Electromagnetic force on a metallic particle in presence of a dielectric surface

P. C. Chaumet and M. Nieto-Vesperinas
Instituto de Ciencia de Materiales de Madrid, Consejo Superior de investigaciones Científicas, Campus de Cantoblanco
Madrid 28049, Spain

By using a method, previously established to calculate electromagnetic fields, we compute the force of light upon a metallic particle. This procedure is based on both Maxwell’s Stress Tensor and the Couple Dipole Method. With these tools, we study the force when the particle is over a flat dielectric surface. The multiple interaction of light between the particle and the surface is fully taken into account. The wave illuminating the particle is either evanescent or propagating depending on whether or not total internal reflection takes place. We analyze the behavior of this force on either a small or a large particle in terms of the wavelength. A remarkable result obtained for evanescent field illumination, is that the force on a small silver particle can be either attractive or repulsive depending on the wavelength. This behavior also varies as the particle becomes larger.

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I. INTRODUCTION

Since the first demonstration of particle manipulation by the action of optical forces,\textsuperscript{1,2} optical tweezers\textsuperscript{3} and other configurations of light beams have been established to hold suspended particles like molecules,\textsuperscript{4} or more recently, dielectric spheres.\textsuperscript{5–7} Also, the possibilities of creating microstructures by optical binding and resonance effects have been discussed\textsuperscript{8–12} as well as the control of particles by evanescent waves.\textsuperscript{13,14} Only a few works exist on the interpretation, prediction and control of the optical force acting on a small particle on a plane surface. To our knowledge, the only theoretical works dealing with this subject are those of Refs. [15,16,17]. In Ref. [15] no multiple interaction of the light between the particle and the dielectric surface is considered. On the other hand, Ref. [16] deals with a 2-D situation. Only recently in Ref.[17] the full 3-D case with multiple scattering was addressed for dielectric particles.

This work, extends the study of Ref.[17] to metallic particles and, as such, this is the first theoretical study of light action on a metallic particle. We shall therefore present a rigorous procedure to evaluate the electromagnetic force in three dimensions. Further, we shall analyze how this force depends on the wavelength, distance between the particle and the surface, angle of incidence (whether the excitation is a plane propagating or an evanescent wave), and on the excitation of plasmons on the sphere. We shall make use of the couple dipole method previously employed, and whose validity was analyzed in detail in Ref.[17].

In section II we introduce a brief outline on the method used to compute the optical force on a particle. We also write its expression from the dipole approximation for a metallic sphere in presence of a surface. Then, in Section III A we present the results and discussion obtained in the limit of a small sphere, and in Section III B we analyze the case of larger spheres compared to the wavelength.

II. COMPUTATION OF THE OPTICAL FORCES

The Coupled Dipole Method (CDM) was introduced by Purcell and Penyapcker in 1973.\textsuperscript{18} In this paper we use this procedure together with Maxwell’s Stress Tensor (MST)\textsuperscript{19} in order to compute the optical forces on a metallic object in the presence of a surface. Since we developed this method in a previous paper,\textsuperscript{17} we shall now outline only its main features. It should be remarked that all calculations next will be written in CGS units for an object in vacuum.

The system under study is a sphere, represented by a cubic array of N polarizable subunits, above a dielectric flat surface. The field at each subunit can be written:

\[
E(r_i, \omega) = E_0(r_i, \omega) + \sum_{j=1}^{N} [S(r_i, r_j, \omega) + T(r_i, r_j, \omega)]\alpha_j(\omega)E(r_j, \omega). \tag{1}
\]

where \(E_0(r_i, \omega)\) is the field at the position \(r_i\) in the absence of the scattering object, \(T\) and \(S\) are the field susceptibilities associated to the free space\textsuperscript{20} and the surface,\textsuperscript{21,22} respectively. \(\alpha_i(\omega)\) is the polarizability of the \(i^{th}\) subunit. Like in Ref. [17] we use the polarizability of the Clausius-Mossotti relation with the radiative reaction term given by Draine:\textsuperscript{23}

\[
\alpha = \frac{\alpha_0}{1 - (2/3)ik_0^3\alpha_0} \tag{2}
\]

where \(\alpha_0\) holds the usual Clausius-Mossotti relation \(\alpha_0 = \alpha^3(\varepsilon - 1)/(\varepsilon + 2)^3\).\textsuperscript{17,24} In a recent paper,\textsuperscript{23} we have shown the importance to compute the optical forces taking into account the radiative reaction term in the equation for the polarizability of a sphere. For a metallic sphere, the polarizability is written as \(\alpha = \alpha_0(1 + (2/3)ik_0^3\alpha_0^*)/D\) with \(D = 1 + (4/3)k_0^33m(\alpha_0) + (4/9)k_0^3|\alpha_0|^2\), where the
The force \( F_\kappa(r_i) \) at each subunit is:
\[
F_\kappa(r_i) = (1/2) \text{Re} \left( \alpha_i E_i(r_j, \omega) \left( \frac{\partial}{\partial \kappa} E^l(r, \omega) \right)_{r=r_j} \right), \tag{3}
\]
where \( k \) and \( l \) stand for the components along either \( x, y \) or \( z \), and \( \text{Re} \) denotes the real part. The object is a set of \( N \) small dipoles so that it is possible to compute the force on each one from Eq. (3). Hence, to obtain the total force on the particle, it suffices to sum the contributions \( F(r_i) \) on each dipole.

![FIG. 1. Geometry of the configuration considered. Sphere of radius \( a \) on a dielectric flat surface (\( \epsilon = 2.25 \)). The incident wave vector \( \mathbf{k} \) is in the \( XZ \) plane.]

Being the object under study a small sphere located at \( r_0 = (0, 0, z_0) \) (see Fig. 1), we can employ the dipole approximation, and hence use directly Eq. (3) with \( N=1 \). Within the static approximation for the field susceptibility associated to the surface (SAFSAS) (that is to say \( k_0 = 0 \)), we have found an analytical expression for \( E(r_0, \omega) \) that yields the force components:
\[
F_x = |E_{0x}| \frac{2 \mu_0 \epsilon_0}{2} \left( \frac{2|E_{0x}|^2}{8z_0^2 + \alpha \Delta} + \frac{|E_{0x}|^2}{4z_0^2 + \alpha \Delta} \right), \tag{4}
\]
\[
F_z = |E_{0x}|^2 \frac{\text{Re}}{2} \left( \frac{8z_0^2 \alpha |\kappa_x|^2}{8z_0^3 + \alpha \Delta} + \frac{12z_0^2 |\alpha|^2 \Delta}{|8z_0^3 + \alpha \Delta|^2} \right) + |E_{0z}|^2 \frac{\text{Re}}{2} \left( \frac{4z_0^2 \alpha |\kappa_x|^2}{4z_0^3 + \alpha \Delta} + \frac{6z_0^2 |\alpha|^2 \Delta}{|4z_0^3 + \alpha \Delta|^2} \right), \tag{5}
\]
for \( p \)-polarization, and
\[
F_z = |E_{0y}|^2 \frac{\text{Re}}{2} \left( \frac{8z_0^2 \alpha |\kappa_x|^2}{8z_0^3 + \alpha \Delta} + \frac{12z_0^2 |\alpha|^2 \Delta}{|8z_0^3 + \alpha \Delta|^2} \right), \tag{6}
\]
\[
F_z = |E_{0y}|^2 \frac{\text{Re}}{2} \left( \frac{8z_0^2 \alpha |\kappa_x|^2}{8z_0^3 + \alpha \Delta} + \frac{12z_0^2 |\alpha|^2 \Delta}{|8z_0^3 + \alpha \Delta|^2} \right), \tag{7}
\]
for \( s \)-polarization, with \( \Delta = (1 - \epsilon)/(1 + \epsilon) \) being the Fresnel coefficient of the surface. We have assumed a dielectric surface, hence \( \Delta \) is real.

From Fig. 1 is easy to see that \( k_x \) is always real whatever the angle \( \theta \), hence we can write Eqs. (4) and (6) for metallic particles as:
\[
F_x = \left( \frac{|E_{0x}|^2}{8z_0^2 + \alpha \Delta} + \frac{|E_{0x}|^2}{4z_0^2 + \alpha \Delta} \right) \times \left[ k_x \Im(\epsilon_0)/(2D) + k_x k_0^3 |\alpha|^2/(3D) \right] \tag{8}
\]
for \( p \) polarization, and
\[
F_x = \left( \frac{|E_{0y}|^2}{8z_0^2 + \alpha \Delta} \right) \times \left[ k_x \Im(\epsilon_0)/(2D) + k_x k_0^3 |\alpha|^2/(3D) \right] \tag{9}
\]
for \( s \)-polarization. In Eq. (8) the factor in front of \( (k_x \Im(\epsilon_0)/(2D) + k_x k_0^3 |\alpha|^2/(3D)) \) for the two polarizations constitutes the field intensity at \( z_0 \). The first term within these square brackets corresponds to the absorbing force whereas the second represents the scattering force on the sphere. We see from Eqs. (8) and (9) that \( F_x \) always has the sign of \( k_x \). Notice that it is not possible to write a general equation for the force along the \( Z \)-direction, as \( k_z \) will be either real or imaginary, according to the angle of incidence.

III. RESULTS AND DISCUSSION

A. Small particles

We first address a small isolated silver particle with radius \( a = 10 \)nm. In this case we can use the dipole approximation, hence we consider Eqs. (4)-(7) with \( \Delta = 0 \). Fig. 2a presents the polarizability modulus \( |\alpha_0| \) of the sphere. The maximum of the curve corresponds to the plasmon resonance, i.e. when the dielectric constant is equal to -2 in Drude’s model. Notice that, in this model the dielectric constant is real, on using experimental values, it is complex and the resonance is not exactly at \( \Re(\epsilon) = -2 \) but slightly shifted. In Fig. 2b we plot the real part of the polarizability \( \Re(\alpha_0) \), and Fig. 2c shows its imaginary part \( \Im(\alpha_0) \). Fig. 2d represents the force in free space computed from an exact Mie calculation (full line) and by the dipole approximation from Eqs. (4)-(7) with \( \Delta = 0 \) (dashed line) and Eq. (2) for \( \alpha \). In this case, the dipole approximation slightly departs from Mie’s calculation between 350nm and 375nm. We can compute the polarizability \( \alpha \) from the first Mie coefficient \( a_1 \) given by Dungey and Bohren (DB). Therefore, the dipole dipole polarizability is \( \alpha = 3a_1/(2\epsilon_0)^2 \). The symbol + in Fig.2d corresponds to the DB polarizability and it is exactly coincident with the Mie calculation. When the optical constant of the metallic sphere is close to the plasmon resonance,
the calculation from the Clausius-Mossotti relation with the radiative reaction term, departs from the exact calculation, even for a small radius. We shall next use the polarizability of DB. Analytical calculations will always be done with Eq. (2) to get simple expressions, and thus a better understanding of the physics involved. The curve of the force obtained by Mie’s calculation has exactly the same shape as the imaginary part of the polarizability, of the force obtained by Mie’s calculation has exactly the same shape as the imaginary part of the polarizability, this is due to the fact that for a small metallic sphere the absorbing force is larger than the scattering force.

having made the approximation $|\alpha| \approx |\alpha_0|$, and $D \approx 1$

since we have a small sphere compared to the wavelength ($k_0a \ll 1$). The factor in front of the bracket corresponds to the intensity of the field at $z_0$. The first and the second terms within brackets represent the interaction between the dipole moment associated to the sphere and the incident field: the first term is the absorbing force whereas the second one corresponds to the scattering force, hence these forces are always positive. The third term is due to the interaction between the dipole and the field radiated by the dipole and reflected by the surface. We can consider this term as a gradient force exerted on the sphere due to itself via the surface. Hence, this force is always negative whatever the relative permittivity $\varepsilon$.

Since this term is proportional to $1/z_0^4$, it becomes more dominant as the sphere approaches the surface, hence the force decreases. To derive the point $z_0$ at which the force vanishes, if such a point exists, let us assume the scattering force smaller than the absorbing force, then from Eq. (10) the zero force is:

$$F_z = \frac{64\pi^5}{8\pi^3} \frac{|E_0|^2}{\alpha^2} \left[ \frac{k_0}{2} \frac{3m(\alpha_0) + k_0^4 |\alpha|^2 + 3|\alpha|^2 \Delta}{32 + \alpha^2 \Delta^2} \right], \quad (10)$$

FIG. 2. From top to bottom: the first three curves represent the modulus, the real part, and the imaginary part of the polarizability of a silver sphere with radius $a = 10\text{nm}$ versus the wavelength. The fourth curve is the force on this particle in free space. Plain line: Mie calculation, dashed line: polarizability of Clausius-Mossotti relation, with the radiative reaction term, symbol +: DB polarizability.

Next, we consider the small sphere on a dielectric plane surface as shown by Fig. 1. Illumination takes place from the dielectric side with $\theta = 0^\circ$, hence in vacuum $k_x = k_0$ and $k_z = 0$. Fig. 3 represents the force in the $Z$-direction from Eq. (7) versus $z$ for different wavelengths. Far from the surface the force tends to the Mie limit. Near the surface the force decreases, and, depending on the wavelength, it can become negative. For a better understanding of this force we write Eq. (7) as:

$$F_z = \frac{64\pi^5}{8\pi^3} \frac{|E_0|^2}{\alpha^2} \left[ \frac{k_0}{2} \frac{3m(\alpha_0) + k_0^4 |\alpha|^2 + 3|\alpha|^2 \Delta}{32 + \alpha^2 \Delta^2} \right], \quad (10)$$

FIG. 3. Force along the $Z$-direction on a silver sphere with a radius of $10\text{nm}$, for different wavelengths and angles of incidence is $\theta = 0^\circ$. With the static approximation (no symbol) and in an exact calculation (+) for three wavelengths used to be: $\lambda = 270\text{nm}, z_0 = 7.9\text{nm}$, $\lambda = 360\text{nm}, z_0 = 12.8\text{nm}, \lambda = 500\text{nm}, z_0 = 21.6\text{nm}$.

Now, $z_0$ (the location of the center of the sphere) must be larger than the radius $a$, or else, the sphere would be buried in the surface. Therefore the first of those values of $z_0$ is not possible. Hence, the force is always positive. Thus, the distance between the sphere and the surface is $z = 2.8\text{nm}$ and $11.6\text{nm}$ for $\lambda = 360\text{nm}$ and $500\text{nm}$, respectively. These values are very close to those shown in the inset of Fig. 2. Notice that near the plasmon resonance for the sphere ($\lambda \approx 360\text{nm}$) both $|\alpha|^2$ and $\Im(\alpha_0)$ are maxima, hence the force is very large when the sphere is far from the surface and, due to the third term of Eq. (10), which depends of $|\alpha|^2$, the decay of this force is very fast. We have used the static approximation for the susceptibility tensor associated to the surface (SAFSAS) at large distance. We plot in Fig. 3 with crosses the force obtained from an extended calculation for small the dipole approximation. As we see these crosses coincide with those curves obtained with the SAFSAS whatever the distance $z$. Yet, we obtain for dielectric spheres the following: near the surface the SAFSAS is valid, whereas
in the Z-directions in the Eqs. (4)-(7) the force on the sphere for the two polarizations have the same behaviour, we take the analytical solution for \( F_z \) with \( k_z = i \gamma \) \( (\gamma > 0) \) for s-polarization. Then Eq. (7) can be written:

\[
F_z = \frac{|E_{0y}|^2}{|z_0| + \alpha \Delta} \Re \left( -\gamma \varepsilon z_0^3 \alpha (8z_0^3 + \alpha \Delta) + 12z_0^2 |\alpha|^2 \Delta \right) \tag{12}
\]

On using the approximation \( D \approx 1 \) and \( |\alpha| \approx |\alpha_0| \) we obtain:

\[
F_z = \frac{64\varepsilon_0^2 |E_{0y}|^2}{|z_0| + \alpha \Delta} \left( -\gamma \Re(\alpha_0) - \frac{\gamma |\alpha_0|^2 \Delta}{16z_0^3} + \frac{3|\alpha_0|^2 \Delta}{32z_0^3} \right) \tag{13}
\]

As shown by Fig. 3, when the sphere is located at \( z_0 - 30\text{nm} \), the influence of the surface becomes negligible, thus we can use Eq. (13) with the hypothesis that \( z_0 \) is large. Hence, \( F_z \approx -|E_{0y}|^2 \gamma \Re(\alpha_0)/2 \). This is the gradient force due to the incident field, and therefore due to the interaction between the dipole associated to the sphere and the applied field. This force exactly follows the behaviour of \( \Re(\alpha_0) \) (cf. Fig. 2). When \( \alpha_0 \) is negative, the dipole moment of the sphere oscillates in opposition to the applied field and so the sphere is attracted towards the weaker field. Notice that the same phenomenon is used to build an atomic mirror: for frequencies of oscillation higher than the atomic frequency of resonance, the induced dipole oscillates in phase opposition with respect to the field. The atom then undergoes a force directed towards the region of weaker field.\(^{30}\) For p-polarization, the force can be written \( F_z \approx -(|E_{0x}|^2 + |E_{0z}|^2) \gamma \Re(\alpha_0)/2 \). As the modulus of the field becomes more predominant in p-polarization, the magnitude of the force becomes more important. We now search more carefully the change of sign in the force. Writing \( \varepsilon = \varepsilon' + i\varepsilon'' \) for the relative permittivity, we get:

\[
\Re(\alpha_0) = a^3 \frac{(\varepsilon' - 1)(\varepsilon' + 2) + \varepsilon''^2}{(\varepsilon' + 2)^2 + \varepsilon''^2} \tag{14}
\]

\[
\Im(\alpha_0) = a^3 \frac{3\varepsilon''}{(\varepsilon' + 2)^2 + \varepsilon''^2} \tag{15}
\]

If the damping \( \gamma \) is weak, in the change of sign of the imaginary polarization \( \approx 1 \) and at the plasmon resonance for the sphere, i.e., \( \varepsilon' \approx -2 \). Between these two values, the gradient force is positive. In fact, the limiting values of the positive gradient force are always strictly in the interval \([-2, 1]\) due to damping. For example, the force vanishes at \( \lambda = 352\text{nm} \) with \( \varepsilon = -1.91 + 0.6i \) and \( \lambda = 317\text{nm} \) with \( \varepsilon = 0.66 + 0.95i \). We notice that the change of sign happens steeply at the plasmon resonance since then the denominator of the real part of the polarizability becomes very weak (see Eq. (14)), hence the zero force is surrounded by the two maxima of the force (one positive and the other negative). At \( \lambda = 317\text{nm} \) the change of sign is smoother as in that case the denominator is far from zero. The third case, \( \lambda = 259\text{nm} \), lies between those two previous cases as the damping of

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**FIG. 4.** Force along the Z-direction on a silver sphere with \( a = 10\text{nm} \). The angle of incidence is \( \theta = 50^\circ \). Plain line: p-polarization; dashed line: s-polarization. 

**FIG. 5.** Force along the X-direction on a silver sphere with \( a = 10\text{nm} \). The angle of incidence is \( \theta = 50^\circ \). Plain line: p-polarization; dashed line: s-polarization.

We next consider the surface illuminated at angle of incidence \( \theta \) larger than the critical angle: \( \theta = 50^\circ > 41.8^\circ = \theta_c \). Now the transmitted electromagnetic wave above the surface is evanescent. We plot in Fig. 4 from Eqs. (4)-(7) the force on the sphere for the two polarizations in the Z-direction versus the wavelength, and in the X-direction in Fig. 5, when the sphere is located at \( z_0 = 30\text{nm} \). In Fig. 4 we see that the force in the Z-direction is also either positive or negative. In a previous work\(^{17}\), we have observed that the force on a small dielectric sphere is always attractive when the sphere is located in an evanescent wave. This is no longer the case for a metallic sphere. To understand this difference, and as far from the surface the sphere does not feel its presence, and, thus, whether using the tensor susceptibility associated to the surface in its exact form, or within the static approximation, has no influence.

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\( |E_{0y}|^2 \)

\( |z_0| + \alpha \Delta \)

\( -\gamma \varepsilon z_0^3 \alpha (8z_0^3 + \alpha \Delta) + 12z_0^2 |\alpha|^2 \Delta \)

\( 64\varepsilon_0^2 |E_{0y}|^2 |z_0| + \alpha \Delta \left( -\gamma \Re(\alpha_0) - \frac{\gamma |\alpha_0|^2 \Delta}{16z_0^3} + \frac{3|\alpha_0|^2 \Delta}{32z_0^3} \right) \)

\( \Re(\alpha_0) = a^3 \frac{(\varepsilon' - 1)(\varepsilon' + 2) + \varepsilon''^2}{(\varepsilon' + 2)^2 + \varepsilon''^2} \)

\( \Im(\alpha_0) = a^3 \frac{3\varepsilon''}{(\varepsilon' + 2)^2 + \varepsilon''^2} \)
the relative permittivity is important: \( \varepsilon = -1.65 + 1.12i \). We have also investigated the cases of gold and copper spheres, where a plasmon easily takes place, but we found not possible change of sign for \( F_z \) for these two materials as the damping is then too important: if \( \varepsilon'' > 3/2 \), then \( \Re(\alpha) \) is always positive whatever \( \varepsilon' \). However, if the particle is embedded in a liquid with a relative permittivity 2, then it is possible to get \( \Re(\alpha) < 0 \) for gold.

Notice that if \( \theta \) is close to \( \theta_c \), then \( \gamma \approx 0 \) and so we only have the third term of Eq. (13), then the force is always negative whatever the wavelength. In Fig. 5 we see, as previously, that the force in the \( X \)-direction has the sign of \( k_x \) and, as the absorbing force is the most predominant one, the curve has the same shape as the imaginary part of the polarizability (cf. Fig. 2). In that case, the maximum of the force \( F_x \) is at the plasmon resonance (see Eq. (15)) namely at \( \lambda = 354 \text{nm} \).

\[
\Delta = \frac{\lambda}{2} \quad \text{and} \quad \gamma z \to 0 \quad \text{as the damping is then too important: if } \varepsilon'' > 3/2, \text{then } \Re(\alpha) \text{ is always positive whatever } \varepsilon'. \text{ However, if the particle is embedded in a liquid with a relative permittivity 2, then it is possible to get } \Re(\alpha) < 0 \text{ for gold. Notice that if } \theta \text{ is close to } \theta_c, \text{ then } \gamma \approx 0 \text{ and so we only have the third term of Eq. (13), then the force is always negative whatever the wavelength. In Fig. 5 we see, as previously, that the force in the } X \text{-direction has the sign of } k_x \text{ and, as the absorbing force is the most predominant one, the curve has the same shape as the imaginary part of the polarizability (cf. Fig. 2). In that case, the maximum of the force } F_x \text{ is at the plasmon resonance (see Eq. (15)) namely at } \lambda = 354 \text{nm}. 
\]

\[
\text{FIG. 6. Force along the } Z \text{-direction on a silver sphere with } a = 10 \text{nm versus distance } z \text{ in the dipole approximation. The angle of incidence is } \theta = 50^\circ. \text{ Without symbol: } p\text{-polarization; with cross (+): } s\text{-polarization. Plain line: } \lambda = 340 \text{nm, dashed line: } \lambda = 317.5 \text{nm}. 
\]

\[
\text{Fig. 6 shows the force in the } Z \text{-direction versus } z \text{ for both } s \text{ (symbol +) and } p\text{-polarization (without any symbol) at three different wavelengths (} \lambda = 265, 340, 317.5 \text{nm) for } \theta = 50^\circ. \text{ The behaviour is the same for both polarizations, only appearing a difference of magnitude, this is due to the component of the field perpendicular to the surface in } p\text{-polarization. All curves manifest that near the surface the force is attractive, this is due, as seen before with a propagating wave, to the term of } \Delta/\varepsilon_0^2 \text{ in Eqs. (10) and (13) which is always negative, irrespective of the kind of wave above the surface. At } \lambda = 317.5 \text{nm (dotted line), we have } \Re(\alpha) = 0 \text{ which is why the force very quickly goes to zero when } z \text{ grows. The two other cases correspond to } \Re(\alpha) > 0 \text{ (} \lambda = 260 \text{nm) and } \Re(\alpha) < 0 \text{ (} \lambda = 340 \text{nm) and far from the surface the force tends to zero. As the force is proportional to } |E_0|^2, \text{ we have } F_z \propto e^{-(2\gamma z)}. 
\]

B. large particles

It is difficult to obtain convergence for the CDM when the relative permittivity of the medium to discretize is large. This imposes a very fine sampling. In this section, we use the range 250-355nm for the wavelength, the real part of the relative permittivity being small. Then, the computation is simplified. In addition, this is the most interesting case as then \( \Re(\alpha) = 0 \) three times in this interval of wavelengths.

\[
\text{FIG. 7. Force along the } Z \text{-direction on a silver sphere versus distance } z \text{ in the dipole approximation. The angle of incidence is } \theta = 50^\circ. \text{ Without symbol: } p\text{-polarization; with cross (+): } s\text{-polarization. Plain line: } \lambda = 340 \text{nm, dashed line: } \lambda = 317.5 \text{nm.} 
\]

\[
\text{In Fig. 7 we plot the force in the } Z \text{-direction for an incident propagating wave (} \theta = 50^\circ) \text{ versus the distance between the sphere and the surface at three different wavelengths: } \lambda = 255 \text{nm, } 300 \text{nm, } 340 \text{nm. The calculations are done without any approximation. The curves have a similar magnitude and behaviour at the three wavelengths. The forces present oscillations due to the multiple reflection of the radiative waves between the sphere and the surface, hence the period of these oscillations is } \lambda/2. \text{ The magnitude of these oscillations depends on the reflectivity of the sphere, so the higher the Fresnel coefficient is, the longer these oscillations are. As expected, they are less remarkable when the sphere goes far from the surface. We notice that the decay of the force when the metallic sphere gets close to the surface is not comparable to that on a dielectric sphere (see Ref.[17]). This is due to strong absorbing and scattering forces on the metallic sphere in comparison to the gradient force induced by the presence of the dielectric plane.} 
\]
In Fig. 8 we plot for $\theta = 50^\circ$ the normalized force in the Z-direction, i.e., $F_z/|E_0|^2$, $E_0$ being the field at $z_0$ in the absence of the sphere. We remark two important facts at this angle of incidence. First, the decay of the force when the sphere is near the surface is more important in $p$-polarization. Notice that with the CDM it is not possible to numerically split the scattering, absorbing, and gradient forces. Therefore, when the sphere is large we shall argument on the set of dipoles forming it. In $p$-polarization, due to the $z$-component of the incident field, the dipoles also have a component perpendicular to the surface, and this is larger than in $s$-polarization. Hence, in agreement with Fig. 6, due to this $z$-component, the attraction of the sphere towards the surface is larger for $p$-polarization. Second, all forces are positive when the sphere is far from the surface except for $\lambda = 300\text{nm}$ in $p$-polarization. At this wavelength, for a small sphere, the force is negative for both $s$ and $p$-polarization, hence we can assume an effect due to the size of the sphere. Just to see this effect, we present in Fig. 9 the force in the Z-direction versus the radius $a$, on a sphere located at $z_0 = 100\text{nm}$ for an angle of incidence $\theta = 50^\circ$ but without taking into account the multiple interaction with the surface (i.e., $S = 0$). We take the previous wavelength of Fig. 8 ($\lambda = 255\text{nm}, 300\text{nm},$ and $340\text{nm}$) more the wavelength at the plasmon resonance, $\lambda = 351.5\text{nm}$ where $\Re(\alpha_0) = 0$. For small radius, we observe the same behaviour as in the previous section for an incident evanescent wave, namely at $\lambda = 255\text{nm}$ and 340nm, the force in the Z-direction is positive, and it becomes negative for $\lambda = 300\text{nm}$ for both polarizations. These curves show a dependence proportional to the cube of the radius as $\Re(\alpha_0) \propto a^3$. At the plasmon resonance, the force is slightly positive as there is no gradient force, but only weak absorbing and scattering forces.

But as shown by Fig. 9, when the radius grows, in $s$-polarization at $\lambda = 300\text{nm}$, the force sign changes and it becomes positive around $82\text{nm}$ as $p$-polarization keep the same behaviour. This confirms the fact that the positive force obtained in Fig. 8 for $\lambda = 300\text{nm}$ is only a size effect. For the cases $\lambda = 255\text{nm}, 340\text{nm}$, the gradient force is positive in the Z-direction, like the scattering and absorbing forces. Hence, the force, is always positive whatever the radius. Nevertheless, for $\lambda = 300\text{nm}$, these is a negative gradient force, the other two forces being positive. As previously said, it is not possible to know the relative contribution of the different forces, but in $s$-polarization the dipoles have a component mainly parallel to the plane. In that case, the dipole associated to each subunit radiates a field both propagating and evanescent along the Z-direction. Hence, we can assume that due to the radiative part, the absorbing and scattering forces acting on the subunits inside the sphere, become important when the radius increases. Concerning $p$-polarization, due to the component of the field perpendicular to the plane, the dipoles have a component along Z and thus only radiate evanescent waves along this direction. In this case, we believe that the gradient force is more important on each subunit and counterbalances the absorbing and scattering forces. At plasmon resonance $\lambda = 351.5\text{nm}$, when the radius grows, the absorbing and scattering forces become larger, but as no gradient force exists, the force is lower than those obtained at $\lambda = 255\text{nm}$ and 340nm. In fact, the curve shown at $\lambda = 340\text{nm}$ is the most important one, due to the onset of the plasmon resonance, thus $|\alpha_0|$ and $3m(\alpha_0)$ being near their maximum, and $\Re(\alpha_0)$ close to its minimum. In this case, the gradient force is maximum and positive.
IV. CONCLUSIONS

We have presented a theoretical study of the optical forces acting upon a metallic particle on a dielectric plane surface either illuminated at normal incidence or under total internal reflection. This study is done both with the coupled dipole method and Maxwell’s stress tensor. We observe that when the incident wave is propagating, the difference between the force acting on a dielectric sphere and that on a metallic sphere stems from the absorbing force. Due to this contribution, the force upon a small silver sphere close to the dielectric surface can be positive in spite of the gradient force. The opposite happens with a dielectric sphere. The main difference between the two cases (dielectric and metallic) arises however on illumination under total internal reflection. In that case, the effect on a small silver sphere is completely different to that observed on a dielectric sphere. Depending on the wavelength, the gradient force due to the incident field can be either repulsive or attractive. The change of sign happens both at the plasmon resonance and when \( \varepsilon \) becomes close to one. In the interval between these two values the gradient force is positive. The explanation is very similar to that on the effect used to build an atomic mirror. At a wavelength where the gradient force on a small sphere is negative, we see that when the sphere radius grows, the force along the \( \text{Z} \)-direction stays negative for \( p \)-polarization, and it becomes positive for \( s \)-polarization due to the size effect. Nevertheless, at any arbitrary wavelength and angle of incidence, as the sphere approaches close to the surface, the attraction of the surface on the sphere increases: repulsive forces diminish and even can change sign, eventually becoming attractive at certain wavelengths. Attractive forces, on the other hand, increase their magnitude.

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17. P. C. Chaumet, and M. Nieto-Vesperinas, accepted at Phys. Rev. B.
24. \( a^3 = 3d^3/(4\pi) \) where \( d \) is the spacing of lattice discretization. In the case of the dipole approximation \( a \) is the radius of the sphere.
26. In fact \( \mathbf{F} \) is not exactly the force but moreover the time averaged force.