It is well known that, when analyzed at the light of current synthesis model predictions, variations in the physical properties of single stellar populations (e.g. age, metallicity, initial mass function, element abundance ratios) may have a similar effect in their integrated spectral energy distributions. The confusion is even worsened when more realistic scenarios, i.e. composite star formation histories, are considered. This is, in fact, one of the major problems when facing the study of stellar populations in star clusters and galaxies. Typically, the observational efforts have been aimed to find the most appropriate spectroscopic indicators in order to avoid, as far as possible, degeneracies in the parameter space. However, from a practical point of view, the most suited observables are not, necessarily, those that provide more orthogonality in that parameter space, but those that give the best balance between parameter degeneracy and sensitivity to signal-to-noise ratio per Å, $SN(Å)$. In order to achieve the minimum combined total error in the derived physical parameters, this work discusses how the functional dependence of typical line-strength indices and colors on $SN(Å)$ allows to define a suitability parameter which helps to obtain better realistic combinations of spectroscopic data. As an example, we discuss in more detail the problem of breaking the well known age-metallicity degeneracy in relatively old stellar populations, comparing the suitability of different spectroscopic diagrams for a simple stellar population of solar metallicity and 12 Gyr old. methods: data analysis – techniques: spectroscopic – galaxies: stellar content

Evolutionary synthesis modeling subsection, models

In order to predict the expected spectral energy distribution (SED) of simple stellar populations (chemically homogeneous and coeval stellar systems), it is possible to use first principles (e.g. initial mass function, star formation rate, stellar isochrones, element abundance ratios) to generate synthetic star systems. This technique, known as evolutionary synthesis modeling, has been widely employed to understand the origin and evolution of star clusters and galaxies (Crampin & Hoyle crampin61, Tinsley tinsley72, tinsley78, tinsley80, Tinsley & Gunn tinsley76, Gunn, Stryker & Tinsley gunn81, Bruzual bruzaual83, bruzaual02; Aragón-Salamanca, Gorgas & Rego aragon87; Charlot & Bruzual charlot91; Bruzual & Charlot bruzaual93; Arimoto & Yoshii arimoto86, arimoto87; Guiderdoni & Rocca-Volmerange guiderdoni87; Buzzoni buzzoni89, buzzoni95; Mas-Hesse & Kunth mmas91; Fritze-v. Alvensleben & Gerhard fritze94; Cerviño & Mas-Hesse cervino94; Worthey worthey94; Worthey & Ottaviani worthey97; Bressan, Chiosi & Fagotto bressan94; Chiosi, Bressan & Fagotto chiosi96; Tantalo et al. tantalo98; Milone, Barbuy & Bica milone95; Leitherer & Heckman leitherer95; Leitherer et al. leitherer96, leitherer99; Fioc & Rocca-Volmerange fioc97; Vazdekis et al. vazdekis96, vazdekis97, vazdekis03; Vazdekis vazdekis99; Mayya mayya95, mayya97; García-Vargas, Mollá & Bressan marisa98; Mollá & García-Vargas molla00; Maraston maraston98; Schiavon, Barbuy & Bruzual schiavon00; Origlia & Oliva origlia00; Zackrisson et al. zackrisson01; Thomas, Maraston & Bender thomas03).

The reliability of model predictions has greatly increased as their developers include more realistic physical ingredients. However, as discussed by Charlot, Worthey & Bressan (charlot96), there are still problems due to uncertainties in the theory of stellar evolution (e.g. post-main-sequence stages), the physics of stellar interiors (e.g. atomic diffusion, helium content, the temperature of the red giant branch), and the lack of complete stellar spectra libraries. It is important to note that although initially it is straightforward to predict spectroscopic indices from this type of models, the inherent problems associated to the SED libraries, either empirical or theoretical, have a non negligible influence in the line-strength predictions. For instance, empirical SED libraries constitute a coarse grained, and usually incomplete (specially for nonsolar metallicities and nonsolar abundance ratios) sampling of the atmospheric stellar parameter space, whereas theoretical libraries usually exhibit systematic discrepancies among themselves and when compared with observational data (e.g. Lejeune, Cuisinier & Buser 1997, 1998).
The use of empirical fitting functions (e.g. Gorgas et al. gorgas93, gorgas99; Worthey et al. wortheyetal94; Cenarro et al. cenarro02) can help to reduce substantially these effects (Worthey worthey94; Vazdekis et al. vazdekis03). They do not only allow the computation of line-strength indices for any given combination of input parameters, but the error in their predictions can be minimized with the use of a large set of stars. However, and since the empirical fitting functions only predict the value of a given line-strength feature for a fixed set of stellar atmospheric parameters, the shape of the spectrum that leads to such value is therefore unknown. To insert the fitting function predictions into the evolutionary synthesis models it is necessary to use the local continuum of each single star in the SED library as a reference continuum level. In this way it is possible to weight the luminosity contribution of each type of star, in the neighborhood wavelength region of each index, to obtain the final line-strength prediction.

In addition, there are also additional sources of biases in model predictions. Cerviño et al. (cervino00, cervino01, cervino02), and Cerviño & Valls-Gabaud (cervino03) have thoroughly analyzed the impact of the actual discreteness of real stellar populations (see also Bruzual bruzual01, and references therein), the Poissonian dispersion due to finite populations in non-time-integrated observables, and the influence of the interpolations in time-integrated quantities, among others.

But far from being a discouraging situation, the recognition of all these problems is providing a solid understanding of the challenging task of modeling stellar populations. In this sense, the collective effort of many modelers (e.g. Leitherer et al. leitherer96) is given strength to the idea that reliable and unbiased model predictions are starting to emerge.

Physical parameter degeneracy

Although spectroscopic data provide a direct way to analyze the integrated light of composite stellar systems, the predictions from simple stellar population synthesis models reveal that variations in the relevant physical properties of such systems may produce quite similar spectral energy distributions (SEDs). This \textit{conspiracy} leads to undesirable degeneracies when passing from the observable space (e.g. that defined by line-strength indices and colors), to the parameter space (age, metallicity, initial mass function, etc.).

Among the best known examples of degeneracy we must highlight the one exhibited by age and metallicity in the study of relatively old stellar populations (O’Connell oconnell76, oconnell80, oconnell94; Aaronson et al. aaronson78; Worthey worthey94; Faber et al. faber94). This outstanding problem drove many authors to seek for spectral line-strength indices which were more sensitive to age than to metallicity and vice versa (e.g. Rose rose85, rose94; Worthey worthey94). In this sense, Worthey (1994) introduced an interesting quantitative measure of the metal sensitivity of each index, computed as the partial derivatives \( \frac{d \log(\text{age})}{d \log(Z)} \) around his model predictions for a 12 Gyr old stellar population with solar metallicity. Since then, large efforts have been focused toward the search of spectral features with very high (e.g. Fe4668) and very low (e.g. H\(\beta\), H\(\gamma\)) metal sensitivities. However, this work has led to the use of individual and narrow absorption features (e.g. Jones & Worthey jones95; Worthey & Ottaviani worthey97; Vazdekis & Arimoto vazdekis99) for which accurate measurements demand high signal-to-noise ratios. In addition, these spectral signatures are usually very sensitive to spectral resolution and, therefore, velocity dispersion.

Compromise between orthogonality and errors

It is important to note that since the problem is to break a degeneracy, in practice the real concern is how uncertain the requested physical parameters are when derived from a particular observable space. In this sense, two circumstances have to be carefully handled. The first is the orthogonality of the iso-parameter lines in the observable space. As we have just mentioned, this is precisely the major concern of previous works. The second condition to be aware of is the propagation of the errors in the spectroscopic indices into the corresponding errors in the parameters. However, and as it is expected, narrow indices (better suited to provide more orthogonality) exhibit larger errors than broad spectral features, for a given signal-to-noise ratio. Summarizing, orthogonality and small errors are magnitudes that can not be, a priori, simultaneously maximized. As a result, it seems clear that the most suited observable space will be that in which the two mentioned requirements are best balanced.

The relevance of finding this equilibrium can hardly be overemphasized, specially when one considers the important observational effort that is being (or is going to be) spent in ambitious spectroscopic surveys, like e.g. DEEP (Mould mould93; Koo koo95), EFAR (Wegner et al. wegner96), CFRS (Lilly et al. lilly95; Hammer et al. hammer97), Sloan (York et al. york00; Kauffmann et al. kauffmann03), VLT-VIRMOS (Le Fèvre et al. lefevre00), SDSS (Bernardi et al. bernardi03). In all these type of surveys, a large amount of
spectroscopic data is collected, although signal-to-noise ratios and spectral resolution are typically moderate. These factors strengthen the need of a quantitative estimation of the reliability of the physical parameters derived from such spectroscopic studies.

With a clear practical sense, in this paper we explore the way to determine those combinations of spectroscopic observables that provide robust tools to constrain physical properties in stellar populations. For this purpose, we are going to assume that evolutionary synthesis model predictions are error free. Although, as we have discussed in Sect. subsections, models, this is not the case, we want to concentrate in the problem of balancing errors and its associated random error spectrum, showing that a common functional dependence of final index error versus signal—noise ratio can be found for different metallicities. In section, age, degeneracy, through the comparison of the suitability of different spectroscopic measurements section, error handling section, error in spectroscopic measurements section, error handling section.

Errors and spectroscopic measurements

Random uncertainties and biases are inherently associated to the physical process of data acquisition. Random errors can be easily derived with the help of statistical methods. Unfortunately, the situation is not so simple when handling systematic effects, where a case by case solution must be sought. In practice, the aim is to obtain reliable quantitative constraints of the total random errors present in the data while having uncorrected systematic effects (if any) well within the range spanned by the former. For this to be the case, possible sources of systematic effects should be identified and alleviated during the measure process. In this paper we are assuming that this is actually the case, and for that reason we are exclusively focusing on the impact of random errors.

Although, as we have just mentioned, appropriate observational strategies can greatly help in reducing the sources of data biases, the unavoidable limited exposure time that can be spent in each target determines the maximum signal-to-noise ratio in practice achievable. The data reduction process, aimed to minimize the impact of data acquisition imperfections on the measurement of data properties with a scientific meaning for the astronomer, is typically performed by means of arithmetical manipulations of data and calibration frames. As a result, the initial random errors present in the raw scientific and calibration data are combined (and thus enlarged) and propagated throughout the reduction procedure.

In a recent paper, Cardiel et al. (cardiel02) have discussed the benefits and drawbacks of different methods to quantify random errors in the context of data reduction pipelines. One of the conclusions of this work is that a parallel reduction of data and error frames is likely the most elegant and general approach, and, in some circumstances, the only way to proceed when observing or computing time demands are specially restrictive. It must be noted, however, that in order to apply this method to compute final errors, it is essential to prevent the introduction of correlation between neighboring pixels, which dangerously leads to underestimated errors. This problem arises when one performs image manipulations involving rebinning or non-integer pixel shifts of data, which is the case of those data reduction steps devoted to correct for geometric distortions, to produce a wavelength calibration into a linear scale, or to correct for differential refraction variations with wavelength, to mention a few. Fortunately, a modification in the way typical reduction processes operates can help to solve this problem. Although we are not going to enter into details (we refer the interested reader to that paper), the key point is to transfer the responsibility of the most complex reduction steps to the analysis tools, which must manipulate data and error frames using a distorted system of coordinates, overriding the orthogonal coordinate system defined by the physical pixels in a detector.

Random errors in spectroscopic measurements

Once it can be assumed that reliable final random error estimates are available, and that, in comparison, systematic biases are not relevant, it is straightforward to obtain a quantitative estimate of the error in a given spectroscopic measurement. Since the information collected by detectors is physically sampled in pixels, the starting point in the analysis of a single spectrum will be the spectrum itself $S(\lambda_i)$ (with $i = 1, ..., N_{\text{pixels}}$) and its associated random error spectrum $\sigma_S(\lambda_i)$. In the following discussion, we are assuming equation

$$\text{cov}[S(\lambda_i), S(\lambda_j)] = \langle S(\lambda_i) S(\lambda_j) \rangle - \langle S(\lambda_i) \rangle \langle S(\lambda_j) \rangle = 0.$$  

One possibility is to estimate numerically the effect of errors via Monte Carlo simulations. In practice, new instances of the spectrum, $\hat{S}(\lambda_i)$, can be generated introducing Gaussian noise in each pixel using, for example, equation

$$\hat{S}(\lambda_i) = S(\lambda_i) + \sqrt{2} \sigma_S(\lambda_i) \sqrt{-\ln(1 - r_1)} \cos(2\pi r_2).$$