Hadron production in heavy ion collisions: Fragmentation and recombination from a dense parton phase

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We discuss hadron production in heavy ion collisions at RHIC. We argue that hadrons at transverse momenta $P_T < 5 \text{ GeV}$ are formed by recombination of partons from the dense parton phase created in central collisions at RHIC. We provide a theoretical description of the recombination process for $P_T > 2 \text{ GeV}$ our results smoothly match a purely statistical description. At high transverse momentum hadron production is well described in the language of perturbative QCD by the fragmentation of partons. We give numerical results for a variety of hadron spectra, ratios and nuclear suppression factors. We also discuss the anisotropic flow $v_2$ and give results based on a flow in the parton phase. Our results are consistent with the existence of a parton phase at RHIC hadronizing at a temperature of 175 MeV and a radial flow velocity of 0.55c.

I. INTRODUCTION

Recent data from the Relativistic Heavy Ion Collider (RHIC) have shown a strong nuclear suppression of the pion yield at transverse momenta larger than 2 GeV/c in central Au + Au collisions, compared to $p+p$ interactions. This is widely seen as the experimental confirmation of jet quenching, the phenomenon that high energy partons lose energy when they travel through the hot medium created in a heavy ion collision, entailing a suppression of intermediate and high $P_T$ hadrons.

However, the experiments at RHIC have provided new puzzles. The amount of suppression seems to depend on the hadron species. In fact, in the production of protons and antiprotons between 2 and 4 GeV/c the suppression seems to be completely absent. Generally, pions and kaons appear to suffer from a strong energy loss while baryons and antibaryons do not. Two stunning experimental facts exemplify this. First, the ratio of protons over positively charged pions is equal or above one for $P_T > 1.5 \text{ GeV/c}$ and is approximately constant up to 4 GeV/c. Second, the nuclear suppression factor $R_{AA}$ below 4 GeV/c is close to one for protons and lambda, while it is about 0.3 for pions.

There have been recent attempts to describe the different behavior of baryons and mesons through the existence of gluon junctions or alternatively through recombination as the dominant mechanism of hadronization. The recombination picture has attracted additional attention due to the observation that the elliptic flow pattern of different hadron species can be explained by a simple recombination mechanism. The anisotropies $v_2$ for the different hadrons are compatible with a universal value of $v_2$ in the parton phase, related to the hadronic flow by factors of two and three depending on the number of valence quarks.

In this work we elaborate on the arguments presented in [10]. We will present the formalism as well as new numerical results. The competition between recombination and fragmentation delays the onset of the perturbative/fragmentation regime to relatively high transverse momentum of 4–6 GeV/c, depending on the hadron species. This is the explanation for several key observations at RHIC:

• the two component form of hadron transverse momentum spectra, including an exponential part and a power law tail with a transition between 4 and 6 GeV/c.

• the very different behavior of the nuclear suppression factors $R_{AA}$ of mesons and baryons.

• the particle dependence of the elliptic flow.

• the unusually large baryon/meson ratios.

In addition, our calculation clarifies the range of applicability of perturbative calculations including energy loss.

The paper is organized as follows. In the next section we explain why fragmentation might not be the dominant mechanism of hadronization at intermediate transverse momenta of a few GeV/c in heavy ion collisions. We discuss the fundamental principles of recombination and fragmentation. In Sec. III we present the theoretical framework of recombination in more detail and also discuss its shortcomings. In particular, we address the question of applicability at low transverse momentum. In Sec. IV we introduce our parametrization of the parton spectrum and discuss further calculational details, and in Sec. V we present numerical results on spectra, hadron ratios, nuclear suppression and elliptic flow. Sec. VI summarizes our work.
II. FRAGMENTATION VS. RECOMBINATION

A. Fragmentation of partons

Inclusive hadron production at sufficiently large momentum transfer can be described by perturbative quantum chromodynamics (pQCD). The invariant cross section for the production of parton $h$ with momentum $P$ can be given in factorized form

$$E_{a} d\sigma_{h} / d^{3}P = \sum_{a} \int_{0}^{1} dz D_{a\rightarrow h}(z) E_{a} d\sigma_{a} / d^{3}P_{a}.$$  \hspace{1cm} (1)

The sum runs over all parton species $a$ and $\sigma_{a}$ is the cross section for the production of parton $a$ with momentum $P_{a} = P/z$. Thus the parton production cross section has to be convoluted with the probability that parton $a$ fragments into hadron $h$. The probabilities $D_{a\rightarrow h}(z)$ are called fragmentation functions. Like parton distributions they are non-perturbative quantities. However they are universal and once measured, e.g. in $e^+e^-$ annihilations, they can be used to describe hadron production in other hard QCD processes.

Using (1) we can estimate the ratio of protons and pions. Taking, e.g., the common parametrization of Kniehl, Kramer and Pöttner (KKP), the ratio $D_{u\rightarrow p}/D_{d\rightarrow \pi^{0}}$ is always smaller than 0.2 for each parton $a$. This reflects the well known experimental fact that pions are much more abundant than protons in the domain where pQCD is applicable. The excess of pions over protons even holds down to very low $P_{T}$, smaller than 1 GeV, where perturbative calculations are no longer reliable. In that domain one can argue that the difference in mass, $M_{\pi} \gg M_{z}$, lays a huge penalty on proton production. The small value of the $p/\pi^{0}$ ratio predicted by these calculations over the entire range of $P_{T}$ is the reason why the ratio $p/\pi^{0} \sim 1$ measured at RHIC is so surprising.

It has been suggested that the fragmentation functions $D_{a\rightarrow h}(z)$ can be altered by the environment. The energy loss of the propagating parton in the surrounding medium leads, in first approximation, to a rescaling of the variable $z$. This would affect all produced hadrons in the same way, and thus cannot explain the observations at RHIC. In a picture with perturbative hadron production and jet quenching alone, the different behavior of hadrons can not be described by one consistent sets of energy loss parameters. To save the validity of the purely perturbative approach species dependent non-perturbative contributions to the fragmentation functions have to be introduced ad hoc to explain the data.

Perturbative hadron production consists of three steps: production of a parton in a hard scattering, propagation and interaction with a medium, and finally hadronization of the parton. Only modifications in our understanding of hadronization are able to provide an explanation of the experimental observations, since the other steps are blind to the hadron species that will eventually be created.

B. From fragmentation to recombination

For the production of a hadron with momentum $P$ via fragmentation we need to start with a parton with momentum $P/z \gg P$. The fragmentation functions favor very small values of $z$, i.e. the situation where the energy of the fragmenting parton is not concentrated in one hadron. On the other hand, the transverse momentum spectrum of partons is steeply falling with $P_{T}$. This makes it clear that fragmentation is a rather inefficient mechanism for the production of high $P_{T}$ hadrons, since it has to overcome the limited availability of partons at even higher transverse momentum. As a result, the average $\langle z \rangle$ is larger than what is expected from the shape of the fragmentation functions. For pion production, $\langle z \rangle$ is about 0.6 for the production from a valence quark, 0.4 for a sea quark and 0.5 for a gluon in the range 2 GeV/c $\leq P_{T} \leq 10$ GeV/c for leading order KKP fragmentation functions.

An outgoing high energy parton is not a color singlet and will therefore have a color string attached. The breaking of the string will initiate the creation of quark antiquark pairs until there is a entire jet of partons, which have to share the energy of the initial parton. They will finally turn into many hadrons. The creation of several hadrons from one fragmenting parton is the reason why fragmentation functions prefer small values of $z$. If phase space is already filled with partons, a single parton description might not be valid anymore. Instead one would have to introduce higher twist (multiple parton) fragmentation functions. In the most extreme case, if partons are abundant in phase space, they might simply recombine into hadrons. This means that a $u$ and a $d$ quark that are “close” to each other in phase space can bind together to form a $\pi^{+}$. The scale of being close will be set by the width of the pion wave function. In this scenario the total pion momentum will be just the sum of the individual quark momenta. We immediately notice that this recombination mechanism is very efficient for steeply falling spectra: in order to produce a 5 GeV pion we can start with two quarks having (on average) about 2.5 GeV/c transverse momentum and each being therefore far more abundant (on average) than a 10 GeV/c parton that could produce the pion via fragmentation. Of course the recombining partons must be close in phase space, i.e. recombination will be suppressed if the phase space density is low.

Recombination can be interpreted as the most “exclusive” form of hadronization, the endpoint of a hypothetical resummation of fragmentation processes to arbitrary twist. We cannot achieve a quantitative understanding of this at the moment. However, we can try to find an effective description which can be tested against observable consequences. In this work we will advocate a simple model for recombination and compare it with single parton fragmentation. These two mechanisms of hadron production compete differently depending on the phase space density of partons. From the above we understand...
that the competition between fragmentation and recombination is dominated by the slope and the absolute value of the phase space distribution of partons. Below we show that recombination always wins over fragmentation for an exponentially falling parton spectrum, but that fragmentation takes over if the spectrum has the form of a power law, as it is provided by pQCD. We will apply this insight to hadron production in relativistic heavy ion collisions at midrapidity and transverse momenta of a few GeV/c where we expect a densely populated phase space. For the recombination of three quarks into a proton the momenta of three partons have to be added up, but only two momenta in the case of a pion. Assuming an exponential parton spectrum this implies for a proton a distribution $\sim \exp(-P_T/3)$ and for pions $\sim \exp(-P_T/2)^2$, predicting a constant $p/\pi^+$ ratio where the value is determined by simple counting of quantum numbers $^{10}$. We will show that some of the surprising experimental results from RHIC can be explained in this way.

C. The recombination concept

The idea of quark recombination was proposed long ago to describe hadron production in the forward region of $p+p$ collisions $^{23}$. This was later justified by the discovery of the leading particle effect, the phenomenon that, in the forward direction of a beam of hadrons colliding with a target, the production of hadrons sharing valence quarks with the beam hadrons are favored. E.g. in the Fermilab E791 fixed target experiment with a 500 GeV $\pi^-$ beam $^{24}$ the asymmetry

$$\alpha(x_F) = \frac{d\sigma_{D^-}/dx_F - d\sigma_{D^+}/dx_F}{d\sigma_{D^-}/dx_F + d\sigma_{D^+}/dx_F}$$

(2)

between $D^-$ and $D^+$ mesons grows nearly to unity when the Feynman variable $x_F$, measuring the longitudinal momentum relative to the beam momentum, approaches 1. Fragmentation would predict this asymmetry to be very close to zero. However, recombination of the $c$ quark from a $c\bar{c}$ pair produced in a hard interaction with a $d$ valence quark from the $\pi^-$, propagating in forward direction with large energy, is highly favored compared to the recombination of the $c$ with a $d$ which is only a sea quark of the $\pi^-$. This leads to the enhancement of $D^-$ over $D^+$ mesons in the forward region.

The leading particle effect is a clear signature for the existence of recombination as a hadronization mechanism and has been addressed in several publications recently $^{25, 26}$. In this case recombination is favored over fragmentation only in a certain kinematic situation (the very forward direction), which is a only a small fraction of phase space.

In central heavy ion collisions many more partons are produced than in collisions of single hadrons. The idea that recombination may then be important for a wide range of rapidities – and at least up to moderate transverse momenta – was advocated before $^{27, 28, 29}$. However, it was only recently that RHIC data indicated that recombination could indeed be a valid approach up to surprisingly high transverse momenta of a few GeV/c.

Charm and heavy hadron production have the advantage that the heavy quark mass provides a large scale that permits a more rigorous treatment of the recombination process $^{26}$. The description of recombination into pions and protons seems to be theoretically less rigorous. However, a simple counting of quantum numbers in a picture where the structure of hadrons is dominated by their valence quarks often provides surprisingly good results. This has been pointed out for particle spectra and ratios $^{11, 30, 31}$ and for elliptic flow $^{12, 13, 14}$. Most of the work so far has stayed on a qualitative level without quantitative predictions. Recombination of $D$ mesons in heavy ion collisions has been investigated in $^{32}$.

We will argue below that the counting of quantum numbers is a good description of the recombination process for intermediate momenta. The formalism will set the stage to obtain quantitative results in this regime by recombining quarks from a possibly thermalized phase. We know that the parton phase will not behave like a perturbative plasma near the hadronization point $^{33}$. Instead quarks at hadronization will be effective degrees of freedom exhibiting a mass and gluons will disappear as dynamical degrees of freedom. We will assume here that the effective quarks behave like constituent quarks and that there are no dynamical gluons.

This is different from the work of Greco et al. $^{11}$ who suggested to recombine one perturbative quark with thermal quarks, leading to an additional contribution at the transition region between the pure thermal phase dominating below 5 GeV/c and the pure fragmentation regime dominating above 5 GeV/c. A similar form of this pick-up reaction of a perturbative parton was recently proposed in the context of the sphaleron model $^{34}$.

One can also attempt to extend the recombination concept to low $P_T$, however, the theoretical situation is much more ambiguous there. The main reasons are that the simple counting of quantum numbers violates energy and entropy conservation at low $P_T$, where the bulk of the hadron yield resides. Nevertheless, once recombination is recognized to be the dominant hadronization mechanism at intermediate $P_T$ from 2 to 5 GeV/c, it is quite reasonable to expect that this mechanism extends down to very small transverse momentum, where quarks are even more abundant. However the theoretical description will only be on solid ground once the problems of energy and entropy conservation are addressed properly.

D. A non-relativistic model

For a first estimate, let us consider a simple static model for the recombination process. We start with a system of quarks and antiquarks which are homogeneously distributed in a fixed three-dimensional volume $V$ with
phase space distributions \( w(p) \), so that the number of quarks or antiquarks with a particular set \( a \) of quantum numbers (color, spin, isospin) is

\[
N_a = V \int \frac{d^3p}{(2\pi)^3} w_a(p) \quad (3)
\]

Hadronization is assumed to occur instantaneously throughout the volume. The distributions \( w(p) \) are supposed not to change during the hadronization process, i.e. the quarks are assumed to be quasi-free.

The spatial wave functions for a two particle quark/antiquark state with momenta \( p_1 \) and \( p_2 \) and a meson bound state with momentum \( P \) are

\[
\langle q | p_1 p_2 | M, P \rangle = V^{-1} e^{i (p_1 x_1 + p_2 x_2)} \quad (4)
\]

\[
\langle x | M, P \rangle = V^{-1/2} e^{iP \cdot R} \varphi_M(y) \quad (5)
\]

respectively, with the center of mass and relative coordinates \( R = (x_1 + x_2)/2 \) and \( y = x_1 - x_2 \) for the two quarks in the meson system. To keep our notation simple we omit the proper antisymmetrization of multi fermion states, since all combinatorial factors will cancel in the final result. The internal meson wave functions is normalized as

\[
\int d^3y |\varphi_M(y)|^2 = 1. \quad (6)
\]

The overlap amplitude is given by

\[
\langle q, p_1 p_2 | \pi, P \rangle = \frac{(2\pi)^3}{V^{3/2}} \delta^3(P - p_1 - p_2) \varphi_M(q). \quad (7)
\]

Here, we have introduced the relative momentum \( q = (p_1 - p_2)/2 \), conjugate to \( y \), and \( \varphi_M(q) \) is the Fourier transformed wave function. The squared amplitude is

\[
|\langle q, p_1 p_2 | M, P \rangle|^2 = \frac{(2\pi)^3}{V^3} \delta^3(P - p_1 - p_2) |\varphi_M(q)|^2. \quad (8)
\]

We conclude that the total number of pions found in the quark/antiquark distribution is

\[
N_M = C_M V^3 \int \frac{d^3P}{(2\pi)^3} \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} \times w(p_1)w(p_2) |\langle q, p_1 p_2 | M, P \rangle|^2 \quad (9)
\]

with a degeneracy factor \( C_M \). The momentum distribution of the pions is given by

\[
\frac{dN_M}{dP} = C_\pi \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \times w\left(\frac{P}{2} + q\right) w\left(\frac{P}{2} - q\right) |\varphi_M(q)|^2 \quad (10)
\]

From above equation the spectra of mesons can be calculated for given quark distributions. This will require knowledge of \( \varphi_M \). The wave function has some width \( \Lambda_M \), and we assume that it drops rapidly for \( |q| > \Lambda_M \). Let us study the kinematic region where \( P \gg \Lambda_M \). The integral over the relative momentum \( q \) is dominated by values \( |q| \sim \Lambda_M \) and thus we can assume that \( |q| \ll |P| \). We apply a Taylor expansion

\[
w\left(\frac{P}{2} + q\right) w\left(\frac{P}{2} - q\right) = w\left(\frac{P}{2}\right)^2 + \sum_{ij} q_i q_j \left[ w\left(\frac{P}{2}\right) \partial_i \partial_j w\left(\frac{P}{2}\right) - \partial_i w\left(\frac{P}{2}\right) \partial_j w\left(\frac{P}{2}\right) \right] + O(q^4), \quad (11)
\]

where the first order term in the expansion vanishes.

From the lowest order term in the expansion we get a contribution to the meson spectrum which is independent of the shape of the wave function. Only the normalization \( C_\pi \) enters and leads to a universal term

\[
C_\pi \frac{V}{(2\pi)^3} w\left(\frac{P}{2}\right)^2. \quad (12)
\]

The second order term (like all higher order terms) in the expansion generates a correction depending on the shape of the wave function. For the sake of simplicity, let us assume a Gaussian shape

\[
\varphi_M(q) = N_M e^{-q^2/2\Lambda_M^2}. \quad (13)
\]

The normalization factor is determined by \( C_\pi \) as \( \Lambda_M^2 = (2\sqrt{\pi}/\Lambda_M)^3 \). For this example we have the second order correction

\[
C_\pi \frac{V}{(2\pi)^3} \frac{\Lambda_M^2}{2} \left[ w\left(\frac{P}{2}\right) \Delta w\left(\frac{P}{2}\right) - (\nabla w\left(\frac{P}{2}\right))^2 \right]. \quad (14)
\]

We want to emphasize once more that in the limit \( P \gg \Lambda_M \) the exact shape of the wave function is not important. In a relativistic framework this statement is softened by the fact that we consider the hadron formation in a boosted frame, where \( P \) is large. Therefore \( \Lambda_M \), which is of order \( \Lambda_{\text{QCD}} \) in the rest frame of the hadron, will be dilated.

As an example, let us assume an exponential parton distribution of the form \( w(p) = e^{-p/P} \): The meson spec-
trum at large \( P \) is then given by

\[
d\mathcal{N}_x = C_\pi \frac{V}{(2\pi)^3} \left[ e^{-P \cdot x / T} \left[ 1 - \frac{2\Lambda_m^2}{TP} \right] \right],
\]

(15)

using (12), (14). The second order term introduces a power correction of order \( \Lambda_m^2 / TP \) to the universal result, depending both on the width of the wave function and the slope of the parton distribution.

For nucleons we start with three quarks at coordinates \( x_i \). We introduce center of mass and relative coordinates \( R = (x_1 + x_2 + x_3)/3, y = (x_1 + x_2)/2 - x_3 \) and \( z = x_1 - x_2 \).

The overlap amplitude between a 3 quark state and a baryon with momentum \( P \) is

\[
\langle q, p_1, p_2, p_3 | B, P \rangle = \frac{(2\pi)^3}{V^2} \delta^3(P - p_1 - p_2 - p_3) \hat{\varphi}_B(q, s).
\]

(16)

Here \( \hat{\varphi}_B(q, s) \) is the baryon wave function in momentum space, depending on the relative momenta \( q \) and \( s \) conjugate to \( y \) and \( z \) respectively.

Hence the baryon distribution is given by

\[
d\mathcal{N}_B = C_B \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \frac{d^3s}{(2\pi)^3} |\hat{\varphi}_B(q, s)|^2
\]

\[
\times w \left( \frac{P}{3} + \frac{q}{2} + s \right) w \left( \frac{P}{3} + \frac{q}{2} - s \right) w \left( \frac{P}{3} - q \right).
\]

(17)

\( C_B \) is the appropriate degeneracy factor. In the region where the nucleon momentum \( P \) is large compared to the intrinsic width of the wave function, we can again expand the product of the quark distribution functions. The leading term is universal and contributes

\[
d\mathcal{N}_B = \frac{V}{(2\pi)^3} w(P/3)^3
\]

(18)
to the nucleon distribution. The second order term provides a correction

\[
C_\pi \frac{V}{(2\pi)^3} \frac{1}{2} \left( \frac{3}{4} \Lambda_B^2 \right) \left[ w(P/3)^2 \triangle w(P/3) - w(P/3) (\nabla w(P/3))^2 \right],
\]

(19)

assuming a normalized Gaussian wave function

\[
\hat{\varphi}_B(q, s) = \mathcal{N}_B e^{-q^2/2\Lambda_B^2} e^{-s^2/2\Lambda_B^2}.
\]

(20)

E. First conclusions

Let us summarize and analyze our first results. The transverse spectrum of mesons from recombination is proportional to \( C_M w^2(P_T/2) \) whereas fragmentation would generate a distribution proportional to \( D(z) \otimes w(P_T/z) \). For an exponential parton spectrum \( w = e^{-P_T/T} \) the ratio of recombination to fragmentation is

\[
\frac{R}{F} = \frac{C_M}{\langle D \rangle} e^{-P_T/(1 + \frac{\alpha}{\langle z \rangle})} - \frac{P_T}{Q} \]

(21)

and fragmentation ultimately dominates at high \( P_T \). Again, this holds both for mesons and baryons. This implies that fragmentation from an exponential spectrum and recombination from a power law spectrum are suppressed. We will thus omit these contributions in our work.

One might ask whether these considerations are still valid in the case of more realistic formulation of recombination. It turns out that the basic formulae obtained above are still valid in a relativistic description. Deviations are less than 20\% for \( P_T > 2 \) GeV/c.

Given an exponential parton spectrum we note that recombination yields a constant baryon-to-meson ratio. The ratio is then only determined by the degeneracy factors

\[
\frac{d\mathcal{N}_B}{d\mathcal{N}_M} = \frac{C_B}{C_M}.
\]

(23)

Just counting the hadron degeneracies, the direct \( p/\pi^0 \) ratio (neglecting protons and pions from secondary hadronic decays) would be \( \sim 2 \), in contrast to \( \sim 0.2 \) from fragmentation in pQCD. We will later see that finite mass effects and superposition with the fragmentation process will bring down the \( p/\pi^0 \) ratio from 2 to approximately 1 in the range between 2 and 4 GeV/c transverse momentum.

III. The recombination formalism

In this section, we turn to a better description of recombination. This will require a more realistic model of the parton phase including longitudinal and transverse expansion as well as an improved space-time picture.

Let us consider a system of quarks and antiquarks evolving in Minkowski space. We choose a spacelike hypersurface \( \Sigma \) on which recombination of these partons
into hadrons occurs. In the simplest scenario, that could be just a slice of Minkowski space with fixed time \( t_0 \), leading back to the case we described in the previous subsection.

It has been discussed in the literature \[ \text{[35, 38]} \], how the freeze-out can be smeared around the hypersurface to account for a finite hadronization time. However, RHIC experiments suggest a very rapid freeze-out. The measured two-particle correlation functions are consistent with an extremely short emission time in the local rest frame, suggesting a sudden transition after which the individual hadrons interact only rarely \[ \text{[37]} \].

This can be understood by the fact that the hadronization time in the laboratory frame is Lorentz contracted. The Lorentz factors \( \gamma_T \) are 3.35 for a 3 GeV/c proton and 22 for a pion of the same \( P_T \). The hadronization time — even if it is more than 1 fm/c in the rest frame of the hadron — will be experienced as considerably shorter by fast hadronizing particles.

### A. Wigner function formalism

Recombination has already been considered before in a covariant form utilizing Wigner functions for the process of baryons coalescing into light nuclei and clusters in nuclear collisions \[ \text{[35, 38]} \]. Here, we will provide a derivation for the recombination of a quark antiquark pair into a meson. The generalization to a three quark system recombining into a baryon is straightforward.

By introducing the density matrix \( \hat{\rho} \) for the system of partons, the number of quark-antiquark states that we will interpret as mesons is given by

\[
N_M = \sum_{ab} \int \frac{d^3P}{(2\pi)^3} \langle M; P | \hat{\rho}_{ab} | M; P \rangle \tag{24}
\]

Here \( | M; P \rangle \) is a meson state with momentum \( P \) and the sum is over all combinations of quantum numbers — flavor, helicity and color — of valence partons that contribute to the given meson \( M \). We insert complete sets of coordinates

\[
N_M = \int \frac{d^3P}{(2\pi)^3} d^3\hat{r}_1 d^3\hat{r}_2 d^3\hat{r}_3 \langle M; P | \hat{\rho} | \hat{r}_1, \hat{r}_2, \hat{r}_3 \rangle \times \langle \hat{r}_1, \hat{r}_2 | \hat{\rho} | \hat{r}_1', \hat{r}_2' \rangle \langle \hat{r}_1', \hat{r}_2' ; M; P \rangle. \tag{25}
\]

and change the variables to \( r_{1,2} = (\hat{r}_{1,2} + \hat{r'}_{1,2})/2 \) and \( r'_{1,2} = \hat{r}_{1,2} - \hat{r'}_{1,2} \). We define the 2-parton Wigner function \( W_{ab}(r_1, r_2; p_1, p_2) \) as

\[
\langle r_1 - \frac{r_1'}{2}, r_2 - \frac{r_2'}{2} | \hat{\rho} | r_1 + \frac{r_1'}{2}, r_2 + \frac{r_2'}{2} \rangle = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} e^{-ip_1 \cdot r_1} e^{-ip_2 \cdot r_2} W_{ab}(r_1, r_2; p_1, p_2) \tag{26}
\]

and introduce a notation for the meson wave function \( \varphi_M \)

\[
\langle r_1 + \frac{r_1'}{2}, r_2 + \frac{r_2'}{2} | \pi; P \rangle = e^{-iP \cdot (R + R'/2)} \varphi_M \left( r - \frac{r'}{2} \right). \tag{27}
\]

It is convenient to change coordinates again to

\[
R^{(t)} = \frac{r^{(t)} + r_2^{(t)}}{2}, \tag{28}
\]

\[
r^{(t)} = r_1^{(t)} - r_2^{(t)} \tag{29}
\]

with conjugated momenta

\[
\hat{P} = p_1 + p_2, \tag{30}
\]

\[
q = (p_1 - p_2)/2. \tag{31}
\]

We arrive at

\[
N_M = \sum_{ab} \int \frac{d^3P}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \int d^3R d^3r d^3r' \times W_{ab} \left( R + \frac{r}{2}, R - \frac{r}{2}; P - q, P + q \right) \times e^{iq \cdot r'} \varphi_M \left( r + \frac{r'}{2} \right) \varphi_M \left( r - \frac{r'}{2} \right). \tag{32}
\]

The integration over \( R' \) has been carried out and provides the three-momentum conservation \( P = \hat{P} \).

We define the Wigner function \( \Phi_M^W \) of the meson as

\[
\Phi_M^W(r, q) = \int d^3r' e^{-iq \cdot r'} \varphi_M \left( r + \frac{r'}{2} \right) \varphi_M \left( r - \frac{r'}{2} \right). \tag{33}
\]

Then

\[
\frac{dN_M}{d^5P} = (2\pi)^{-3} \sum_{ab} \int d^3R \int \frac{d^3q}{(2\pi)^3} d^3r \left( \Phi_M^W(r, q) \right). \tag{34}
\]
To evaluate this expression, we have to model the Wigner functions of the parton system and of the meson. We assume that the 2-parton Wigner function can be factorized into a product of classical one-particle phase space distributions \( w \). Furthermore the color and helicity states for each flavor will be degenerate. We can therefore replace the sum over quantum numbers \( a \) and \( b \) by a degeneracy factor \( C_M \).

We introduce the following simplifications: the spatial width \( \Delta r \) of the hadron wave function, translating into a width of \( \Phi_M^W \), will be small compared to the nuclear size of the system at hadronization. The phase space distributions \( w(r; p) \) are steeply falling functions of the momentum \( p \), but vary much less with the spatial coordinate \( r \) within the typical size \( \Delta r \) of a hadron. We shall assume that the spatial variation of the phase space distribution is small on this scale, replacing \( R \pm r/2 \) with \( R \).

In our derivation we implicitly chose an equal time formalism by introducing the spatial coordinate states \( |\vec{r}\rangle \) at a fixed time. However, in our result the integration over the final state phase space

\[
d^3Pd^3r = d^3Pd^3R P \cdot u(R)/E
\]

is manifestly Lorentz invariant. Here \( E \) is the energy of the four vector \( P \) and \( u(R) \) is the future oriented unit vector orthogonal to the hypersurface defined by the hadronization volume. This form can be easily generalized for an arbitrary hadronization hypersurface \( \Sigma \). We have

\[
E d^3N_M = C_M \int_{\Sigma} \frac{d^3P \cdot u(R)}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \times w_a \left( R; \frac{P}{2} - q \right) \Phi_M^W(q) \ w_b \left( R; \frac{P}{2} + q \right).
\]

Here we have rewritten the spatially integrated Wigner function of the meson in terms of light cone coordinates for quark \( a \) and approximated it by a squared light cone wave function of the meson

\[
\Phi_M(x, k_\perp) = |\phi_M(x, k_\perp)|^2.
\]

The light cone wave functions we introduce here do not necessarily coincide with the light cone wave functions \( \Phi_{LC}(x, k_\perp, \mu) \) used for exclusive processes in QCD. There, a hadron is decomposed into a series of Fock states of perturbative partons, starting from the valence states of perturbative partons, starting from the valence quarks in meson \( M \) and

\[
\Phi_M^W(q) = \int d^3r \ \Phi_M^W(r, q)
\]

is the spatially integrated Wigner function of the meson.

### B. Local light cone coordinates

The structure of hadrons is best known in the infinite momentum frame which is described in light cone coordinates. If we let the hadron momentum \( P \) define the \( z \) axis of the hadron light cone (HLC) frame, we can introduce the light cone coordinates e.g. \( q^-, q_\perp \) for the relative momentum \( q \). Note that we denote transverse momenta in the HLC frame — i.e. the component orthogonal to \( P \) — by the label \( \perp \), but transverse momenta in the center of mass (CM) frame of the heavy ion collision — orthogonal to the beam axis — by the label \( T \). The labels + and − always refer to light cone coordinates in the HLC frame for a given \( P \). We fix the HLC frame by a simple rotation from the CM frame, i.e. \( P_{\perp} = (E + |P|)/\sqrt{2} \). We reintroduce the momentum \( k = P/2 - q \) of parton \( a \) in the meson. Assuming a mass shell condition \( k^2 = m_a^2 \) with arbitrary but fixed virtuality \( m_a \), we can rewrite the integral

\[
d^3k = d^3k_\perp \delta(k^2 - m_a^2) = dk_\perp d^2k_\perp \frac{k^0}{k^+}
\]

in HLC coordinates. \( m_a \) will be of order \( \Lambda_{\text{QCD}} \), or more precisely, of the order of a constituent quark mass. In the HLC frame, where we assume that formally \( P^+ \to \infty \), we parametrize \( k^+ = xP^+ \) with \( 0 \leq x \leq 1 \). Since \( k^- \ll k^+ \), we have \( k^0/k^+ \approx 1/\sqrt{2} \). We end up with

\[
E d^3N_M = C_M \int_{\Sigma} \frac{d^3P \cdot u(R)}{(2\pi)^3} \int \frac{dx P^+ d^2k_\perp}{\sqrt{2}(2\pi)^3} \ \times w_a \left( R; xP^+, k_\perp \right) \Phi_M(x, k_\perp) \ w_b \left( R; (1-x)P^+, -k_\perp \right).
\]
fer (higher twist). This usually permits a fairly good description of hard exclusive processes in terms of the lowest Fock state.

As we will discuss below, we expect the parton spectrum of a heavy ion collision at freeze out to be composed of an exponential part at small transverse momentum and a power law tail, given by pQCD, at high transverse momentum. We will study recombination in the pQCD domain in a forthcoming publication.

Here, we want to focus on recombination from the exponential part of the parton spectrum. It will be given by a slope $1/T^*$ with a temperature-like parameter $T^*$. $T^*$ also sets the scale for the typical momentum transfer in the parton medium before hadronization. If $T^*$ is an effective blue shifted temperature $T^* = \sqrt{(1+\beta)/(1-\beta)}T$ in an expanding medium with physical temperature $T$ and flow velocity $\beta$, then the typical scale $T < T^*$ will even be smaller. Here we use the phase transition temperature at zero baryon density $T \approx 175$ MeV.

We cannot expect that perturbative QCD will work as a description of partons at the phase transition. What are the quanta that recombine? We know that in pQCD for decreasing scales the non-perturbative (long-range) matrix elements describing hadrons get more "valence like", though we cannot seriously extend this study to scales below 1 GeV. We will assume here that we recombine effective constituent partons, taking into account only the valence structure of the hadron. Gluons are no dynamic degrees of freedom in this picture, and the quarks and antiquarks will have an effective mass.

This picture is supported by the recent discovery of "magical factors" of 2 and 3 in measurements of spectra and the elliptic flow of mesons and baryons respectively at RHIC. Later we will apply our assumptions to partons in the exponential spectrum having as much as 2 GeV/c of transverse momentum. This might raise doubts about the validity of an effective description. However, we have to keep in mind that the momentum is in principle meaningless, only the momentum transfer experienced by a particle in a reaction sets the scale at which we resolve its structure.

From the normalization condition

$$\langle M; P | M; P' \rangle = (2\pi)^3 \delta(P - P')$$  \hspace{1cm} (41)

we infer $\int d^3r \varphi_M(r) = 1$ and finally

$$\int \frac{dk^+ d^2k_\perp}{\sqrt{2(2\pi)^3}} |\varphi_M(x, k_\perp)|^2 = 1.$$  \hspace{1cm} (42)

This is different from the light cone wave functions in exclusive processes which involve a dimensionful quantity connected to the weight of the Fock state, e.g. the pion decay constant $f_\pi$ in case of the pion.

We further utilize the fact that we work in a kinematic regime where $P^+$ is large compared to all non-perturbative quantities. Of course, our main concern here is that we do not really know the shape of the wave function $\varphi_M$. We can choose a factorized ansatz

$$\varphi_M(x, k_\perp) = \varphi_M(x) \Omega(k_\perp)$$  \hspace{1cm} (43)

with a longitudinal distribution amplitude $\varphi_M(x)$ and a transverse part. We know that the transverse shape should be quite narrow, e.g. given by a Gaussian with a width $\Lambda_\perp \ll P^+$. Considering hadrons at mid rapidity, $k_\perp$ will mainly be pointing in the longitudinal and azimuthal directions in the CM frame, where the variation of the parton distributions $w$ is small. This implies $w(R; xP^+, k_\perp) \approx w(R; xP^+)$ for typical transverse momenta $k_\perp \sim \Lambda_{QCD} \ll P^+$. We can then integrate the $k_\perp$ dependence of the wave function. This leaves us with

\begin{equation}
E \frac{dN_M}{d^3P} = C_M \int \frac{d^3R P \cdot u(R)}{(2\pi)^3} \int_0^1 dx \, w_a(R; xP^+)|\varphi_M(x)|^2 \, w_b(R; (1-x)P^+).  \hspace{1cm} (44)
\end{equation}

The amplitude $\varphi_M(x)$ encodes the remaining QCD dynamics. We expect it to be peaked around $x = 1/2$, meaning that the two quarks will carry roughly the same amount of momentum. But the width of the distribution, since it is formulated in terms of momentum fractions, could be quite broad in momentum space. Thus we cannot use the same argument as for the transverse coordinates in order to integrate out this degree of freedom. However, for an exponential parton spectrum we have

$$w_a(R; xP^+) w_b(R; (1-x)P^+) \sim e^{-xP^+/T} e^{-(1-x)P^+/T} = e^{-P^+/T}.$$  \hspace{1cm} (45)

Hence the product of parton distributions is independent of $x$ and we can perform the integral over $x$, which just gives the trivial normalization of the wave function from $\Omega(k_\perp)$. There will be corrections to that from momentum components other than $P^+$ which are not additive because energy is not conserved (see next subsection).
Where we want to take into account wave functions, we adopt the asymptotic form of the perturbative pion distribution amplitude
\[ \phi_M(x) = \sqrt{30}x(1-x) \] (46)
as a model.

C. Energy conservation

Energy conservation is not manifest in the recombination approach. Since we are dealing with a 2→1 process, one of the particles in general needs to be off mass shell \[35\]. This does not pose a problem in the physical environment where recombination takes place. Both in the quark phase and in the hadronic phase we expect interactions with the surrounding medium to occur. Since recombination, as described in this work, has been simplified to a counting of quantum numbers and momenta, without real QCD dynamics, we neglect effects of these additional interactions that ensure energy conservation. This is tolerable due to the small time scale of hadronization and that changes in the parton distribution \[w\] by interactions during this time are negligible. We also neglect effects that final state interactions between the hadrons could have on the spectra – these are expected to contribute mainly in the low momentum domain, \(P_T < 1 \text{ GeV}\).

For large momenta \(P\), the energies of the particles are dominated by the kinetic energies and not by the masses. The light cone wave functions have a transverse width \(\Lambda_\perp\) which is a non-perturbative momentum scale. Therefore the momenta of the recombining quarks will be collinear up to transverse momenta of order \(\Lambda_\perp\). This implies that the energy is conserved up to terms of order \(\Lambda_\perp^2/P^+\) and \(m^2/P^+\), where \(m\) stands for the masses of the participating particles.

Energy conservation is a problem at low transverse momentum and can only be overcome if one takes further interactions between the partons into account. This would require a much more sophisticated formulation that includes nonperturbative initial- and final-state effects.

Restricting ourselves to large transverse momentum finally permits to use light cone fractions \(x\) for the spatial momentum \(P\) in the CM frame instead of the momentum \(P^+\) in the HLC frame. The relation \(p^+ = xP^+\) between the quark momentum \(p\) and the hadron momentum \(P\) in light cone coordinates translates to
\[ p = xP + O(\frac{M^2}{|P|}) + O(\Lambda_\perp). \] (47)

This is a leading order expansion in \(M^2/P^2\) where \(M\) is the hadron momentum. At mid rapidity this further translates to the simple formula \(p_T = xP_T\) for the transverse momenta.

D. Low transverse momentum and hadron thermodynamics

It is well known that total hadron yields at RHIC can be described very accurately by a purely statistical model, using only hadronic properties, such as masses, chemical potentials and spin degeneracies \[12, 13, 14\]. The hadron yields are given by the \(P_T\)-integrated spectra and are naturally dominated by particles with less than 2 GeV/c transverse momentum. The picture of thermal hadron production does not necessarily require the existence of a parton phase.

However, one can show that recombination of a thermalized parton phase is consistent with thermal hadron production in the limit \(P_T \to \infty\). Although pQCD will eventually dominate over both mechanisms at large \(P_T\), this nevertheless suggests that the recombination mechanism connects a thermal parton phase with the observed thermal hadron phase. Therefore it is justified to call recombination from a thermal parton phase the microscopic manifestation of statistical hadron production.

E. Summary of the formalism

We want to summarize what we have so far. From \[14\] the meson spectrum is given by

\[ E \frac{d^3N_M}{d^3P} = C_M \int d\sigma_R \frac{P \cdot u(R)}{(2\pi)^3} \int_{0}^{1} dx \, w_a(R; xP) |\phi_M(x)|^2 \, w_b(R; (1-x)P). \] (48)

where \(d\sigma_R\) measures the volume of the hypersurface \(\Sigma\) and \(R \in \Sigma\). For baryons, the same steps result in the expression

\[ E \frac{d^3N_B}{d^3P} = C_B \int d\sigma_R \frac{P \cdot u(R)}{(2\pi)^3} \int \mathcal{D}x_1 \, w_a(R; x_1P) w_b(R; x_2P) w_c(R; x_3P) |\phi_B(x_1, x_2, x_3)|^2. \] (49)

\(a, b\) and \(c\) are the valence partons and \(\phi_B(x_1, x_2, x_3)\) is the effective wave function of the baryon in light cone coordinates. We use the short notation
\[ \int \mathcal{D}x_i = \int_{0}^{1} dx_1 \, dx_2 \, dx_3 \delta(x_1 + x_2 + x_3 - 1) \] (50)
for the integration over three light cone fractions. Inspired by the asymptotic form of the light cone distribution amplitudes for pions and nucleons we choose

\[ \phi_M(x) = \sqrt{30}x(1-x), \quad (51) \]
\[ \phi_B(x_1, x_2, x_3) = 12\sqrt{35}x_1x_2x_3. \quad (52) \]

These wave functions are broad in momentum space. In order to study the effect of the width of the wave functions on our results it will be interesting to alternatively explore the case of narrow wave functions in the spirit of Sec. III. The limiting case are \( \delta \)-shaped wave functions

\[ |\phi_M(x)|^2 = \delta \left( x - \frac{1}{2} \right) \]
\[ |\phi_B(x_1, x_2, x_3)|^2 = \delta \left( x_1 - \frac{1}{3} \right) \delta \left( x_2 - \frac{1}{3} \right). \quad (54) \]

The spectra are then given by

\[ E \frac{N_M}{d^3P} = C_M \int d\sigma_R \frac{P \cdot u(R)}{(2\pi)^3} w_a \left( R; \frac{P}{2} \right) w_b \left( R; \frac{P}{2} \right) \]
\[ E \frac{N_B}{d^3P} = C_B \int d\sigma_R \frac{P \cdot u(R)}{(2\pi)^3} \]
\[ \times w_a \left( R; \frac{P}{3} \right) w_b \left( R; \frac{P}{3} \right) w_c \left( R; \frac{P}{3} \right). \quad (56) \]

It is an important observation that in the case of exponential parton distributions, the shape of the wave function is almost negligible. We have to be aware that there will be corrections to the above equations of order \( m/|P| \) and \( \Lambda_{QCD}/|P| \), where \( m \) is the mass of the hadron or the partons, reducing their range of applicability to large \( P \).

On the other hand the spectrum of hadrons from fragmentation is given by

\[ E \frac{dN_h}{d^3P} = \sum_a \int_0^1 \frac{dz}{z^2} D_{a-h}(z) E_a \frac{dN_a}{d^3P_a}. \quad (57) \]

The number of partons can be obtained from the cross section via the impact parameter dependent nuclear thickness function \( dN/d^3P = T_{A \to A}(b)d\sigma_R/d^3P \). We take \( T_{A \to A}(0) = 9 \Lambda^2/8\pi R_A^2 \) for central collisions. \( R_A = A^{1/3}1.2 \) fm is the radius of the nucleus.

**IV. HADRON AND PARTON SPECTRA**

In the following we want to discuss the parton phase created in collisions of gold nuclei at RHIC with \( \sqrt{s} = 200 \) GeV per nucleon pair. We will then proceed to calculate the hadron spectra emerging from recombination and fragmentation of this parton phase.

A. Modeling the parton phase

Assuming longitudinal boost invariance, we fix the hypersurface \( \Sigma \) by choosing \( \tau = \sqrt{t^2 - z^2} = \text{const.} \) for

\[ R^a = (t, x, y, z) = (\tau \cosh \eta, \rho \cos \phi, \rho \sin \phi, \tau \sinh \eta). \quad (58) \]

It is convenient to introduce the space time rapidity \( \eta \) and the radial coordinate \( \rho \), since the measure for the hypersurface \( \Sigma \) then takes the simple form \( d\sigma_R = \tau \, d\eta \, d\rho \, d\rho \, d\phi \) and the normal vector is given by

\[ u^\mu(R) = (\cosh \eta, 0, 0, \sinh \eta). \quad (59) \]

Using a similar parametrization of the parton momentum

\[ p^\mu = (m_T \cos \psi, p_T \cos \Phi, p_T \sin \Phi, m_T \sinh \psi) \]

with rapidity \( \eta \) and transverse mass \( m_T = \sqrt{m^2 + p_T^2} \), we obtain \( p \cdot u(R) = m_T \cosh(\eta - \psi) \). We remind the reader that we will use capitalized variables \( (P, P_T, M, M_T, \ldots) \) for hadrons.

Now we have to specify the spectrum of partons. As already discussed above we assume that the parton spectrum consists of two domains. At large \( P_T \), the distribution of partons is given by perturbative QCD and follows a power law. For the transverse momentum distribution at midrapidity for central collisions \((b=0)\) we use the parametrization \[45, 46\]

\[ \left. \frac{dN_0^{\text{pert}}}{d^2p_T dy} \right|_{y=0} = K \frac{C}{(1 + p_T/B)^\beta}. \quad (61) \]

The parameters \( C, B \), and \( \beta \) are taken from a leading order (LO) pQCD calculation and can be found in \[46\] for the three light quark flavors and gluons. A constant \( K \) factor of 1.5 is included to roughly account for higher order corrections in \( \alpha_s \). The calculation includes nuclear shadowing of the parton distributions, but no higher twist initial state effects. Higher twist effects, like the Cronin effect, will fade like \( A^{1/3} \Lambda_{QCD}^2/P_T^2 \) for high transverse momentum \([48]\). Since we will show that fragmentation and pQCD are only dominant for transverse momenta above 5 GeV/c in the hadron spectrum for RHIC, it is safe to omit the Cronin effect.

Energy loss of partons, resulting in a shift of the transverse momentum spectrum \([3, 4]\), is taken into account and parameterized as

\[ \Delta p_T(b, p_T) = \epsilon(b) \sqrt{p_T} \left( \frac{L}{R_A} \right). \quad (62) \]

For central collisions we take \( \langle L \rangle = R_A \) and therefore \( \Delta p_T(0, p_T) = \epsilon(0) \sqrt{p_T} \) (we postpone the discussion of the impact parameter dependence to Subsection IV.C). The choice of \( \langle L \rangle = R_A \) neglects the fact that for the strong quenching observed at RHIC energies, jet emission becomes a surface effect \([3, 49]\). However, we note
that only the product $\epsilon \langle L \rangle$ as a whole is a parameter. We have no ambition here to make a connection to the microscopic parameters of jet quenching, therefore we will not disentangle $\epsilon$ and $\langle L \rangle$. This would require a more sophisticated model of the emission geometry. We also do not use a radial profile for the emission and the density of the medium. This can be found discussed elsewhere in the literature \cite{4, 21, 22}. Nevertheless, our “minimal” description of energy loss is quite successful to describe the available data on high-$P_T$ spectra ($P_T > 5 \text{ GeV/c}$) for $\pi^0$, $K_L^0$ and charged hadrons ($h^+ + h^-)/2$ in central Au+Au collisions. From a fit to these data we find $\epsilon_0 = \epsilon(0) = 0.82 \text{ GeV}^{1/2}$ for central Au+Au collisions. This value corresponds to an average energy loss of 3 GeV for a 10 GeV parton in a central Au+Au collision. The perturbative spectrum for up and strange quarks at midrapidity is shown in Fig. 1. For fragmentation of pions, kaons, protons and antiprotons we use LO KKP fragmentation functions with the scale set to the hadron transverse momentum $P_T$ \cite{12}. A fragmentation is calculated with the LO fragmentation functions of de Florian, Stratmann and Vogelsang \cite{30}.

Besides the perturbative tail of the parton spectrum that will turn into hadrons via fragmentation, we assume the existence of a spectrum of thermalized partons that are recombining at hadronization and dominate at low and intermediate values of $P_T$. In this phase we assume the effective degrees of freedom to be constituent quarks without dynamical gluons. We take the spectrum to be exponential with a given temperature $T$

$$w_a(R; p) = \gamma_a e^{-p-v(R)/T} e^{-\eta^2/2\Delta^2} f(\rho, \phi). \quad (63)$$

$\gamma_a$ is a fugacity factor for each parton species $a$. We also include longitudinal and radial flow through the velocity vector

$$v^\mu(R) = (\cosh \eta_L \cosh \eta_T, \sin \eta_T \cos \phi, \sinh \eta_T \sin \phi, \sinh \eta_L \cosh \eta_T). \quad (64)$$

$\eta_L(R)$ and $\eta_T(R)$ are the rapidities of the longitudinal and radial flow which still could depend on the space time point $R \in \Sigma$. For the longitudinal expansion we choose a Bjorken scenario where the longitudinal rapidity is simply fixed by the space time rapidity

$$\eta_L(R) = \eta. \quad (65)$$

The transverse flow is given by a velocity $v_T(R)$ with $v_T = \tanh \eta_T$. For practical purposes we will not work with a radial profile but assume $v_T$ to be independent of $\rho$ and $\phi$. However, for collisions with finite impact parameter $b$ we will later allow a dependence of $v_T$ on the azimuthal angle $\phi$ in order to describe the measured elliptic asymmetry in the spectra. The space-time structure of the parton source in \cite{13} is given by a transverse distribution $f(\rho, \phi)$ and a wide Gaussian rapidity distribution with a width $\Delta$.

We assume that hadronization occurs at $\tau = 5 \text{ fm}$ at a temperature $T = 175 \text{ MeV}$ in the parton phase. This is consistent with predictions of the phase transition temperature at vanishing baryon chemical potential from lattice QCD \cite{41}. For the spread of the parton distribution in longitudinal direction we choose $\Delta = 2$. The constituent quark masses are taken to be 260 MeV for $u$ and $d$ quarks and 460 MeV for $s$ quarks.

The two component model of the parton spectrum with an exponential bulk and a power law tail is also predicted by parton cascades like VNI/BMS \cite{51}, although the interactions in that case are purely perturbative. This implies that an exponential shape of the spectrum does not necessarily mean that the parton system is in thermal equilibrium.

In the region where contributions from recombination and fragmentation are of the same size we expect other mechanisms to play a role, which interpolate between the two pictures. This could include partial recombination and higher twist fragmentation. In the absence of a consistent description of these mechanisms we simply add both contributions to the hadron spectrum — recombination from the exponential part and the fragmentation from the pQCD part — for $P_T > 2 \text{ GeV/c}$.

### B. Degeneracy factors

It is not a priori clear from QCD what the degeneracy factors $C_h$ for each hadron $h$ are. In principle every quark has 3 color and 2 spin degrees of freedom. On could argue that 3 quarks of any color and spin can form a proton and that quantum numbers can be “corrected”
at no cost by the emission of soft gluons. That would lead to degeneracy factors \( C_p = 2 \times (3 \times 2)^3 \) and \( C_{\alpha} = (3 \times 2)^3 \). On the other hand there are no dynamical gluons in our picture and it would be consistent to require recombining partons to have the right quantum numbers at the beginning.

Surprisingly this is supported from work on recombination in pQCD, where the contributions from color octets and spin-flip states to the recombination of D mesons were found to be small [20]. Using this assumption, the degeneracies are only determined by the degrees of freedom of the hadron, e.g. \( C_p = 2, C_{\alpha} = 1 \). These are exactly the degeneracies used in the statistical model. We will not take into account feeddown from decays of resonances, except for the \( \Lambda \), where the \( \Sigma^0 \) is too close in mass to be suppressed. Hence we use \( C_{\Lambda} = 4 \).

This is different from [10] where we counted \( \Delta \) resonances to give nucleons with weight 1. However this overestimates the correction from \( \Delta \) decays. Nevertheless the degeneracy of 5/3 given in [10] for the proton was of the right size due to a mistake in the normalization of the baryon states, that gave an additional factor of 1/3!. Therefore the numerical results given in [10] are still valid.

We should add that due to the small but probably non-vanishing color octet and spin flip contributions and due to feeddown corrections we expect all degeneracy factors to have an error of at least 20%.

### C. Central collisions

For the momentum spectrum of quarks

\[
E \frac{dN_a^{th}}{d^3p} = g\gamma_a \int \frac{d\sigma_R}{(2\pi)^3} p \cdot u(R) w_a(R; p) 
\]

we rewrite the exponent in (66) as

\[
p \cdot v(R) = m_T \cosh(\eta - \gamma) \cosh \eta_T - p_T \cos(\phi - \Phi) \sinh \eta_T
\]

where \( m_T \) is the transverse mass of quark \( a \). The factor \( g = 6 \) in (66) is counting the color and spin degeneracies. In the case of central collisions (impact parameter \( b = 0 \)) we can assume that \( \nu_T \) and \( f \) are independent of \( \phi \). For simplicity we furthermore assume that \( f \) is also independent of the radial coordinate \( \rho \) and that the radial distribution of the partons is homogeneous up to a radius \( \rho_0 \): \( f(\rho) = \Theta(\rho_0 - \rho) \). We can then easily perform the \( \phi \) and \( \rho \) integrals in (66). If we consider the region around \( y = 0 \), we can also neglect the Gaussian profile function in \( \eta \), since \( \eta^2/\Delta^2 \ll (m_T/T) \cosh \eta \), and integrate over this variable analytically.

The transverse momentum spectrum of partons in the thermal phase is then given by

\[
\frac{N_a^{th}}{d^2p_Tdy} = 2g\gamma_a m_T \frac{\tau_{T}}{(2\pi)^3} \int_0^1 dx_1 |\phi_A(x)|^2 k_A(x, P_T),
\]

\[
A_T = \rho_0^2 \pi
\]

is the transverse area of the parton system and \( I_0 \) and \( K_1 \) are modified Bessel functions. Fig. 1 also shows the thermal spectrum of up and strange quarks. The radial flow velocity \( \nu_T = 0.55c \), fugacities \( \gamma_u = \gamma_d = 1, \gamma_s = 0.9, \gamma_{\bar{s}} = 0.8 \) and the radius \( \rho_0 = 9\ \text{fm} \) were determined by fits of our calculation to the measured hadron spectra in central collisions, see below.

The transverse momentum spectrum of hadrons formed by recombination from the thermal parton spectrum can be derived from Eqs. (68) to be

\[
\frac{N_M}{d^2p_Tdy} = C_M M_T \frac{\tau_{T}}{(2\pi)^3} 2\gamma_a \gamma_\beta I_0 \left[ \frac{P_T \sinh \eta_T}{T} \right] \int_0^1 dx_1 |\phi_M(x)|^2 k_M(x, P_T),
\]

\[
\frac{N_B}{d^2p_Tdy} = C_B M_T \frac{\tau_{T}}{(2\pi)^3} 2\gamma_a \gamma_\beta \gamma_\delta I_0 \left[ \frac{P_T \sinh \eta_T}{T} \right] \int_0^1 dx_1 |\phi_B(x_1, x_2, x_3)|^2 k_B(x_1, P_T)
\]
for mesons and baryons respectively. We introduced the short notations

\[
k_M(x, P_T) = K_1 \left[ \frac{\cosh \eta_T}{T} \left( \sqrt{m_a^2 + x^2 P_T^2} + \sqrt{m_b^2 + (1-x)^2 P_T^2} \right) \right],
\]

\[
k_B(x, P_T) = K_1 \left[ \frac{\cosh \eta_T}{T} \left( \sqrt{m_a^2 + x^2 P_T^2} + \sqrt{m_b^2 + x^2 P_T^2} + \sqrt{m_c^2 + x^2 P_T^2} \right) \right].
\]

(71)

(72)

Here \(m_a\), \(m_b\) and \(m_c\) are the masses of the valence quarks and \(M_T\) is the transverse mass of the hadron. Fig. 2 shows the result of our calculation for the spectra of pions, protons, antiprotons, kaons, Lambdas, Xis and Omegas. See Sec. \(\text{V}\) for discussion.

D. Peripheral Collisions

In order to describe peripheral collisions, we have to scale the perturbative part of the parton spectrum given in (61) by the ratio of thickness functions, or equivalently by the number of binary nucleon collisions

\[
dN_{\text{pert}}^a(b) = \frac{T_{\text{AuAu}}(b)}{T_{\text{AuAu}}(0)} dN_{\text{pert}}^a = \frac{N_{\text{coll}}(b)}{N_{\text{coll}}(0)} dN_{\text{pert}}^a.
\]

(73)

Our values for the number of collisions \(N_{\text{coll}}\) as a function of impact parameter are listed in Table I and are close to the values used by the PHENIX collaboration [52].

<table>
<thead>
<tr>
<th>(b [\text{fm}])</th>
<th>0</th>
<th>5.5</th>
<th>7.5</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>13.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{\text{coll}})</td>
<td>1146</td>
<td>594</td>
<td>350</td>
<td>199</td>
<td>120</td>
<td>61.6</td>
<td>26.0</td>
<td>10.0</td>
<td>5.3</td>
</tr>
</tbody>
</table>

TABLE I: Average number of binary nucleon collisions \(N_{\text{coll}}\) for some values of the impact parameter \(b\) in collisions of gold nuclei.

The length and width of the overlap zone of two nuclei with radius \(R_A\), colliding at impact parameter \(b\), are \(l(b) = \sqrt{R_A^2 - (b/2)^2}\) and \(w(b) = R_A - b/2\). We scale the average length entering the energy loss in (62) as \(\langle L \rangle = (l(b) + w(b))/2\). On the other hand the density of the hot medium is decreasing with increasing impact parameter. We choose the simple ansatz

\[
\epsilon(b) = \epsilon_0 \frac{1 - e^{-(2R_A-b)/R_A}}{1 - e^{-2}}
\]

(74)

for the \(b\) dependence of the energy loss parameter, which describes the data surprisingly well. We refer to [42] for more detailed studies of the jet quenching effect.

For the thermal phase of the parton spectrum, we keep the temperature \(T\) and the hadronization time \(\tau\) independent of the impact parameter \(b\), but adjust the size of the volume according to the profile function \(f(\rho)\). We scale the transverse area \(A_T\) of the parton phase at hadronization with the transverse area of the overlap zone of the two nuclei

\[
A_T(b) = \frac{l(b)w(b)\pi}{R_A^2 \pi} A_T(0) = \frac{l(b)w(b)\rho_0^2}{R_A^2}.
\]

(75)

In principle, the radial flow velocity \(v_T\) is expected to vary with impact parameter. However, it turns out that the slope of the measured hadron spectra above 2 GeV/c is consistent with a constant flow velocity \(v_T = 0.55c\) up to very large impact parameters, so that we fix this value for all \(b\). It may be questionable whether partons are produced in equilibrium in peripheral collisions. We therefore introduce an additional impact parameter dependent fugacity \(\gamma(b)\) common to all quark flavors. However, the measured hadron spectra favor \(\gamma(b) = 1\) up to high values of \(b\). Corrections are only necessary for very peripheral collisions. We take \(\gamma(b) = 1\) for \(b \leq 10.5\) fm, \(\gamma(11\) fm) = 0.7, \(\gamma(12\) fm) = 0.4 and \(\gamma(13\) fm) = 0.4.

E. Elliptic Flow

For hadrons from fragmentation we assume that the azimuthal anisotropy in peripheral collisions is induced by the azimuthal dependence of the energy loss [4, 49, 53, 54]. To determine the coefficient \(v_2\) we generalize (62) to

\[
\Delta p_T(b, p_T, \Phi) = \epsilon_0 \sqrt{p_T} \frac{(L)}{R_A} (1 - \alpha \cos 2\Phi),
\]

(76)

where \(\epsilon(b)\) is given by (74). This leads to a spectrum \(\frac{d^2N}{dP_T^2}\) with non trivial dependence on \(\Phi\). From that we obtain the elliptic flow by applying the definition

\[
v_2(P_T) = \langle \cos 2\Phi \rangle = \frac{\int d\Phi \cos 2\Phi \frac{d^2N}{dP_T^2}}{\int d\Phi \frac{d^2N}{dP_T^2}}.
\]

(77)

The parameter \(\alpha\) is given by the collision geometry. It has the value

\[
\alpha = \frac{w(b) - l(b)}{w(b) + l(b)}
\]

(78)

for given impact parameter \(b\).

In the thermal phase, the hydrodynamic expansion from an originally anisotropic overlap zone of both nuclei (for \(b \neq 0\)) induces an elliptic anisotropy of the parton spectrum. This is leading to a dependence of the transverse flow on the azimuthal angle \(\phi\). To model the
FIG. 2: Hadron spectra at midrapidity as a function of transverse momentum $P_T$ for central Au+Au collisions at $\sqrt{s} = 200$ GeV. We show fragmentation (dotted), recombination (dashed) and the sum of both contributions (solid line) versus data from PHENIX and STAR (for $\Phi$, $\Xi^+$ + $\Xi^-$ and $\Omega$ + $\bar{\Omega}$ recombination only). $\Omega$ + $\bar{\Omega}$ data is minimum bias, therefore the result of a calculation with impact parameter $b = 10$ fm is also shown. All data is preliminary except $\pi^0$, $p$ and $\bar{p}$ data from PHENIX. All error bars give statistical errors only, except for $\pi^0$ data from PHENIX which give the total error. See Sec. IV for more details.
anisotropy in the parton phase we take the azimuthal
dependence of the transverse rapidity to be
\[ \eta_T(\phi) = \eta^0_T (1 - f(p_T) \cos 2\phi) . \] (79)
Here \( \eta^0_T \) is the rapidity given by the flow velocity \( \nu_T = 0.55 c \), so that \( \tanh \eta^0_T = 0.55 \). The amplitude of the
anisotropy \( f(p_T) \) is given by the geometrical anisotropy
\[ \alpha \text{ at low transverse momenta, but faster partons will ex-} \]
perience the anisotropy in the expansion less than slower ones. Therefore we choose an ansatz
\[ f(p_T) = \frac{\alpha}{1 + (p_T/p_0)^2} . \] (80)
From Eqs. (65-67) one obtains (55, 56)

\[ v_2^a(T) = \langle \cos(2\Phi) \rangle = \frac{\int d\phi \cos 2\phi I_2 [p_T \sinh \eta_T(\phi)/T] K_1 [m_T \cosh \eta_T(\phi)/T]}{\int d\phi I_0 [p_T \sinh \eta_T(\phi)/T] K_1 [m_T \cosh \eta_T(\phi)/T]} . \] (81)

Our assumptions imply \( v_2^a = v_2^b = v_2^c = v_2^d \) in the ther-
al phase, but \( v_2^a = v_2^b \) differ slightly at small \( p_T \) because
of the bigger strange quark mass. We determine the pa-
rameter \( p_0 \) in the parametrization of the parton phase from a comparison to the PHENIX measurement of the
elliptic flow of pions (57). We obtain \( p_0 = 1.1 \text{ GeV}/c \).

After having fixed the coefficient \( v_2^a(p_T) \) for each par-
ton species \( a \) we write the azimuthally anisotropic phase
space distribution for thermal partons at midrapidity as
\[ w^a_\text{a}(R; p) = w_a(R; p) \left[ 1 + 2v_2^a(p_T) \cos 2\Phi \right] . \] (82)

Here \( w_a(R; p) \) is the phase space distribution without
anisotropy from Eq. (82). Substituting this into the basic recombination formulae (48, 49) we obtain

\[ v_2^M(P_T) = \frac{\int dx |\phi_M(x)|^2 \left[ v_2^a(xP_T) + v_2^b(xP_T) + v_2^c(xP_T) + v_2^d(xP_T) \right]}{\int dx |\phi_M(x)|^2 \left[ 1 + 2v_2^a(xP_T)v_2^b(xP_T) \right]} k_M(x, P_T) \] (83)

\[ v_2^B(P_T) = \frac{\int dx |\phi_B(x)|^2 \left[ v_2^a(x_1P_T) + v_2^b(x_2P_T) + v_2^c(x_3P_T) + 3v_2^d(x_1P_T)v_2^a(x_2P_T)v_2^b(x_3P_T) \right]}{\int dx |\phi_B(x)|^2 \left[ 1 + 2v_2^a(x_1P_T)v_2^b(x_2P_T) + v_2^c(x_1P_T)v_2^b(x_2P_T) + v_2^d(x_1P_T)v_2^a(x_2P_T)v_2^b(x_3P_T) \right]} k_B(x_1, P_T) \] (84)

for the anisotropies in the meson and baryon spectra respectively. Using the \( \delta \)-function approximation this re-
duces to the relations already given before in the litera-
ture (14, 15)

\[ v_2^M(P_T) = \frac{v_2^a(1/2P_T) + v_2^b(1/2P_T)}{1 + 2v_2^a(1/2P_T)v_2^b(1/2P_T)} , \] (85)
\[ v_2^B(P_T) = \frac{v_2^a(1/2P_T) + v_2^b(1/2P_T) + v_2^c(1/2P_T) + 3v_2^d(1/2P_T)v_2^a(1/2P_T)v_2^b(1/2P_T)}{1 + 2v_2^a(1/2P_T)v_2^b(1/2P_T) + 2v_2^b(1/2P_T)v_2^a(1/2P_T) + 2v_2^d(1/2P_T)v_2^a(1/2P_T)} . \] (86)

If we assume one universal partonic \( v_2 \) for the recom-
bining quarks the above expressions simplify to
\[ v_{2,M}(P_T) = \frac{2v_2(1/2P_T)}{1 + 2v_2(1/2P_T)^2} , \] (87)
\[ v_{2,B}(P_T) = \frac{3v_2(1/2P_T) + 3v_2(1/2P_T)^3}{1 + 6v_2(1/2P_T)^2} . \] (88)
Since the maximal values of $v_2$ will be of the order of 0.1, we can neglect the quadratic and cubic terms and arrive at the following simple scaling law, which connects the elliptic flow of hadrons $v_2$ to those of the partons $v_2^r$:

$$v_2^r(P_r) = n v_2 \left( \frac{1}{n} P_r \right)$$  \hspace{1cm} (89)

with $n$ being the number of valence quarks and antiquarks contained in hadron $h$. This scaling law was indeed already found to hold in STAR data on the elliptic flow of $A$ and $K^0_s$ down to transverse momenta of about 500 MeV/c [16]. This is a very strong support for the recombination picture. Apparently a part of the uncertainty in the recombination mechanism at low $P_T$, introduced by the violation of energy conservation, cancels after taking the ratios in Eqs. (88, 89). The recombination formalism seems to give valid results for $v_2$ down to transverse momenta of several hundred MeV/c.

We combine the contributions to the anisotropic flow from recombination and fragmentation by using the relative weight $r(P_T)$ for the recombination process

$$v_2(P_T) = r(P_T)v_{2,R}(P_T) + \left( 1 - r(P_T) \right)v_{2,F}(P_T).$$  \hspace{1cm} (90)

$r(P_T)$ is defined as the ratio of the recombination contribution to the spectrum and the total yield.

$$r(P_T) = \frac{dN^R/d^2P_T}{dN^R/d^2P_T + dN^F/d^2P_T}.$$  \hspace{1cm} (91)

### F. The statistical thermal model

In this subsection we give a brief account of the statistical model following variant I of [42]. For further details we refer the reader to the comprehensive literature [14, 42].

The hadron spectrum at is supposed to emerge from a hypersurface $\Pi$ and has the form

$$E^2 \frac{dN_h}{d^3P} = \int d\sigma_R \frac{P \cdot v(R)}{(2\pi)^3} G_h(R; P).$$  \hspace{1cm} (92)

We use the same parametrization for the four velocity $v(R)$ as in [41]. The hypersurface $\Pi$ is determined by the condition $\sqrt{v^2} = \tau_{\text{SM}} = \text{const.}$ The hadronic phase-space distribution functions are given by

$$G_h(R; P) = \frac{C_h f_{\text{SM}}(r)}{e^{-(P \cdot v - \mu_B B_h - \mu_S S_h - \mu_I I_h)/\tau_{\text{SM}}} \pm 1},$$  \hspace{1cm} (93)

for bosons and fermions respectively. $r = \tau_{\text{SM}} \sinh \eta_T$ is the radial coordinate and $f_{\text{SM}}(r) = \Theta(r_0 - r)$ is a radial profile function providing a cylindrical shape. $C_h$ is the degeneracy factor and $B_h$, $S_h$ and $I_h$ are baryon number, strangeness and third component of the isospin for hadron species $h$.

Equation (92) can be evaluated analogous to [60]. We note that in the limit $P_T \to \infty$ Eqs. (90, 91) are equivalent to (92) if the same hypersurface and the same temperature and chemical potentials are used. This is an indication that recombination from a thermal parton phase is the underlying microscopic picture of hadron production in a statistical model. While we will not elaborate on this in more detail, we will quote some results of the statistical model for hadron ratios and compare with our calculation.

The geometric parameters are fixed to be $\tau_{\text{SM}} = 7.66$ fm and $r_0 = 6.69$ fm for for most central collisions at RHIC in Ref. [42]. Particle ratios at mid rapidity in a boost-invariant model are not influenced by the expansion of the system [12], thus we can use the parameters which are determined by particle ratios from the entire phase space. We follow [43] and set $T_{\text{SM}} = 177$ MeV, $\mu_B = 29$ MeV, $\mu_S = 10$ MeV and $\mu_I = -0.5$ MeV.

### G. Note on the parameters in our model

We want to give a brief summary of all the parameters for the parton phase. Essentially we have three degrees of freedom for central collisions. Theses are the energy loss given by $\epsilon_0(L)$, the slope of the exponential part given by temperature $T$ and radial flow velocity $v_T$ and the normalization of the recombination spectrum by the volume $\tau A_T$. In addition there are the parton fugacities. After fixing ($L$), $T$ and $\tau$ to physical or at least reasonable values, we retain $\epsilon_0$, $v_T$ and $\rho_0$ as true parameters that were determined by fitting to the final data given by PHENIX for the inclusive $\pi^0$ spectrum [52]. This is in contrast to our previous study where the parameters of the parton spectrum were fixed by the preliminary charged hadron spectrum [10].

The light quark fugacity was set to 1 in accordance with the measured $p/\pi^0$ ratio and the fugacities for antiquarks and strange quarks were obtained from other ratios. The ratio of fugacities $\gamma_b/\gamma_u = 0.9$ can be translated into a baryon chemical potential $\mu_B = 27$ MeV. For other impact parameters, the simple geometric scaling of the volume and the number of collisions with $b$ and a reasonable ansatz for $\epsilon(b)$ describe the data up to $b = 10$ fm. Only for very peripheral collision there is the need to introduce the new parameter $\gamma(b)$.

### V. NUMERICAL RESULTS

In this section we are going to discuss our numerical results on hadron production.

#### A. Hadron spectra

In Fig. [2] we show our results for hadron production from fragmentation and recombination for impact parameter $b = 0$ in central Au+Au collision at $\sqrt{s} = 200$ GeV. We compare to available experimental data from
the PHENIX and STAR collaborations at RHIC. The \( \pi^0 \) spectrum has been measured by PHENIX up to 10 GeV/c. The final data were released very recently \cite{52} together with final data for neutral pion production in \( p+p \) collisions up to 14 GeV/c \cite{55}. Error bars show the total error in the \( \pi^0 \) yield. All our calculations use the expressions \cite{18,19} with realistic light cone wave functions \cite{61}. For protons and neutral pions we also performed the calculations in the \( \delta \)-function approximation for the wave functions \cite{55,60}. Fig. 3 shows the relative deviation \( r_5 \) of the \( \delta \) function approximation from calculations using realistic wave functions \cite{55,61}. For protons and neutral pions we also performed the calculations in the \( \delta \)-function approximation for the wave functions \cite{55,60}. Fig. 3 shows the relative deviation \( r_5 \) of the \( \delta \) function approximation from calculations using realistic wave functions \cite{55,61}. The deviation is less than 22% for protons and less than 12% for pions at small \( P_T \). It becomes considerably smaller at larger \( P_T \) since the violation of energy conservation is less important there. This explains why calculations using \cite{55,60} are often satisfactory.

Preliminary data on \( \pi^+ \) and \( \pi^- \) production are only available up to 2 GeV/c \cite{60}, but we can expect that the global behavior of charged pions is similar to that of neutral pions. All data except for \( \pi^0 \), \( p \) and \( \bar{p} \) shown in Fig. 2 are preliminary. Only statistical errors are given for all sets beside \( \pi^0 \). In the pion spectra we clearly see the two \( P_T \) domains of hadron production. Above 4–5 GeV/c the spectrum is dominated by fragmentation and follows a power law. Below 4 GeV/c the spectrum is exponential and dominated by recombination from the thermal phase. In our calculation the contribution from fragmentation is artificially cut off below 2 GeV/c (corresponding to a lower cut-off of about 4 GeV/c in the parton spectrum) because perturbative QCD loses its validity at low \( P_T \).

In the crossover region the yield is slightly underestimated, due to our simplified treatment not allowing for recombination involving perturbative partons and mixed mechanisms \cite{11}. We also note that below 2 GeV/c the calculated recombination spectrum bends down and underestimates the data. This effect is caused by neglecting the large binding energy of pions in our recombination formalism, in which pions have an effective mass of \( 2m_u \approx 520 \) MeV compared to the true pion mass of 140 MeV. In addition, pions from secondary decays of hadronic resonances are an important contribution at very low \( P_T \).

The same effect can be seen in the kaon spectra, where we again underestimate the yield for \( P_T < 2 \) GeV/c. The \( K^0 \) spectrum was measured by STAR up 6 GeV/c \cite{61}. The preliminary \( K^+ \) and \( K^- \) spectra up to 2 GeV/c are available from PHENIX \cite{60}, and first results on \( K^- \) from STAR up to 4 GeV/c were shown recently \cite{61}. The \( K^0 \) data above 2 GeV/c can be described very well by our calculations. We note that the last four data points of the preliminary STAR data on \( K^- \) follow a different systematics than the rest of the points and also seem to deviate from the \( K^0 \) data. This could indicate a failure of proper particle identification in this momentum range.

Protons and antiprotons have been identified by the PHENIX collaboration up to 4.5 GeV/c and the final data was published very recently \cite{62}. In contrast to the Goldstone bosons considered before, the mass of \( p \) and \( \bar{p} \) in our constituent quark picture is closer to the physical mass of 938 MeV and secondary protons and antiprotons are less abundant. Hence our calculation provides a satisfactory description of the spectra even between 1 and 2 GeV/c. This is true for all hadrons that are not Goldstone bosons. The crossover between the recombination and the fragmentation process is shifted upward for \( p \) and \( \bar{p} \) compared with pions, to around 6 GeV/c. We alert the reader to the apparent suppression of the fragmentation process below 6 GeV/c compared with recombination. This is much more prominent here than in the case of pions. In Fig. 4 we show the ratio \( r(P_T) \) of recombined hadrons to the full calculation, defined in Eq. \cite{61}. The shift of the crossover to higher \( P_T \) from pions over kaons to protons is obvious. The 50% mark changes from 4 to 4.5 to 6 GeV/c.

Here we need to emphasize an important point regarding the perturbative calculation. The fragmentation functions are a non-perturbative input to these calculations, and are derived from other experiments. Most data about fragmentation functions are from \( e^+e^- \) annihilation experiments which do not allow to distinguish between quark and antiquark fragmentation. Quarks have to be created in pairs, so that only fragmentation functions like \( D_{u+\bar{u}\rightarrow p+\bar{p}} \) can be deduced. Additional input from semi-exclusive reactions helps to separate the contributions, but fragmentation functions still require improvement for applications in hadron interactions. The KKP parametrization seems to work well for \( \pi^0 \) production at RHIC \cite{50} but we anticipate more problems for other hadrons, in particular for protons and antiprotons. This suggests a considerably larger theoretical uncertainty for the results from fragmentation of all hadrons other than pions. For this reason, and due to the lack of appropriate fragmentation functions, we do not show...
shown in a previous publication [10] that recombination from the PHENIX collaboration [62]. We have already from RHIC and allow us to make predictions for future

ter set are consistent with all the currently existing data in detail in the section on centrality dependence. 

kaons are taken into account. This spectrum, including its impact parameter dependence, will be discussed separately in Fig. 5: pions, protons, antiprotons and kaons are taken into account. This spectrum, including its impact parameter dependence, will be discussed in detail in the section on centrality dependence.

In summary, our calculations using one fixed parameter set are consistent with all the currently existing data from RHIC and allow us to make predictions for future measurements.

B. Hadron ratios

In Fig. 6 we show hadron ratios. Only statistical errors are shown. The systematic errors can be quite large – we refer the reader to the cited experimental publications for further details.

One of the main motivations at the onset of our investigation was to find an explanation for the surprisingly large proton over pion ratios that are of order one above 1.5 GeV/c. The data on the \( p/\pi^0 \) and \( \bar{p}/\pi^0 \) ratios stem from the PHENIX collaboration [62]. We have already shown in a previous publication [10] that recombination naturally provides a \( p/\pi^0 \) of order one for hadron transverse momenta up to 4 GeV. In addition we predict that a sharp drop beyond 4 GeV/c should be seen when the fragmentation process takes over. The value predicted for the ratio in the fragmentation domain is about 0.1. At small transverse momenta, the statistical model describes the data well but continues to rise beyond 4 GeV/c.

The \( K^+/\pi^+ \) and \( K^-/\pi^- \) ratios have been measured by BRAHMS [64], and the \( K^-/K^+ \) ratio is compared to data from PHENIX [60]. For the statistical model we probe an additional strangeness fugacity and provide curves for values of 1.0, 0.9 and 0.8. For the recombination part \( \gamma_s = 0.8 \) was used as discussed above. The interesting feature of the preliminary \( \bar{p}/p \) data from STAR [63] and PHENIX [60] is that PHENIX implies a flat \( \bar{p}/p \) ratio up to 4 GeV/c, while STAR sees a decrease from 2 GeV/c on. (This may be related to the apparent surplus of \( K^- \) measured by STAR in the same momentum range, indicating a possible failure of charged particle identification beyond 2 GeV/c.) Our calculation predicts a flat ratio in this transverse momentum range in agreement with the PHENIX data. A linear sum of statistical and systematic error is shown for the STAR data.

Preliminary results for the ratios \((\Lambda + \bar{\Lambda})/4K^0\) and

![Image of Fig. 4: The ratio \( r(P_T) = R/(R+F) \) of recombined hadrons to the sum of recombination and fragmentation for \( \pi^0 \) (solid), \( K^0_s \) (dashed) and \( p \) (dotted lines). For protons and pions different impact parameters \( b = 0, 7.5 \) and 12 fm (from top to bottom) are shown. \( K^0_s \) is for \( b = 0 \) fm only.]

![Image of Fig. 5: The spectrum of charged hadrons \((h^+ + h^-)/2\) for four different impact parameters 0, 7.5 (divided by 25), 10 (/200) and 12 fm (/1000) [from top to bottom] in comparison with data from STAR and PHENIX in different centrality bins. Contributions from fragmentation only (dotted) and the sum of recombination and fragmentation (R+F, solid lines) are shown.]

2(Ξ^− + Ξ^+)/((Λ + Λ) were also presented from the STAR collaboration \cite{61}. Our calculations (for Ξ recombination only) are in rather good agreement. Recombination predicts a large peak in the Λ/kaon ratio and a sharp decrease beyond 4 GeV/c similar to the p/π^0 ratio. First indications of such a sharp transition can be seen in the STAR data. This observation supports the recombination picture including a transition to the fragmentation regime beyond 4 GeV/c.

In summary, our calculations are in good agreement with the available RHIC data. The predicted decrease in the proton/pion ratio, if confirmed, and first observations of a similar drop in the Λ/kaon ratio will be strong arguments in favor of the recombination+fragmentation picture.
C. Centrality dependence

Figs. 7 and 8 show the centrality dependence of the $\pi^0$ and $K^0_S$ spectra. Final results in various centrality bins for $\pi^0$ have been published by PHENIX [52]. The $K^0_S$ data are preliminary results from STAR [61]. The impact parameter dependence of the charged hadron spectrum $(h^+ + h^-)/2$ was already shown in Fig. 3 with data from STAR [67] and PHENIX [68].

The impact parameter dependence of the parameters in our calculation was fixed by a fit to the $\pi^0$ data but it is consistent with the kaon and charged hadron data. We notice that with increasing impact parameter the hadrons from fragmentation come ever closer to the data points below the crossover point. Thus fragmentation becomes more and more important in peripheral collisions in accordance with our expectations. For the most peripheral bin in $\pi^0$, with an impact parameter of 13.9 fm we refrained from extracting a recombination contribution. In principle the data can be explained by fragmentation alone down to 2 GeV/c. We give a recombination contribution for $b = 13$ fm although its contribution might be disputable already in this centrality bin.

FIG. 7: The spectrum of neutral pions for impact parameters 0, 5.5 (divided by 5), 7.5 (/25), 9 (/100), 10 (/200), 11 (/500), 12 (/1000), 13 (/2000) and 13.9 fm (/5000) [from top to bottom] in comparison with data from PHENIX in different centrality bins. Contributions from fragmentation only (dotted) and the sum of recombination and fragmentation (R+F, solid lines) are shown. 13.9 fm calculation is fragmentation (F) only.

FIG. 8: The spectrum of neutral kaons for impact parameters 0, 5.5 (divided by 5), 7.5 (/25), 9 (/100), 10 (/200), 11 (/500), 12 (/1000), 13 (/2000) and 13.9 fm (/5000) [from top to bottom] in comparison with data from PHENIX in different centrality bins. Contributions from fragmentation only (dotted) and the sum of recombination and fragmentation (R+F, solid lines) are shown.

FIG. 9: The ratio $p/\pi^0$ for three different impact parameters 0, 7.5 and 13 fm (solid lines, top to bottom) compared to the statistical model (dashed dotted line) and PHENIX data.

Fig. 4 shows the ratio of $r(P_T)$ for three different impact parameters (0, 7.5 and 12 fm) for protons and pions. The systematics confirms that the crossover point shifts to smaller $P_T$ for increasing $b$ and that the weight of fragmentation becomes more and more important. In Fig. 9 we display the impact parameter dependence of the $p/\pi^0$.
ratio. As expected the proton/pion ratio is decreasing with increasing $b$ in the recombination region and is unaltered where fragmentation is dominating.

It is an interesting question at which impact parameter the recombination mechanism becomes negligible for transverse momenta above 2 GeV/$c$. This will happen at smaller impact parameter for pions than for protons. Related to this issue is the question whether recombination contributes to hadron production at central rapidity in $p + A$ or $d + A$ reactions, where the produced matter is less dense. We still expect recombination to be an important mechanism at low $p_T$. However, there will probably be no chemical and thermal equilibration in the parton spectrum, and much less flow. The $b$ dependence in Au+Au collisions is not a good basis for extrapolation since flow and equilibration seem to diminish only in very peripheral collisions. That makes it difficult to give quantitative predictions without data. However we would expect that no recombination effects in pion production are visible above 2 GeV/$c$ in $d+Au$ collisions.

D. Nuclear modification factors

The nuclear modification factor is defined as the ratio of the hadron yield in Au+Au collisions to the one in $p + p$ scaled with the number of collisions

$$R_{AA} = \frac{d^2N_{Au+Au}(b)/dP_T^2}{N_{coll}(b) d^2N_{p+p}/dP_T^2}. \quad (94)$$

Similarly one can consider scaled ratios of different centrality bins like central to peripheral

$$R_{CP} = \frac{N_{coll}(b) d^2N_{Au+Au}(0)/dP_T^2}{N_{coll}(0) d^2N_{Au+Au}(b)/dP_T^2}. \quad (95)$$

In Fig. 10 we show the nuclear modification factor for neutral pions for two impact parameters, $b = 0$ and $b = 10$ fm. We provide ratios taken both with our own $p + p$ calculation and with the $p + p$ results from PHENIX and compare to final data from PHENIX. We notice that there is an apparent uncertainty in the perturbative calculation which makes the two curves using our own $p + p$ calculation and the PHENIX $p + p$ results deviate. However, both curves are consistent with the data for central collisions. The problem is amplified for peripheral collisions, where the curve using our $p + p$ calculation overestimates $R_{AA}$ below 4 GeV/$c$. The spread between both curves can be interpreted as a typical error to be expected in a lowest-order perturbative calculation. The nuclear modification factor for pions shows the strong jet quenching effect that suppresses the pion yield by a factor of 5 for the highest $P_T$ bins in central collision. Recombination predicts a slight increase below 4 GeV/$c$ which can be observed in the data. However, recombination is not able to compensate the loss of pions through jet quenching at high $P_T$. In peripheral collisions jet quenching effects are much weaker.

Fig. 10 displays the scaled ratio $R_{CP}$ for neutral pions and protons. The ratio of impact parameters 0 and 12 fm is used and compared to data from the PHENIX collaboration. The data for protons shows $R_{CP}$ to be between 0.8 and 1 below 4 GeV/$c$, which is quite surprising, considering the strong suppression suffered by the pions in that momentum domain. As already noticed above, recombination is more effective for protons than for pions. Therefore our calculation for the protons yields a similarly value of 0.8 as observed by the experiment. This implies that protons from recombination make up for the loss suffered from jet quenching at intermediate transverse momenta. Our calculations predicts sharp drops in $R_{CP}$ and $R_{AA}$ for protons and antiprotons beyond 4 GeV/$c$ where fragmentation with jet quenching start to dominate.

In Fig. 11 we give $R_{CP}$ for charged hadrons compared to data from STAR and PHENIX. The contribution from recombination below 4 GeV/$c$ leads to a value of about 0.6 which is between the values for protons and pions. The observed steep drop to the value attributed to jet quenching is well described by our theory.

Finally in Fig. 12 we compiled $R_{CP}$ for $K^0_s$ and $Λ + Λ$ together with data from STAR. The different behavior of mesons and baryons is again impressively confirmed. For the first time experimental data indicate that a steep decrease in $R_{CP}$ for baryons will occur beyond 4 GeV/$c$. The data on $Λ + Λ$ suggest a drop to the perturbative value even sharper than what our results show. This could be due to too less $Λ$ baryons from fragmentation using this particular set of fragmentation functions.

![Fig. 10: Nuclear modification factor $R_{AA}$ for impact parameters 0 (bottom) and 10 fm (top). Normalization by $p + p$ is taken via our own calculation (solid) or via PHENIX $p + p$ results (dashed lines). Data on $R_{AA}$ are from the PHENIX collaboration with point-to-point errors only.](image-url)
E. Results on elliptic flow

Fig. 14 shows the elliptic flow $v_2(p_T)$ of $u$ quarks before hadronization. The contributions from jet quenching and from anisotropic flow in the thermal phase are shown separately as well. Due to the very different behavior in the two domains, $v_2$ is more sensitive to mechanisms which interpolate between the perturbative domain and the soft domain. The range of this theoretical uncertainty is highlighted by the shaded region in Fig. 14.

In Fig. 15 we provide results for $\pi^+$. The contributions from fragmentation and recombination are shown separately. The full calculation interpolates between these two curves in the interval between 2 and 4 GeV/c. We also compare the result of a calculation using the $\delta$-function approximation for the wave functions. As expected the deviations are small. Our calculations agree with PHENIX data on $v_2$ of $\pi^+$ and $\pi^-$.

Our results on the particle dependence of elliptic flow are summarized in Fig. 16. One can see the different behavior of mesons and baryons by comparing protons with pions and kaons with $\Lambda$s. $v_2$ for baryons saturates at a higher value than for mesons in the recombination domain. At higher $p_T$, when fragmentation takes over, the results rapidly approach each other. In our calculation, where we do not take into account the binding energies, we cannot resolve the splitting between protons and mesons coming from the mass difference. Nevertheless the agreement with data from PHENIX [57] and preliminary data from STAR [70] is good. In Fig. 17 $v_2$ for charged hadrons is compared with preliminary STAR data [71]. This is interesting since for charged hadrons the measurements extend up to 7 GeV/c and constrain $v_2$ in the pQCD domain. Note that in this case $v_2$ was extracted by STAR from 4 particle correlations. This is supposed to reduce non-flow effects to $v_2$ [72].
FIG. 15: $v_2$ for positively charged pions. We show recombination only (dashed), fragmentation only (dotted) and the full calculation (solid line). The result of a calculation in the $\delta$-function approximation (NW) for the wave function is also shown (dash-dotted line). Data are taken from the PHENIX collaboration [57].

In Fig. 18 we test the scaling law from Eq. (89) for protons and pions. Protons and pions follow one universal curve below 1.5 GeV/$c$, which is very similar to the flow of thermal partons given in Fig. 14. Beyond 1.5 GeV/$c$ we predict a transition to the values given by pQCD. The scaling law is no longer valid in that domain.

We would like to emphasize that we expect modifications from other hadronization mechanisms in the region where we interpolate between the recombination and the fragmentation dominated domains. These could be quite important in the case of $v_2$ and alter the results in the interpolation region, e.g. they could smoothen the transition between both domains. Interactions in the hadronic phase could alter $v_2$ further.

VI. CONCLUSIONS

In this work we have presented extensive evidence that recombination is the dominant hadronization mechanism for central Au+Au collisions at RHIC up to about 4 GeV/$c$ for pions and 6 GeV/$c$ for protons. We have described a covariant framework that permits the calculation of recombination from a dense thermal parton phase using light-cone wave functions for the produced hadrons. This formalism is adequate for momenta much larger than the non-perturbative scales involved. At lower energies, energy and entropy conservation pose a serious problem, the solution of which requires a dynamical, rather than purely kinematic, treatment of the recombination process. We have found that, for practical purposes, hadron spectra are well described down to transverse momenta of 2 GeV/$c$ for Goldstone bosons ($\pi, K$) and 1 GeV/$c$ for other hadrons.

Recombination competes with fragmentation from perturbatively scattered partons. The large energy loss of these partons leads to sizable quenching factors, which reduce the fragmentation contribution and cause it to be buried under soft physics at scales which one would not generally attribute to soft physics. However, 4 GeV/$c$ in the pion and 6 GeV/$c$ in the proton spectrum correspond to a transverse momentum of only 2 GeV/$c$ on average for the coalescing partons.

The interplay of recombination and fragmentation leads to interesting effects in particle ratios and nuclear modification factors. The proton/pion ratio is naturally around one in the recombination regime. For protons the unquenching effect by recombination below 4 GeV/$c$ is so strong that essentially no nuclear suppression can be observed at all in this momentum range. For the proton/pion ratio and suppression factors $R_{AA}$ and $R_{CP}$ we expect a sharp drop beyond 4 GeV/$c$ indicating the beginning of the perturbative regime.

For the azimuthal asymmetry $v_2$, our calculations describe the data well. The different behavior of baryons and mesons is above 1 GeV/$c$ can be explained. The scaling law [59], derived from the recombination formalism, is consistent with data up to 1.5 GeV/$c$. We predict...
FIG. 17: $v_2$ for charged hadrons. Again we show the contributions from different mechanisms as in Fig. 15. Data are preliminary and taken from the STAR collaboration.

FIG. 18: The anisotropy $v_2/n$ for pions (bottom) and protons (top) as a function of transverse momentum $p_T/n$ using the scaling law (89) with $n = 2$ for pions and $n = 3$ for protons. Data points are pions and protons from PHENIX using the same scaling law.

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