A “Littlest Higgs” Model with Custodial $SU(2)$ Symmetry

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In this note, a “littlest higgs” model is presented which has an approximate custodial $SU(2)$ symmetry. The model is based on the coset space $SO(9)/(SO(5) \times SO(4))$. The light pseudo-goldstone bosons of the theory include a single higgs doublet below a TeV and a set of three $SU(2)_W$ triplets and an electroweak singlet in the TeV range. All of these scalars obtain approximately custodial $SU(2)$ preserving vacuum expectation values. This model addresses a defect in the earlier $SO(5) \times SU(2) \times U(1)$ moose model, with the only extra complication being an extended top sector. Some of the precision electroweak observables are computed and do not deviate appreciably from Standard Model predictions. In an S-T oblique analysis, the dominant non-Standard Model contributions are the extended top sector and higgs doublet contributions. In conclusion, a wide range of higgs masses is allowed in a large region of parameter space consistent with naturalness, where large higgs masses requires some mild custodial $SU(2)$ violation from the extended top sector.

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Introduction

In the near future, experimental tests at the LHC will begin to map out physics at the TeV energy scale. With this data, a determination of the higgs sector, and more importantly, discovering the physics that stabilizes the weak scale from radiative corrections should be achievable goals. However, in the interim, the industry of precision electroweak observables has given us some indirect evidence on what the theory beyond the standard model must look like. And given the unreasonably good fit of the standard model to these observables, these constraints generically suggest a theory with perturbative physics at the TeV scale.

For many years, the only models that could stabilize the weak scale and be weakly perturbative were supersymmetric models, most notably the MSSM. In the past two years, it has been shown that there is a new class of perturbative theories of electroweak symmetry breaking, that of the “little higgs” Arkani-Hamed:2001nc, Arkani-Hamed:2002pa, Arkani-Hamed:2002qx, Gregoire:2002ra, Arkani-Hamed:2002qy, Low:2002ws, Kaplan:2002px, Chang:2003un, Skiba:2003yf. For reviews of the physics, see Wacker:2002ar, Schmaltz:2002wx, and for more detailed phenomenology see Burdman:2002us, Han:2003wu, Dib:2003zj, Han:2003gf. Little Higgs theories protect the higgs boson from one-loop quadratic divergences because each coupling treats the higgs boson as an exact goldstone boson. However, two different couplings together can break the non-linear symmetries protecting the higgs mass, and thus the higgs is a pseudo-goldstone boson with quadratic divergences to its mass pushed to two-loop order. This allows a separation of scales between the cutoff and the electroweak scale, so that physics can be perturbative until the cutoff is reached at $\Lambda \approx 10$ TeV.

Having weakly perturbative physics at the TeV scale is probably necessary but definitely not sufficient to guarantee a theory is safe from precision electroweak constraints. Currently precision observables have been measured beyond one-loop order in the standard model, and since little higgs model corrections are parameterically of this order, these observables can put constraints on these theories Chivukula:2002ww, Hewett:2002px, Csaki:2002qg, Csaki:2003yi, Kribs:2003yu, Gregoire:2003kr. However, these constraints are not unavoidable, and isolating the strongest constraints can point to the necessary features to make little higgs models viable theories of electroweak symmetry breaking. First of all, there are modifications of the original models which address these strongest constraints and greatly ameliorate the issue Csaki:2003si. However, just recently, a little higgs model was introduced containing a custodial $SU(2)$ symmetry, the $SO(5) \times SU(2) \times U(1)$ moose model Chang:2003un. In the limit of strong coupling for the $SO(5)$ gauge group, the precision electroweak constraints due to the T parameter were softened and in general, there is a large region of parameter space consistent with precision electroweak constraints and naturalness Csaba:talk.

Let’s briefly summarize the physics that gives the custodial $SU(2)$ symmetry. The important point is that in models with a gauged $U(1) \times U(1)$ subgroup and standard model fermions gauged under just one of the $U(1)$’s, the massive $B'$ of these theories provides two constraints. The first constraint is that integrating out the $B'$ generates a custodial $SU(2)$ violating operator that after electroweak symmetry breaking corrects the standard model formula for the mass of the $Z$ gauge boson. This gives corrections to the $\rho$ parameter, and vanishes as the two $U(1)$ gauge couplings become equal. However, the second
constraint pulls in the opposite direction in gauge parameter space. This is because the coupling of the $B'$ to standard model fermions generates corrections to low energy four-fermi operators and also to coefficients of the $SU(2)_W \times U(1)_Y$ fermion currents. These corrections vanish in the limit in which the $U(1)$ that the standard model fermions is not gauged under becomes strong. Thus, these two constraints prefer different limits in parameter space and can constrain the model.

As pointed out before Chang:2003un, there are simple modifications that evade these two constraints, such as only gauging $U(1)_Y$, charging the SM fermions equally under both $U(1)$’s, or through fermion mixing. Another simple approach that gives custodial $SU(2)$ symmetry is to complete the $B'$ into a custodial $SU(2)$ triplet. If the triplet is exactly degenerate in mass, integrating it out does not contribute to a custodial $SU(2)$ violating operator. To include these new states, instead of gauging two $U(1)$’s, $SU(2)_R \times U(1)$ is gauged. After being broken down to the diagonal $U(1)_Y$, $B'$ and $W^{\pm}$ are put into a “$SU(2)$R” triplet. Integrating out the $W^{\pm}$ generates an operator which only gives mass to the $W$ giving a $\rho$ contribution of the opposite sign of the $B'$ contribution. Numerically, the total $\rho$ contribution from the gauge sector cancels in the strong $SU(2)_R$ coupling limit (where the triplet becomes degenerate), which is the same limit that reduces corrections to fermion operators.

However, this cancelation is not quite exact for the $SO(5) \times SU(2) \times U(1)$ moose model. The higgs quartic potential of that theory has a flat direction when the two higgs vevs have the same phase, thus viable electroweak symmetry breaking requires the higgs vevs to have different phases. This phase difference changes the $\rho$ contribution due to the $W^{\pm}$ gauge bosons. The higgs currents of the $W^{\pm}$ are not invariant under a vev phase rotation, and thus the cancelation in the strong coupling limit only occurs if the phase is 0 or $\pi$. Indeed, this remnant of custodial $SU(2)$ violation puts the strongest constraint on the theory.

The situation can be easily resolved if the little higgs theory contains only a single light higgs doublet. In this case, the $W^{\pm}$ current just transforms by a phase under the vev phase rotation, which cancels out of the contribution. It turns out that the $SO(5) \times SU(2) \times U(1)$ moose’s defect can be removed by imposing a $Z_4$ symmetry inspired by orbifold models Wacker:toappear, which leaves only a single light higgs doublet that still has an order one quartic coupling. In this paper, we will take a different approach and construct a “littlest higgs” model with custodial $SU(2)$ symmetry and just one higgs doublet.

This “littlest higgs” model will be based on an $SO(9) \times SO(5)$ coset space, with an $SU(2)_L \times SU(2)_R \times SU(2) \times U(1)$ subgroup of $SO(9)$ gauged. The pseudo-goldstone bosons are a single higgs doublet, an electroweak singlet and a set of three $SU(2)_W$ triplets, precisely the content of one of the original custodial $SU(2)$ preserving composite higgs models Georgi:1984af. The global symmetries protect the higgs doublet from one-loop quadratic divergent contributions to its mass. However, the singlet and triplets are not protected, and will be pushed to the TeV scale. Integrating out these heavy particles will generate an order one quartic coupling for the higgs. To complete the theory with fermions, the minimal top sector contains two extra colored quark doublets and their charge conjugates.

Since the primary motivation of the model is to improve consistency with precision electroweak observables, the model’s corrections to these observables will be calculated. First, we will see that aside from some third generation quark effects, a limit will exist where non-oblique corrections vanish. This limit was recently described as “near-oblique” Gregoire:2003kr and we will continue to use this terminology. The existence of this limit allows a meaningful S and T analysis of the oblique corrections, which will be performed in this model to order $(v^2/f^2)$. The dominant contributions come from the extended top sector and the higgs doublet, which are quite mild in most of parameter space. In fact, this analysis will show that there is a wide range of higgs masses allowed in a large region of parameter space consistent with naturalness.

The outline of the rest of the paper is as follows: in section Sec: Model we describe the model’s coset space, light scalars and the symmetries that protect the higgs mass. We also analyze the gauge structure and then describe the minimal candidate top sectors. In section Sec: Potential, we will show how the quartic higgs potential is generated as well as describe the log enhanced contributions to the higgs mass parameter. There will be vacuum stability issues, and we will point out ways which these can be resolved. Also as usual, the top sector contributions will generically drive electroweak symmetry breaking. In section Sec: PEWO, some precision electroweak observables will be calculated and the constraints on the theory will be detailed. In section Sec: Conclusion, we conclude and finally in appendix Sec: Gens, we describe our specific generators and representations of $SO(4)$. The Model Sec: Model The first ingredient necessary for custodial $SU(2)$ symmetry is the breakdown of $SU(2)_L \times SU(2)_R \times SU(2) \times U(1)$ down to the diagonal
SU(2)_W \times U(1)_Y subgroup. Therefore the global symmetry group must be at least rank 4. Two rank 4 groups are easy to eliminate--SU(5) does not contain the gauged group and the SO(8) adjoint contains no higgs doublets. This leaves SO(9), Sp(8) and F_4 as the only remaining rank 4 candidates. In this paper, we'll focus on the SO(9) group as it is the easiest to analyze. However, we do mention here that it appears to be difficult to get a single light higgs doublet in the Sp(8), F_4 groups.

Isolating our attention to SO(9), it is straightforward to implement the "little higgs" construction. Using the vector representation, the top four by four block will contain the gauged SO(4) \cong SU(2)_L \times SU(2)_R and the bottom four by four block will contain the gauged SU(2) \times U(1) \subset SO(4). The coset space should break these two SO(4)'s down to their diagonal subgroup, which can be achieved by an off-diagonal vev for a two-index tensor of SO(9). In order to have the largest unbroken global symmetry (and thus reduce the amount of light scalars), a symmetric two-index tensor should be chosen.

This construction can be described in the following way: take an orthogonal symmetric nine by nine matrix, representing a non-linear sigma model field Σ which transforms under an SO(9) rotation by Σ \rightarrow VΣV^T. To break the SO(4)'s to their diagonal, we take Σ's vev to be \langle Σ \rangle = \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}.

It is now simple to determine the global symmetries that protect the higgs mass at one loop. Under the upper five by five SO(5)_1 symmetry, the scalars transform as: \begin{equation}
\delta \hat{h} = \delta \hat{\alpha} + \cdots 0.4im\delta \Phi = -12f(\hat{\alpha}^T \hat{h} + \hat{h}^T \hat{\alpha}) + \cdots
\end{equation}