The value of the $\alpha$ spectroscopic factor ($S_\alpha$) of the 6.356 MeV $1/2^+$ state of $^{17}\text{O}$ is believed to have significant astrophysical implications due to the importance of the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ reaction as a possible source of neutron production for the s process. To further study this effect, an accurate measurement of the $^{13}\text{C}(^{6}\text{Li},d)^{17}\text{O}$ reaction at $E_{\text{lab}} = 60$ MeV has been performed recently by Kubono et al., who found a new value for the spectroscopic factor of the 6.356 MeV $1/2^+$ state of $^{17}\text{O}$ based on a distorted wave Born approximation (DWBA) analysis of these data. This new value, $S_\alpha \approx 0.011$, is surprisingly much smaller than those used previously in astrophysical calculations ($S_\alpha \approx 0.3 - 0.7$) and thus poses a serious question as to the role of the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ reaction as a source of neutron production. In this work we perform a detailed analysis of the same $^{13}\text{C}(^{6}\text{Li},d)^{17}\text{O}$ data within the DWBA as well as the coupled reaction channel (CRC) formalism. Our analysis yields an $S_\alpha$ value of over an order of magnitude larger than that of Kubono et al. for the 6.356 MeV $1/2^+$ state of $^{17}\text{O}$.

Key words: NUCLEAR REACTIONS, $^{13}\text{C}(^{6}\text{Li},d)$, $E_{\text{lab}} = 60$ MeV, DWBA and CRC analyses, deduced $S_\alpha$ and reduced $\gamma_\alpha^2$  

PACS: 24.10.Eq, 25.70.Hi, 27.20.+n

1 Introduction

The slow neutron capture, or s process is thought to be the production mechanism for approximately half of all heavy elements in the universe [1,2]. The

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asymptotic giant branch (AGB) phase of low and intermediate mass stars is thought to be the most likely astrophysical site for this process [2]. Although the neutron source for the s process has still not been well identified, the most recent studies seem to prefer the $^{13}$C($\alpha,n$)$^{16}$O reaction as the main neutron source in AGB stars at low temperatures [2,3].

Models of the s process depend critically on the neutron flux produced by the $^{13}$C($\alpha,n$)$^{16}$O reaction. However, the astrophysical S factor for this reaction, defined by:

$$S(E) = \sigma(E)E \exp(2\pi\eta)$$  \hspace{1cm} (1)

where $\sigma(E)$ is the cross section and $\eta$ the Coulomb parameter, has been determined experimentally down to a centre of mass energy of only 270 keV [4], whereas the reaction takes place predominantly at energies below this point.

As the cross section for the $^{13}$C($\alpha,n$)$^{16}$O reaction decreases extremely rapidly as the incident $\alpha$ energy gets lower, direct measurement of the reaction rate at lower energies is very difficult. Extrapolations [4,5] suggest a rapid increase of the S factor as the incident $\alpha$ energy reduces to zero, which has been ascribed to the effect of resonances in $^{17}$O [5], the $J^\pi = 1/2^-$ level at 5.94 MeV and the $1/2^+$ level at 6.356 MeV, both very close to the $\alpha + ^{13}$C threshold of 6.359 MeV. However, due to the experimental uncertainties, the data are also consistent with a constant, horizontal extrapolated S factor [4].

In a recent paper, Kubono et al. [1] suggested that a better way to determine the S factor at lower energies is via a direct $\alpha$ transfer measurement. The reaction chosen for this study was the $^{13}$C($^6$Li,$d$)$^{17}$O transfer at $E_{\text{lab}} = 60$ MeV and angular distributions for transfers leading to the 0.0 MeV $5/2^+$, 0.87 MeV $1/2^+$, 3.055 MeV $1/2^-$, 3.84 MeV $5/2^-$, 4.55 MeV $3/2^-$ and 6.356 MeV $1/2^+$ states of $^{17}$O were accurately measured. The $\alpha$ spectroscopic factor $S_\alpha$ of each state was determined from a finite-range distorted-wave Born approximation (DWBA) analysis [1] of these data. Using the value of $S_\alpha$ obtained for the 6.356 MeV $1/2^+$ state, its $\alpha$ width was determined and the cross section of the $^{13}$C($\alpha,n$)$^{16}$O reaction through the tail of this sub-threshold resonance calculated. The astrophysical S factor was then calculated using Eq. (1).

The DWBA analysis of Kubono et al. gave a very small value of $S_\alpha \approx 0.011$ for the 6.356 MeV $1/2^+$ state of $^{17}$O when normalised by the same scaling factor used to obtain $S_\alpha = 0.25$ for the 3.055 MeV $1/2^-$ state. This leads to a very small contribution of the $1/2^+_2$ sub-threshold state to the reaction rate at low energies and thus to an astrophysical S factor that is essentially constant below $E_{\text{c.m.}} = 300$ keV. Since such a result poses serious questions about the mechanism of neutron production for the s process, a reanalysis of the Kubono et al. data was undertaken. In the present work we present the
results of detailed DWBA and coupled reaction channels (CRC) analyses of these same data to determine whether $S_\alpha$ for the 6.356 MeV $1/2^+$ state of $^{17}$O is actually so small. Our results show that the original DWBA analysis of Kubono et al. led to incorrect conclusions, and that their data are compatible with a much larger value of $S_\alpha$ for the 6.356 MeV $1/2^+$ state. The present analysis is consistent with previous extrapolations of the astrophysical $S$ factor for the $^{13}$C($\alpha,n$)$^{16}$O reaction that indicate a rapid increase as the incident $\alpha$ energy approaches zero.

2 The DWBA calculations

Initially, we carried out a DWBA analysis as similar as possible to that of Kubono et al. [1], but using the code FRESCO [6]. We employed the same $\alpha + ^{13}$C binding potential and the same $^6$Li + $^{13}$C and $d + ^{17}$O optical potentials as in the original analysis [1]. However, all calculations presented here were carried out including the full complex remnant term unless otherwise stated. The remnant term occurs in the residual interaction $W$ in the expression for the transition amplitude in the DWBA. For the reaction $A + \alpha (= b + x) \rightarrow B (= A + x) + b$ the residual interaction is defined as follows for post form DWBA:

$$W_\beta = V_{bB} - U_\beta = V_{bx} + (V_{bA} - U_\beta)$$  \hspace{1cm} (2)

and in the prior form:

$$W_\alpha = V_{aA} - U_\alpha = V_{xA} + (V_{bA} - U_\alpha).$$  \hspace{1cm} (3)

The remnant terms are the quantities in parentheses, $U_\alpha$ and $U_\beta$ are the (complex) optical model potentials in the entrance and exit channels, respectively and $V_{bx}$ and $V_{xA}$ are the potentials binding the transferred particle ($x$) to the projectile and target cores, respectively. The quantity $V_{bA}$ is a (complex) optical potential operating between the projectile and target cores. See Satchler [7] for a full discussion of these terms.

The use of the full remnant term enables good agreement between post and prior formulations of the DWBA to be obtained, which is not possible when no remnant term is included. We show results for the post formulation, although we did perform test calculations using the prior form to ensure that agreement was obtained. As a further check we compared calculations performed using FRESCO with no remnant term (using the post formulation) with calculations carried out with the code DWUCK5 [8]. For each transfer considered here excellent agreement between the two codes was obtained.
Table 1
Values of $N$, $L$ and $S_\alpha$ used in the DWBA calculations. Values of $S_\alpha$ are given for calculations with the full complex remnant term and calculations with no remnant term included.

<table>
<thead>
<tr>
<th>State $J^\pi$</th>
<th>$N$</th>
<th>$L$</th>
<th>$S_\alpha$ (full complex remnant)</th>
<th>$S_\alpha$ (no remnant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 MeV 5/2$^+$</td>
<td>2</td>
<td>3</td>
<td>0.36</td>
<td>0.15</td>
</tr>
<tr>
<td>0.87 MeV 1/2$^+$</td>
<td>3</td>
<td>1</td>
<td>0.42</td>
<td>0.18</td>
</tr>
<tr>
<td>3.055 MeV 1/2$^-$</td>
<td>4</td>
<td>0</td>
<td>0.81</td>
<td>0.32</td>
</tr>
<tr>
<td>3.84 MeV 5/2$^-$</td>
<td>3</td>
<td>2</td>
<td>0.64</td>
<td>0.28</td>
</tr>
<tr>
<td>4.55 MeV 3/2$^-$</td>
<td>3</td>
<td>2</td>
<td>0.90</td>
<td>0.38</td>
</tr>
<tr>
<td>6.356 MeV 1/2$^+$</td>
<td>4</td>
<td>1</td>
<td>0.49</td>
<td>0.24</td>
</tr>
</tbody>
</table>

The entrance channel $^6$Li + $^{13}$C optical potential was set 1 of Kubono et al. [1] and we used their set 3 for the exit channel $d + ^{17}$O potential [9]. The $\alpha + d$ binding potential was that of Kubo and Hirata [10], with the $\alpha$ particle assumed to be in a relative $2s$ state with respect to the $d$ core. Throughout we adopt the convention that the number of radial nodes includes that at the origin but excludes that at infinity. The $\alpha + ^{13}$C binding potential had a radius of $1.25 \times (4^{1/3} + 13^{1/3})$ fm and diffuseness 0.65 fm, the depth being adjusted to give the correct binding energy for each $^{17}$O state considered. The number of nodes $N$ and orbital angular momentum of the transferred $\alpha$ particle with respect to the $^{13}$C core $L$ were fixed by the oscillatory energy conservation relation $2(N - 1) + L = \sum_{i=1}^4 2(n_i - 1) + l_i$, where $(n_i, l_i)$ are the corresponding single-nucleon shell quantum numbers. The values of $N$ and $L$ used and the spectroscopic factors $S_\alpha$ obtained in the analysis are given in Table 1. The spectroscopic factor for the $\alpha+d$ overlap was taken to be 1.0. Note that the $S_\alpha$ values implicitly contain the $C^2$ term, where $C$ is the isospin Clebsch-Gordan coefficient; however, in this case $C = 1$. As can be seen from Table 1 the contribution from the remnant term in the DWBA calculations is substantial and accounts for more than 60% of the $S_\alpha$ strength for each $^{17}$O state. The results of the DWBA calculations are compared with the data in Fig. 1.

Figure 1 shows that we obtain reasonably good descriptions of the data, with the exception of that for transfers to the 3.84 MeV 5/2$^-$ and 4.55 MeV 3/2$^-$ states at forward angles. Unlike the original analysis of Kubono et al. [1], we obtain a reasonable description of the 0.0 MeV 5/2$^+$ state; as the $\alpha$ particle is in an $L = 3$ state relative to the $^{13}$C core the DWBA angular distribution should be relatively structureless, like the data. As we obtain such an angular distribution (both with FRESCO and DWUCK5), in contrast to the structured angular distribution for this state shown in Ref. [1], we are forced to conclude that these authors have made an error in their calculations for this state, either through plotting the wrong curve or by using the wrong $L$ value.
Fig. 1. DWBA calculations compared to the data. The dashed curves denote calculations including the full complex remnant term, while the dotted curves denote calculations with no remnant term. All calculations use the post form of the DWBA.

It will be noted from Table 1 that if one adopts the procedure of Kubono et al. [1] of re-normalizing all the $S_\alpha$ values with the scaling factor that gives a
value of $S_\alpha = 0.25$ for the 3.055 MeV $1/2^-$ state, derived from a microscopic cluster calculation [11], one obtains from our DWBA results an $S_\alpha$ value for the 6.356 MeV $1/2^+$ state of 0.15 or 0.19, depending on whether the remnant term is included in the calculations or not. Either value is over an order of magnitude greater than that ($S_\alpha = 0.011$) found by Kubono et al. [1]. As we have used the same potentials in our DWBA calculations as those used in Ref. [1], we are unable to account for this discrepancy in our results for $S_\alpha$ compared to theirs. Note that the use of the remnant term cannot account for the discrepancy (it is not clear from their paper whether Kubono et al. [1] included a remnant term in their calculations) as is apparent from Table 1. The only uncertainty is in the values used for the number of nodes $N$ in the $\alpha + ^{13}\text{C}$ wave functions used by Kubono et al., as these are not given in their original paper.

There is some ambiguity in the choice of $N$, depending on the structure assumed for the state in question. In deriving the $N$ values for the 3.055 MeV $1/2^-$, 3.84 MeV $5/2^-$ and 4.55 MeV $3/2^-$ states given in Table 1 we have assumed a 2p-1h configuration for these states. However, it has been suggested that these states are of 4p-3h character [12], and if one assumes this structure one obtains $N$ values of 5, 4 and 4 for the 3.055 MeV $1/2^-$, 3.84 MeV $5/2^-$ and 4.55 MeV $3/2^-$ states, respectively. These larger $N$ values lead to $S_\alpha$ values of 0.30, 0.25 and 0.36, respectively, which would yield a normalised $S_\alpha$ value of 0.41 for the 6.356 MeV $1/2^+$ state. In reality, these states are probably a mixture of 2p-1h and 4p-3h configurations [11,13]; clearly, differences in the assumed structure of the 3.055 MeV $1/2^-$ state cannot account for the discrepancy in normalised $S_\alpha$ values between the current work and that of Kubono et al. [1].

For the 6.356 MeV $1/2^+$ state itself, the value of $N$ is more clearly defined; any physically reasonable choice of structure for this state results in a value of $N = 4$, the value we have adopted. Thus, we find that the smallest value of $S_\alpha$ for this state consistent with the $^{13}\text{C}(^6\text{Li},d)^{17}\text{O}$ transfer data is 0.15 (after applying the normalisation procedure discussed above) and that it could be as high as 0.41, depending on the structure assumed for the 3.055 MeV $1/2^-$ state, used to obtain the normalisation factor. It should be emphasised that there is no solid justification for the normalisation procedure used in Ref. [1] to obtain spectroscopic factors for all states based on the $\alpha$-cluster structure of the 3.055 MeV $1/2^-$ state. Furthermore, this procedure is model-dependent as it relies strongly on the accuracy of the calculated $S_\alpha$ of Furutani et al. [11] for the 3.055 MeV $1/2^-$ state.

We are thus forced to conclude that the original DWBA analysis of Kubono et al. [1] is flawed, and that the $S_\alpha$ value obtained by them for the 6.356 MeV $1/2^+$ state is much too small. Our DWBA calculations using the same optical potential parameters as those used by Kubono et al. show that the
Table 2
Parameters of the real DPP and volume WS imaginary potentials (5)-(6). The negative value for $V$ indicates a repulsive potential. Radius parameters are given as: $R_p = r_p \times 13^{1/3}$.

<table>
<thead>
<tr>
<th>$V$ (MeV)</th>
<th>$r_V$ (fm)</th>
<th>$a_V$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$r_W$ (fm)</th>
<th>$a_W$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-18.0403</td>
<td>1.5079</td>
<td>0.6823</td>
<td>37.2152</td>
<td>1.3770</td>
<td>1.0150</td>
</tr>
</tbody>
</table>

$^{13}$C($^6$Li,$d$)$^{17}$O transfer data measured by them are consistent with an (unrenormalised) $S_\alpha \approx 0.49$ for the 6.356 MeV $1/2^+$ state of $^{17}$O, well within the range used in astrophysical calculations ($S_\alpha \approx 0.3 - 0.7$).

We now discuss a subtle but quite important issue: the sensitivity of the DWBA calculation to the choice of optical potential for the entrance channel. This potential defines the door-way state of the transfer reaction and it should be determined as accurately as possible. The $^6$Li+$^{13}$C elastic scattering was measured by Kubono et al. at forward angles only [1], which gives rise to some ambiguity in the choice of the optical potential. In Fig. 2 we plot the optical model fit to the $^6$Li+$^{13}$C elastic scattering data using their parameter set 1. One can see that for large angles the predicted cross section exhibits unphysical behaviour compared to data for $^6$Li + $^{12}$C elastic scattering at the same $^6$Li incident energy which extend to backward angles [14].

The $^6$Li+$^{12}$C elastic scattering has been extensively studied in the past, and a very accurate systematics for the (energy dependent) optical potential has been established in a folding model analysis of these data over a wide range of incident energies [15]. An important result of this study is that the so-called dynamic polarisation potential (DPP) caused by $^6$Li breakup has a strong repulsive contribution to the real potential. This DPP contribution can be rather well represented by a surface Woods-Saxon (WS) potential. We have, therefore, performed an additional optical model analysis of the $^6$Li+$^{13}$C elastic scattering data at $E_{lab} = 60$ MeV using an entrance channel optical potential of the following form:

$$U(R) = V_{Fold}(R) + \Delta V(R) + iW(R),$$  \hspace{1cm} (4)

where $\Delta V(R) = 4V_a \frac{d}{dR} \left[1 + \exp \left(\frac{R - R_V}{a_V}\right)\right]^{-1}$ \hspace{1cm} (5)

and $W(R) = -W \left[1 + \exp \left(\frac{R - R_W}{a_W}\right)\right]^{-1}$. \hspace{1cm} (6)

The DPP and volume WS imaginary potential parameters obtained (see Table 2) are quite close in shape to those determined from the Folding model analysis [15] of the $^6$Li+$^{12}$C elastic scattering data at the same incident $^6$Li energy but over a much wider angular range.
The Folding potential was calculated using the CDM3Y6 interaction (whose density-dependent parameters were fine-tuned to reproduce the bulk properties of cold nuclear matter [16]) and the ground-state densities of $^6$Li and $^{13}$C obtained in the independent-particle model by Satchler and Love [17] and Satchler [18], respectively. The predicted elastic scattering obtained using this potential is shown as the dotted curve in Fig. 2. Note that the volume integral of the real optical potential per interacting nucleon pair $J_V$ (an important key to distinguish discrete potential families) is around -410 and -329 MeV fm$^3$ for the set 1 potential taken from Ref. [1] and our Folding + DPP potential, respectively. The empirical energy dependence of $J_V$ is well-known from the global optical model analysis of light heavy-ion elastic data [19], and one can find from this systematics that $J_V$ should be around -330 MeV fm$^3$ for an energy of 10 MeV/nucleon (see Fig. 6.7 in Ref. [19]) which is in perfect agreement with our Folding + DPP calculation. We conclude, therefore, that the Folding + DPP potential should be more appropriate for the entrance channel optical potential. While both Kubono et al.’s set 1 and our Folding + DPP potential produce similar elastic scattering cross sections at the forward angles for which data are available, they differ considerably at larger angles, and a future measurement of the $^6$Li+$^{13}$C elastic scattering at larger angles would be extremely helpful in determining a realistic entrance channel optical potential.

We show the results of the DWBA calculations using this new $^6$Li + $^{13}$C optical potential in Fig. 3 and give the $S_\alpha$ values obtained from this analysis in Table 3. As can be seen from Fig. 3, with the exception of the 3.055 MeV 1/2$^-$ state, the agreement between the DWBA calculations and the data is much better than that obtained using the set 1 potential of Kubono et al. This is particularly noticeable for the two 1/2$^+$ states. Table 3 also shows that the $S_\alpha$ values obtained from this analysis are considerably smaller than those given in Table 1. The absolute $S_\alpha$ value obtained for the 6.356 MeV 1/2$^+$ state is around 0.36 in this case. If we adopt the (controversial) normalisation procedure discussed above, we obtain $S_\alpha$ values of 0.3 or 0.5 for the 6.356 MeV 1/2$^+$ state, depending on whether a 2p-1h or 4p-3h structure is assumed for the 3.055 MeV 1/2$^-$ state. All these estimates of the $S_\alpha$ value are over an order of magnitude larger than that ($S_\alpha \approx 0.011$) determined in Ref. [1].

The influence of the entrance channel optical potential on the $S_\alpha$ values extracted from the DWBA analysis stresses again the importance of obtaining elastic scattering data over a sufficiently wide angular range for light heavy-ion systems such as that currently under consideration.
Fig. 2. Optical model calculations compared to the $^6\text{Li} + ^{13}\text{C}$ elastic scattering data. The dashed curve denotes the predicted elastic scattering angular distribution using the optical model potential set 1 of Kubono et al. [1]. The dotted curve denotes the prediction of the present Folding + DPP potential.

3 Coupled reaction channels calculations

In order to test the possible influence of multi-step transfer paths on the $S_\alpha$ values we also carried out a limited coupled reaction channels (CRC) analysis which included couplings between the ground state of $^{17}\text{O}$ and its $1/2^+_1$, $1/2^+_1$, $5/2^-_1$, $3/2^-_1$ and $1/2^+_2$ excited states as well as the direct $\alpha$ transfer route. We did not consider multi-step paths proceeding via the excited states of $^{13}\text{C}$ as a physically meaningful analysis including these paths is not possible without some prior estimate of the necessary $\alpha$ strengths.

We used the Folding + DPP optical potential described in the previous section in the entrance channel and parameter set 3 of Kubono et al. [1] in the exit channel. The $^{17}\text{O}$ Coulomb coupling strengths were determined from the mea-
Fig. 3. DWBA calculations using the Folding + DPP optical potential for the $^6\text{Li}+^{13}\text{C}$ system compared to the data. For the 3.055 MeV $1/2^-$, 3.84 MeV $5/2^-$ and 4.55 MeV $3/2^-$ states the dashed curves denote calculations assuming a 2p-1h structure, while the solid curves denote calculations assuming a 4p-3h structure. All calculations use the post form of the DWBA.
Table 3
Values of $S_\alpha$ obtained from the DWBA and CRC calculations using the Folding + DPP optical potential for $^6\text{Li}+^{13}\text{C}$ system.

<table>
<thead>
<tr>
<th>State</th>
<th>$J^\pi$</th>
<th>$N$</th>
<th>DWBA $S_\alpha$</th>
<th>CRC (2p-1h) $S_\alpha$</th>
<th>CRC (4p-3h) $S_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 MeV</td>
<td>5/2$^+$</td>
<td>2</td>
<td>0.15</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>0.87 MeV</td>
<td>1/2$^+$</td>
<td>3</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>3.055 MeV</td>
<td>1/2$^-$</td>
<td>4</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.18</td>
<td></td>
<td>0.18</td>
</tr>
<tr>
<td>3.84 MeV</td>
<td>5/2$^-$</td>
<td>3</td>
<td>0.34</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.19</td>
<td></td>
<td>0.19</td>
</tr>
<tr>
<td>4.55 MeV</td>
<td>3/2$^-$</td>
<td>3</td>
<td>0.48</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.27</td>
<td></td>
<td>0.27</td>
</tr>
<tr>
<td>6.356 MeV</td>
<td>1/2$^+$</td>
<td>4</td>
<td>0.36</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

sured $B(E\lambda)$ values [20] and the nuclear coupling potentials were obtained by deforming the $d+^{17}\text{O}$ optical potential with nuclear deformation lengths obtained from the corresponding $B(E\lambda)$ values (assuming an $^{17}\text{O}$ radius of $1.2 \times 17^{1/3}$ fm). As the $^{17}\text{O}$ coupling strengths are weak, it was not found necessary to alter the $d+^{17}\text{O}$ optical potential to compensate for the inclusion of these couplings.

The results of the CRC analysis are shown in Fig. 4. The CRC description of the elastic scattering is almost identical to that shown by the dotted curve in Fig. 2. The $S_\alpha$ values extracted from this CRC analysis are given in Table 3 in the two columns labelled “CRC(2p-1h)” and “CRC(4p-3h)”, which denote the values obtained from the calculations assuming a 2p-1h and 4p-3h structure, respectively, for the $1/2^+_1$, $5/2^-_1$ and $3/2^-_1$ states.

As Fig. 4 shows, with the exception of the transfer to the $5/2^+_1$ ground state, the shapes of the predicted angular distributions are unchanged by the inclusion of couplings between the ground state and the excited states of $^{17}\text{O}$. The $S_\alpha$ values are also either unchanged or, for the $5/2^-_1$ and $1/2^+_2$ states, very slightly altered by these extra couplings. The changes in the $S_\alpha$ values for the $5/2^-_1$ and $1/2^+_2$ states are smaller than those produced by, for example, the use of the Folding + DPP optical potential for the $^6\text{Li}+^{13}\text{C}$ system compared to Kubono et al.’s set 1 potential or by the inclusion of the full complex remnant term compared to a calculation with no remnant term. Therefore, we may conclude that couplings within $^{17}\text{O}$ will have a negligible effect on the $S_\alpha$ values extracted from an analysis of the $^{13}\text{C} (^6\text{Li},d)^{17}\text{O}$ transfer reaction. For the crucial 6.356 MeV $1/2^+$ state, an $S_\alpha$ value around 0.4 is given by

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Fig. 4. CRC calculations using the Folding + DPP optical potential for the $^6$Li+$^{13}$C system compared to the data. The dashed curves denote the result of a calculation assuming a 2p-1h structure for the 3.055 MeV $1/2^-$, 3.84 MeV $5/2^-$ and 4.55 MeV $3/2^-$ states, while the solid curves denote the result of a calculation assuming a 4p-3h structure for these states.
both DWBA and CRC calculations which is fully compatible with those used earlier \((S_\alpha \approx 0.3 - 0.7)\) in the \(s\)-process model calculations.

However, multi-step effects can be important and lead to anomalous population of non \(\alpha\)-cluster states in both the \((^6\text{Li},d)\) and \((^7\text{Li},t)\) reactions, as was noted by Debevec \textit{et al.} [21] for the \(^{12}\text{C} (^6\text{Li},d)^{16}\text{O}\) reaction. They found that for 32 MeV incident \(^6\text{Li}\) the 10.35 and 11.1 MeV \(4^+\) states are populated with relative strengths of \(\sigma(11.1)/\sigma(10.35) \approx 0.5\) despite the fact that their \(\alpha\)-particle widths are in the ratio \(\Gamma_\alpha(11.1)/\Gamma_\alpha(10.35) \approx 0.01\). Kemper and Ophel [22] have shown that this anomalous population of the 11.1 MeV \(4^+\) state is non-statistical in origin and must therefore be due to multi-step transfer paths. This problem is not so marked for the \(^{12}\text{C} (^7\text{Li},t)^{16}\text{O}\) reaction and appears to become less important as the incident \(^6\text{Li}\) or \(^7\text{Li}\) energy increases. Nevertheless, it is an indication that multi-step paths can be important and must be considered carefully if one wishes to extract meaningful \(S_\alpha\) values from the analysis of \(\alpha\) transfer reactions.

4 Reduced \(\alpha\) widths

One may also derive the reduced \(\alpha\) width, \(\gamma_\alpha^2\), of a state from a DWBA analysis. The reduced \(\alpha\) width is related to the spectroscopic factor and the \(\alpha+^{13}\text{C}\) bound state radial wave function used in the DWBA calculations as follows

\[ \gamma_\alpha^2 = S_\alpha \frac{\hbar^2 R}{2\mu_\alpha} |u_L(R)|^2 \]  

where \(\mu_\alpha\) is the \(\alpha+^{13}\text{C}\) reduced mass, \(R\) is the channel radius and \(u_L(R)\) is the \(\alpha+^{13}\text{C}\) bound state radial wave function, normalised such that \(\int_0^\infty u_L^2(r)r^2dr = 1.0\). We take \(R = 5.5\) fm as our channel radius, the value used by Kubono \textit{et al.} [1] and Furutani \textit{et al.} [11]. Such a choice for the channel radius is reasonably compatible with the ‘strong absorption’ radius of the \(\alpha+^{13}\text{C}\) system of around 5.2–5.3 fm. The reduced \(\alpha\) width is often scaled by the Wigner limit, defined as

\[ \gamma_{\alpha W}^2(R) = \frac{3\hbar^2}{2\mu_\alpha^2 R^2}, \]  

to obtain the dimensionless reduced \(\alpha\) width, \(\theta_\alpha^2(R)\) [23]. It should be noted that Kubono \textit{et al.} [1] have made assumptions about the \(\alpha+^{13}\text{C}\) bound state wave function that are equivalent to having \(S_\alpha = \theta_\alpha^2\), which is not always the case; we have not made this assumption here.
Table 4
\( \theta_\alpha^2 \) and \( \gamma_\alpha^2 \) values obtained from DWBA analyses using the \(^6\text{Li} + ^{13}\text{C}\) optical potentials of Kubono et al. [1] (Set 1) and the present work (Folding + DPP) at a channel radius \( R = 5.5 \text{ fm} \). Also shown are values for \( \theta_\alpha^2(5.5) \) from the microscopic cluster-model calculation of Furutani et al. [11].

<table>
<thead>
<tr>
<th>State</th>
<th>( J^\pi )</th>
<th>( N )</th>
<th>( \theta_\alpha^2 ) (keV)</th>
<th>( \gamma_\alpha^2 ) (keV)</th>
<th>( \theta_\alpha^2 )</th>
<th>( \gamma_\alpha^2 ) (keV)</th>
<th>( \theta_\alpha^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 MeV</td>
<td>( 5/2^+ )</td>
<td>2</td>
<td>0.209</td>
<td>142</td>
<td>0.087</td>
<td>59.1</td>
<td>0.084</td>
</tr>
<tr>
<td>0.87 MeV</td>
<td>( 1/2^+ )</td>
<td>3</td>
<td>0.331</td>
<td>224</td>
<td>0.134</td>
<td>90.8</td>
<td>0.23</td>
</tr>
<tr>
<td>3.055 MeV</td>
<td>( 1/2^- )</td>
<td>4</td>
<td>0.723</td>
<td>490</td>
<td>0.268</td>
<td>182</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.254</td>
<td>172</td>
<td>0.153</td>
<td>104</td>
<td>0.23</td>
</tr>
<tr>
<td>3.84 MeV</td>
<td>( 5/2^- )</td>
<td>3</td>
<td>0.554</td>
<td>375</td>
<td>0.294</td>
<td>199</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.216</td>
<td>146</td>
<td>0.164</td>
<td>111</td>
<td>0.23</td>
</tr>
<tr>
<td>4.55 MeV</td>
<td>( 3/2^- )</td>
<td>4</td>
<td>0.760</td>
<td>515</td>
<td>0.406</td>
<td>275</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.288</td>
<td>195</td>
<td>0.216</td>
<td>146</td>
<td>0.23</td>
</tr>
<tr>
<td>6.356 MeV</td>
<td>( 1/2^+ )</td>
<td>4</td>
<td>0.272</td>
<td>184</td>
<td>0.200</td>
<td>136</td>
<td>0.23</td>
</tr>
</tbody>
</table>

In Table 4 we give values for \( \gamma_\alpha^2 \) and \( \theta_\alpha^2 \) calculated using the \( S_\alpha \) values of Tables 1 and 3. Note that we do not use the normalised \( S_\alpha \) values in calculating \( \gamma_\alpha^2 \) and \( \theta_\alpha^2 \), as it is uncertain what the effect of the scaling process on the \( \alpha + ^{13}\text{C} \) bound state wave function should be.

We have checked our procedures against those of Becchetti et al. [23] for the much more widely studied \(^{12}\text{C}(^7\text{Li},t)^{16}\text{O}\) reaction and obtain good agreement with their values for \( \gamma_\alpha^2 \) for states in \(^{16}\text{O}\) using the same \( \alpha + ^{12}\text{C} \) wave functions and \( S_\alpha \) values. It should be emphasised that we obtain values of \( \gamma_\alpha^2 \) for the crucial 6.356 MeV \( 1/2^+ \) state considerably greater than the 7.4 keV value given in Ref. [1].

We also give in Table 4 \( \theta_\alpha^2 \) values for the 3.055 MeV \( 1/2^- \), 3.84 MeV \( 5/2^- \) and 4.55 MeV \( 3/2^- \) states from the microscopic cluster-model calculation of Furutani et al. [11]. The calculations using Kubono et al.’s set 1 for the \(^6\text{Li} + ^{13}\text{C}\) optical potential yield \( \theta_\alpha^2 \) values for the 3.055 MeV \( 1/2^- \) state that are much smaller than the cluster model value, while for the 3.84 MeV \( 5/2^- \) and 4.55 MeV \( 3/2^- \) states our \( \theta_\alpha^2 \) values assuming a 4p-3h structure for these states are reasonably close to Furutani et al.’s cluster model results. For the calculations using our Folding + DPP optical potential for the \(^6\text{Li} + ^{13}\text{C}\) system, the agreement is better and the cluster-model \( \theta_\alpha^2 \) values agree reasonably well with our values assuming a 4p-3h structure for these three states. This agreement with the calculation of Furutani et al. [11] shows that the \( \theta_\alpha^2 \) values that we have extracted from our DWBA calculations are at least plausible. Furthermore,
it supports the suggestion of Bethge et al. [12] that these states (the 3.055 MeV $1/2^-$, 3.84 MeV $5/2^-$ and 4.55 MeV $3/2^-$) are of predominantly 4p-3h character.

5 Conclusions

A recent DWBA analysis of new 60 MeV $^{13}$C($^{6}$Li, $d$)$^{17}$O transfer data [1] found a very small $\alpha$-spectroscopic factor ($S_\alpha \approx 0.011$) for the 6.356 MeV $1/2^+$ state. The authors of Ref. [1] concluded that there is no large enhancement of the $^{13}$C($\alpha$, $n$)$^{16}$O reaction rate at energies of astrophysical interest due to this sub-threshold state based on the small reduced $\alpha$ width they obtained, contrary to previous suggestions [4,5]. Since such a result poses serious questions about the $^{13}$C($\alpha$, $n$)$^{16}$O reaction as a neutron source for the $s$ process, we have performed DWBA and CRC analyses of their data using a realistic choice for the entrance channel optical potential. We concluded that the transfer data are consistent with a much larger $S_\alpha$ value for the 6.356 MeV $1/2^+$ state and consequently a much larger $\gamma_\alpha^2$ value. Thus, the transfer data are compatible with a large contribution from this state to the $^{13}$C($\alpha$, $n$)$^{16}$O reaction rate at very low energies.

We have also shown that uncertainties in the assumed $\alpha+^{13}$C structure of the $^{17}$O states can have significant effects on the extracted spectroscopic factors and reduced $\alpha$ widths. While this problem does not directly affect the 6.356 MeV $1/2^+$ state, one needs to be aware of it. As relative $S_\alpha$ factors are reasonably well determined one may use the normalisation procedure employed by Kubono et al. to scale the DWBA results to a calculated value for a state that is believed to have significant $\alpha$ clustering structure in order to obtain absolute $S_\alpha$ values. However, such a method is highly questionable as the fragmentation of the $\alpha$-cluster states over the low-energy region of $^{17}$O excitations is unknown. Moreover, there are difficulties in this method as it relies on the accuracy of the structure calculation used for the normalisation, plus it is not clear what effect this process should have on the $\alpha +$ core wave function if one wishes to extract $\gamma_\alpha^2$ values.

We also noted the effect of the entrance channel optical potential on the $S_\alpha$ values extracted from the DWBA analysis, which was considerable. This demonstrates the need to have a realistic choice for the optical potential in the entrance channel. It is, therefore, necessary to obtain elastic scattering data over a large angular range (possibly much larger than the angular range of the transfer data themselves) if one wishes to use this type of indirect approach to obtain reliable estimates of astrophysical reaction rates.

A limited coupled reaction channels study found that couplings between the
ground and excited states of $^{17}\text{O}$ had a negligible effect on the $S_\alpha$ values. Other multi-step transfer paths, in particular, those proceeding via the excited states of $^{13}\text{C}$, were not investigated due to the need for some prior estimate of the necessary $\alpha$ strengths. Without such estimates one merely introduces extra adjustable parameters that need to be determined from the same data set.

In summary, we have highlighted some of the pitfalls of DWBA analyses of $\alpha$ transfer reactions and their use to extract spectroscopic factors. We have not considered in detail the more fundamental question of whether such transfer reactions may be adequately described by a simple direct $\alpha$ transfer, as is assumed in the DWBA. If multi-step paths contribute significantly to the $\alpha$ transfer strength the $S_\alpha$ values extracted from such a DWBA analysis will not be reliable, regardless of how carefully it is carried out. However, an extensive investigation of these multi-step transfer paths still cannot be carried out in a meaningful way for the $^{13}\text{C}(^6\text{Li},d)^{17}\text{O}$ reaction due to the lack of prior estimates of the required $\alpha$ strengths for transfers proceeding via the excited states of $^{13}\text{C}$. Thus, while the indirect approach to astrophysical reaction rates can be a valuable tool one needs to be aware of possible uncontrolled parameters contained in the analysis.

ACKNOWLEDGEMENTS

The authors thank Prof. S. Kubono for providing them with the data in tabular form and Prof. D. Robson for many helpful discussions. One of the authors (N.K.) would like to thank the Institute for Nuclear Science and Technique, Hanoi, for hospitality during the period in which this work was initiated. The present research was supported, in part, by the Natural Science Council of Vietnam, the U.S. National Science Foundation and the State of Florida.

References
