Axial anomaly in the reduced model:
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The axial anomaly arising from the fermion sector of \((N)\) or \((N)\) reduced model is studied under a certain restriction of gauge field configurations (the “(1) embedding” with \(N = L^d\)). We use the overlap-Dirac operator and consider how the anomaly changes as a function of a gauge-group representation of the fermion. A simple argument shows that the anomaly vanishes for an irreducible representation expressed by a Young tableau whose number of boxes is a multiple of \(L^2\) (such as the adjoint representation) and for a tensor-product of them. We also evaluate the anomaly for general gauge-group representations in the large \(N\) limit. The large \(N\) limit exhibits expected algebraic properties as the axial anomaly. Nevertheless, when the gauge group is \((N)\), it does not have a structure such as the trace of a product of traceless gauge-group generators which is expected from the corresponding gauge field theory. Renormalization Regularization and Renormalons, Lattice Gauge Field Theories, Gauge Symmetry, Anomalies in Field and String Theories

Introduction

It has been unclear for long time how to define the topological charge in the reduced model for large \(N\) QCD Eguchi:1982nm–Das:1984nb. One may try to define the topological charge in a “fermionic way” through the index theorem. However, since the reduced model is given by a zero-volume limit of a field theory, it is a system of finite degrees of freedom as long as \(N\) is finite even very large. It is then obvious that one cannot have the axial anomaly, unless a certain source of an explicit breaking of axial symmetry is introduced from an onset. Then one may ask: What is a good way to simulate the (quantum) axial symmetry breaking in a system of finite degrees of freedom?

Recently, motivated by a success of the overlap-Dirac operator Neuberger:1998fp in lattice gauge theory (which is also a finite system when the lattice size is finite), authors of a paper Kiskis:2002gr proposed a use of the overlap-Dirac operator in the quenched reduced model Bhanot:1982sh. The overlap-Dirac operator satisfies the Ginsparg-Wilson relation Ginsparg:1982bj and this relation ensures remarkable properties concerning the chiral symmetry. For example, an index relation which is analogous to that in the continuum holds even with finite degrees of freedom Hasenfratz:1998ri. Hence, the overlap Dirac operator provides a natural definition of the topological charge in the reduced model. In fact, explicit gauge configurations which have a non-trivial topological charge have been given in ref. Kiskis:2002gr.

A similar problem to define the topological charge (or the index) may be posed in the context of a matrix model for the type IIB superstring Ishibashi:1996xs which is a zero-dimensional reduction of the \(N = 1\) super Yang-Mills theory in ten dimensions. In this context, a use of the overlap-Dirac operator or the overlap Narayanan:1993wx has been proposed Kitsunezaki:1997iu. (More precisely, this is for a compact version of the IIB matrix model. See also ref. Tada:1999nn.) In a related context, a use of Ginsparg-Wilson relation has been actively investigated recently Aoki:2002fq,Iso:2002jc.

In a paper Kikukawa:2002ms, two of us studied chiral anomalies arising from a fundamental fermion in the “naive” Eguchi:1982nm or the quenched Bhanot:1982sh reduced model. There, it was pointed out that a certain restriction of reduced gauge fields (the (1) embedding) allows a mapping of the problem to that of lattice gauge theory. By using available techniques in the latter, we determined a general form of the topological charge resulted by a use of the overlap-Dirac operator. Also, in chiral-gauge reduced models, it was shown that a single fundamental fermion gives rise to an obstruction for a smooth fermion integration measure, which is analogous to the gauge anomaly in the original theory before reduction.

An important question postponed in ref. Kikukawa:2002ms is how chiral anomalies (which was evaluated only for a fundamental fermion) change as a function of a gauge-group representation of the fermion. In particular, we are interested in if the anomaly cancellation in the original theory is realized in the chiral-
gauge reduced model. As a step toward this investigation, in this paper we study the topological charge (or the axial anomaly) in the reduced model for various gauge-group representations.

In Section 2, we present a general setting of our problem in the naive or the quenched reduced model. Next, in Section 3, we recapitulate basic reasoning and results of ref. Kikukawa:2002ms concerning the axial anomaly. In Section 4, on the basis of a structure of the overlap-Dirac operator, we present general properties of the axial anomaly which can be stated without any approximation. In subsequent sections, we perform a large $N$ calculation of the axial anomaly. To illustrate the idea of this calculational scheme, we first re-derive the result of ref. Kikukawa:2002ms within this approximation. Then, in Section 6, this scheme is applied to general gauge-group representations. Section 7 is devoted to the conclusion.

Reduced model with the overlap-Dirac operator

The fermion sector of the vector-like reduced model is defined by equation $$\langle \mathcal{O} \rangle = \int \bar{\psi} \psi \mathcal{O} \exp(-\bar{\psi}D\psi)$$.