV}$ and $A_0 = 0$ together with the results in the standard model with the see-saw mechanism in Eq. (8). The resultant magnetic moments are decreasing as $m_0$ becomes large, according to Eq.(15) with $X_{AX} \sim M^2_{1/2}/m^2_0$ for $m_0 > M_{1/2}$. We find that the SUSY contributions to TMM with the input soft SUSY breaking masses being of order 1 TeV can be an order of magnitude larger than those in the standard model. In Fig. 3, $|\mu_{ij}|$ as a function of the universal gaugino mass $M_{1/2}$ is depicted for fixed $m_0 = 400\,\text{GeV}$ and $A_0 = 0$. We can read off the fact that the resultant TMM is roughly proportional to $M_{1/2}^2$. For fixed $m_0 = 600\,\text{GeV}$ and $M_{1/2} = 800\,\text{GeV}$, TMM is plotted in Fig. 4 as a function of $A_0$. It is found that TMM is not so sensitive to $A_0$.

In conclusion, we have examined the SUSY contributions to TMM of Majorana neutrinos. In concrete evaluations, informations of neutrino mass matrix elements including CP phases are necessary. Using unambiguously determined neutrino mass matrices in the minimal SUSY SO(10) model in [13], TMM has been calculated. We have found that the SUSY contributions to TMM with the input soft SUSY breaking masses being of order 1 TeV can be an order of magnitude larger than those calculated in the standard model extended to incorporate the see-saw mechanism. Unfortunately, the calculated TMMs are found to be too small to cause interesting astrophysical phenomena. This fact can be understood in intuitive way with a rough estimation of TMM. We consider neutrinos as the Majorana particles via the see-saw mechanism with heavy right-handed Majorana neutrinos. At weak scale, the right-handed neutrinos are decoupled, and only the light Majorana neutrinos appear in low energy effective theory and only their left-handed components can couple to SUSY particles as in Eq. (11). In this case, the chirality flip between in and
out neutrino states necessary for TMM can occur only by mass insertions in the external lines. Therefore, TMM is always proportional to the light Majorana masses. Taking this fact into account, we can estimate the order of magnitude of the SUSY contributions such as

\[ \mu_\nu \sim e \times 10^{-2} \frac{m_\nu}{M_{\text{SUSY}}^2} \sim 10^{-18} \left( \frac{m_\nu}{1 \text{eV}} \right) \left( \frac{100 \text{GeV}}{M_{\text{SUSY}}} \right)^2 \mu_B \]  

(21)

where \( e \) is the electric charge, \( 10^{-2} \) is a loop factor, and \( m_\nu \) and \( M_{\text{SUSY}} \) are neutrino mass and typical soft SUSY breaking mass, respectively. We can see that this formula gives a good approximation.

**References**


[12] K.L. Ng, Z. Phys. C48, 289 (1990), and see the references for the previous works.
Figure 1: The 1-loop Feynman diagrams which generate the transition magnetic moments of the Majorana neutrinos. Because of the Majorana property of the external neutrino, we have to sum contributions of two diagrams.
Figure 2: The transition magnetic moments of the Majorana neutrinos, $\log_{10}|\mu_{23}|$, $\log_{10}|\mu_{12}|$ and $\log_{10}|\mu_{13}|$ from top to bottom as a function of $m_0$ (GeV) with fixed $M_{1/2} = 600\text{GeV}$ and $A_0 = 0$. The horizontal lines denote the results from the standard model with see-saw mechanism with the same MNS matrix, and the lines correspond to $\log_{10}|\mu_{23}|$, $\log_{10}|\mu_{12}|$ and $\log_{10}|\mu_{13}|$ from top to bottom, respectively.
Figure 3: The transition magnetic moments of the Majorana neutrinos, $\log_{10}[|\mu_{23}|]$, $\log_{10}[|\mu_{12}|]$ and $\log_{10}[|\mu_{13}|]$ from top to bottom as a function of $M_{1/2}$ (GeV) with fixed $m_0 = 400\,\text{GeV}$ and $A_0 = 0$. 
Figure 4: The transition magnetic moments of the Majorana neutrinos, \( \log_{10}[|\mu_{23}|] \), \( \log_{10}[|\mu_{12}|] \) and \( \log_{10}[|\mu_{13}|] \) from top to bottom as a function of \( A_0 \) (GeV) with fixed \( m_0 = 600 \text{GeV} \) and \( M_{1/2} = 800 \text{GeV} \).