Strings In Baryons And Matrix Coordinates

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Abstract

It is argued that the internal dynamics of a baryon, as a bound state of QCD-strings and quarks, may be captured by a theory of matrix coordinates.

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There is a relatively common belief that gauge theories have some kinds of string theory as their dual description. The supports for this belief come from some phenomenological facts, such as Regge trajectories, and also some theoretical considerations, like those coming from large-$N$ and lattice gauge theories [1][2]. In the theoretical side, the gauge/gravity duality in the AdS limit of brane backgrounds [3] has realized the idea by a definite relation between the objects in two sides [4]. The loop-space formulation [5] is another example of efforts for string theoretic presentation of gauge theory dynamics, still exciting theoretical research [6].

In providing a dictionary between gauge and string theories, it is usually assumed that the dynamics of the electric fluxes in confined phase is presented by a string theory, probably in non-critical dimensions. These electric fluxes are usually referred under the names of “flux-tubes” or “QCD-strings.” In this picture, the QCD-strings are stretched between and are responsible for the confinement of electric charges; similar to the scenario we expect, via the Meisner effect, for the confinement of a monopole and anti-monopole in a superconductor. One justification for giving a role to these kinds of strings is the linear potential that is usually supposed between the quark and anti-quark of a mesonic state; a potential which can generate Regge trajectories, and also is supported by lattice gauge theory calculations [2].

Although the QCD-strings are usually considered for mesons, there are quite reasonable proposals to introduce these strings to baryons too. In fact the results by lattice calculations for the nature of the inter-quark potential in a baryon strongly suggest that these kinds of strings should be present in baryons. There are two main proposals for the potential, the so-called: 1) Δ-shape, and 2) Y-shape. In the Δ-shape ansatz, the total potential of quarks is the sum of linear two-body ones, for three quarks as:

$$V(r_1, r_2, r_3) = \frac{\sigma}{2}(r_{12} + r_{23} + r_{31})$$

(1)

in which $r_{ij} = |r_i - r_j|$, and $\sigma$ is the tension of QCD-string in a mesonic state. The name Δ clearly comes from the proposed geometry by the potential, and also the suggested form for the QCD-strings in a baryon. This behavior is supported by some lattice calculations [7], at least for short distances. In the Y-shape ansatz, the total potential is introduced with the help of an extra point $r_s$, for which the length $(r_{1s} + r_{2s} + r_{3s})$ is minimum ($r_{is} = |r_i - r_s|$); the so-called Steiner (or Fermat) point. Then the total potential for three quarks is given by:

$$V(r_1, r_2, r_3) = \sigma(r_{1s} + r_{2s} + r_{3s}).$$

(2)

The Y-shape ansatz has been used to extract the properties of baryons [8], and also has found supports by the lattice calculations [9]. The field correlator method also
produces results closer to Y-ansatz [10]. Besides, the stringy shape of electric fluxes has been revealed also by this method [10].

Due to geometrical reasonings [11], the disagreement between ∆- and Y- ansatzes is at most about %15, which is still small to provide a final conclusion for lattice based calculations. Also both proposals are almost equally suitable for phenomenological purposes. One may state that by the present results the potential approaches the ∆-ansatz at short distances, but rises like the Y-ansatz at large distances [12, 11], and hence the ∆ model is more appropriate inside a hadron [12, 11]. The deviations from the Y-ansatz in small separations is interpreted in [10] by the depletion of the electric field at and around the Steiner point (see Figs. by [10]). The depletion of electric field can be understood as the cancellation of different components of electric fields when they arrive the Steiner point. So by this interpretation we see that, quite interestingly, there is a very preferred ∆-shape for the profile of electric field and also for the configuration of QCD-strings inside baryons, specially for small separations of quarks.

The picture of baryons as bound states of quarks and QCD-strings has been used for many years for phenomenological aims, and has been able to produce a lots of considerable ‘numbers’. The natural guess about the relevant theory is the formulation for the system of “three point-like masses bounded by relativistic strings” [8] [13]. By this formulation one can explain somehow directly some known facts, such as Regge behavior. In spite of long history of the efforts based on string-quark picture, however, there are difficulties with this formulation. One is that the string theory in use is in non-critical dimensions, and so in principle, one expects that in the level of complete quantum theory some kinds of anomalies or infinities appear. The source of anomalies or infinities simply is that, after all, we are dealing with a field theory, living on the world-sheets of some strings. The other problem is related to high non-linearity of the system; a non-linearity which can not be get rid of easily due to non-critical dimension.

As the existence of QCD-strings is well accepted, and as the models based on quark-string picture have shown to be practically useful, it is apposite to suggest models that while they capture the essential features of the quark-string picture, they avoid difficulties that one usually is faced in studying the highly non-linear field theory on the world-sheet. To avoid unwanted infinities one way is dealing with a quantum mechanical system rather than a quantum field theory. In this direction, one analogy with the picture we imagined for baryons and the introduction of D0-branes in string theory may be suggestive. D0-branes are defined as massive point-like objects to whom the open strings end [14]. While the open strings are stretched
between the D0-branes, they join D0-branes together, making a bound state of D0-branes and strings. Here two cases can be considered, as one assumes the open strings are oriented or unoriented. Eventually it appears that in a bound state of $N$ D0-branes the relevant degrees of freedom in each direction of space, rather than $N$, are $N^2$ in the case of oriented strings, and $N(N + 1)/2$ for unoriented case. These degrees of freedom may be represented by matrices belonging to U($N$) and O($N$) algebras, for oriented and unoriented cases, respectively. So one may argue that the degrees of freedom for system of $N$ D0-branes, rather than $d.N$ numbers in $d$ dimensions, are $d$ matrices of dimension $N \times N$ [15][16]. The matrix coordinates find this interpretation that the diagonal elements capture the dynamics of D0-branes, and the dynamics (oscillations) of strings are encoded in off-diagonal elements; for example, the dynamics of the string(s) stretched between $a$-th and $b$-th D0-branes has (have) been encoded in the element $X_{ab}$ ($a \neq b$).

In [17, 18] the action for D0-branes was considered as a model for QCD purposes. The first motivations for these studies originated by some early results which appear quite suggestive to take serious analogy between quark-strings systems and bound state of D0-branes. For this one should assume that the theory for matrix coordinates, perhaps coming from a critical string theory, should be also suitable for non-critical dimensions. This needs justification, though since the theory in its quantized form is just a quantum mechanics, one might be so hopeful that the results should be free from very bad and unwanted behaviors. The concerned model in [17, 18] has shown its ability to reproduce and recover some features and expectations in hadron physics. Some of these features and expectations are: the linear inter-quark potentials, the behavior of total scattering amplitudes, rich polology of scattering amplitude, behavior in large-$N$ limit, and the whiteness of baryons with respect to the SU($N$) sector of the external fields. The purpose of this Letter is to restate and refresh the main idea, in particular, via appreciating the suggestive and insightful presence of QCD-strings in a baryon, revealed by the recent lattice calculations and the field correlator method.

The dynamics for the matrix coordinates of D0-branes is formulated as a matrix theory. Although the exact form of the action should come from stringy calculations, based on some general arguments one suggestion is:

$$S[X, a_t] = \int dt \, \text{Tr} \left( \frac{1}{2} m D_t X \cdot D_t X + q D_t X \cdot A(X, t) - q A_0(X, t) + \frac{m}{4 \hbar^4} [X^i, X^j]^2 + \cdots \right),$$  \hspace{1cm} (3)

where Tr acts on the matrix structure, and $D_t X = \dot{X} + i[a_t, X]$ is the covariant
velocity, with $a_t(t)$ as the one dimensional $N \times N$ gauge field. $l$ in the language of string theory is order of the string length. The potentials $(A_0(X, t), A(X, t))$ are functionals of symmetrized products of the matrix coordinates, and “…” is for $O(X^6)$ and higher non-symmetrized terms, and also non-linear terms in velocity $D_t X$. We note that the fields $(A_0(X, t), A(X, t))$ appear as $N \times N$ hermitian matrices due to their functional dependence on the matrix coordinate $X$. One can check easily that action (3) is invariant under the symmetry transformations [19, 20, 21]:

$$X \rightarrow X' = UXU^\dagger,$$

$$a_t \rightarrow a'_t = U a_t U^\dagger - iU \frac{d}{dt} U^\dagger,$$

$$A_i(X, t) \rightarrow A'_i(X', t) = U A_i(X, t) U^\dagger + iU \delta_i U^\dagger,$$

$$A_0(X, t) \rightarrow A'_0(X', t) = U A_0(X, t) U^\dagger - iU \partial_t U^\dagger,$$  

where $U \equiv U(X, t) = \exp(i\Lambda)$ is arbitrary up to the condition that $\Lambda(X, t)$ is totally symmetrized in the $X^i$’s. In above $\delta_i$ is the functional derivative $\frac{\delta}{\delta X^i}$. We recall that in approving the invariance of the action, the symmetrization prescription on the matrix coordinates plays an essential role [19, 20]. The above transformations on the gauge potentials are similar to those of non-Abelian gauge theories, and we mention that it is just the consequence of enhancement of degrees of freedom from numbers to matrices. In other words, we are faced with a situation in which ‘the rotation of fields’ is generated by ‘the rotation of coordinates’ [20]. Despite the non-Abelian behavior of the gauge transformations, since they are defined by just one function $\Lambda(x, t)$ after replacing ordinary coordinates by their matrix partners, i.e. $x \rightarrow X$, the transformations are not equivalent to non-Abelian ones [22]. After all, it is quite natural to interpret the fields $(A_0, A)$ as the external gauge fields that the constituents, whose degrees of freedom are included in the matrix coordinate, interact with them [21].

By ignoring the external fields $(A_0, A)$, one can find the effective theory for the matrices

$$X(t) = \text{diag. } (x_1(t), \cdots, x_N(t)),$$

$$a_t(t) = \text{diag. } (a_{t1}(t), \cdots, a_{tN}(t)),$$  

(5)

with $x_a = x_{a0} + v_a t$, $a = 1, \cdots, N$. This configuration solves the equations of motion, and describes the “classical” free motion of $N$ D0-branes, neglecting the effects of the strings. The situation is different when we consider quantum effects, and it will be realized that the dynamics of the off-diagonal elements affect the dynamics of D0-branes significantly. Concerning the effect of the strings, one may
try to extract the effective theory for D0-branes. In particular, it will be found out
that the commutator potential is responsible for the formation of the bound state,
and by a simple dimensional analysis we understand that the size of the bound state,
$\ell$, is $\sim m^{-1/3}\ell^{2/3}$. For the static configuration [17] one can easily calculate one-loop
effective potential between the D0-branes, getting [18, 17]:

$$V_{\text{one-loop}} \sim \sum_{a>b=1}^{N} \frac{|x_a - x_b|}{\ell^2}. \quad (6)$$

This result shows that the matrix theory, up to this order of expansion in use, is in
accordance with $\Delta$-ansatz for potential. Since the original theory is invariant under
the rotation among the indices $1, \cdots, N$, only the states which are singlets under the
global rotation among the indices can be accepted as the physical states of effective
theory for diagonal elements.

The concerned model above is in the non-relativistic limit, and though it may
appear suitable for heavy quarks, for light or massless quarks we should change our
approach. The formulation in use is that of the M(atrix) model conjecture [23],
accompanied with the gauge field terms. To approach the covariant formulation,
following finite-$N$ interpretation of [24], it is reasonable to interpret things in the
DLCQ (Discrete Light Cone Quantization) framework, see [21] and Appendix of
[18].

In [22] a construction of matrix coordinates from some basic tools and expecta-
tions in QCD was proposed. In particular, by giving an equivalent role to color labels
in gauge theory, and the so-called Chan-Paton labels in open strings, one observes
that by the wave functions of individual quarks, say in a Hartree-Fock approxima-
tion for the internal dynamics of baryon, one can define the matrix coordinates by
their elements:

$$X_{ab}(t) = \langle \psi_b(t)|\hat{x}|\psi_a(t) \rangle \quad (7)$$

It was argued in [22] that the amount of simultaneous diagonalizability of matrix
coordinates, and their generalizations to higher moments, can be used as a criteria
for identifying the confined phase of a gauge theory.

One may wonder why a formulation of a non-Abelian gauge theory in confined
phase, according to a proposal for covering the stringy behavior, should be involved
by matrix coordinates. In [20, 18] a conceptual relation between appearance of ma-
trix coordinate in formulation of a non-Abelian gauge theory and one interpretation
of special relativity is mentioned. According to an interpretation of the special relativity, it is meaningful if the ‘coordinates’ and the ‘fields’ that are involved in a theory have some kinds of similar characters. Accordingly, we see that both the space-time coordinates $x^\mu$ and the electro-magnetic potentials $A^\mu (x)$ transform as a four-vector under the boost transformations. Also by this way of interpretation, the superspace formulations of supersymmetric field and superstring theories are the continuation of the special relativity program: adding spin-half coordinates as representatives of the fermionic degrees freedom of the theory. It may be argued that the relation between ‘matrix coordinates’ and ‘matrix fields’ is one of the expectations which is supported by the spirit of the special relativity. By previous discussion we recall, 1) the matrix character of gauge fields is the result of dependence of them on matrix coordinates [20], 2) the symmetry transformations of gauge fields are induced by the transformations of matrix coordinates [20], 3) the transformations of fields in the theory on matrix space appeared to be similar to those of non-Abelian gauge theories. We note that by the picture supposed in this work, we expect that the matrix coordinates become relevant just for studying the internal dynamics of bound states, a situation referred in [19, 20, 21, 22] as ‘confined non-commutativity.’ This way of interpretation may lead us to conclude that the non-Abelian gauge fields in a confined theory do not have an independent character, and they are introduced to the formalism just through the functional dependence of Abelian gauge fields on the matrix coordinates of ‘bounded quarks’. It is very interesting when we note that by the present status of the experimental data, the existence of pure gluonic states, the so-called glueballs, is quite doubtful. This lack of detection, while it is quite unexpected by the masses we expect for glueballs, may be taken as a support for this interpretation.

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References


