Collision of large nuclei at nonzero impact parameter exhibit the special feature of a strongly deformed overlap region. The subsequent dynamical evolution of the collision zone converts the initial spatial eccentricity into an azimuthal anisotropy of the final state observables. In particular anisotropies in momentum space [1] have been analyzed experimentally in great detail in recent years, as they have been shown to be generated during the earliest and hottest stages of the reaction [2]. As momentum anisotropies are generated, the eccentricity in coordinate space is smoothed out, bringing further generation of momentum anisotropies to a stall, a process that occurs within about the first 5 fm/c of the total 15 fm/c lifetime of the fireball [2–4]. In Au+Au collisions at RHIC (the Relativistic Heavy Ion Collider at Brookhaven National Laboratory), those anisotropies have been found [5] to be as large as predicted by hydrodynamic calculations that assume a short equilibration time $\tau_{\text{equ}} \leq 1 \text{ fm/c}$ [4,6]. Further theoretical investigation has revealed the sensitivity of anisotropic flow on the equation of state of the expanding medium [7–9], indicating the necessity of a hard phase at high temperatures with a soft transition region of width $\Delta e \sim 1 \text{ GeV/fm}^3$ to lower temperatures in order to describe the data in more detail. For these reasons anisotropic flow became one of the most attractive tools to study the nuclear equation of state of the matter created in the collision by utilizing the most abundant particles, alternatively to study statistically disadvantaged rare probes with which one tries to infer the properties of the bulk by modeling their mutual interaction with the medium they traverse. Other predictions from hydrodynamic concepts such as centrality and mass dependence of momentum anisotropies [8] have subsequently been confirmed to be in qualitative and quantitative agreement with experiments [10,11], as long as the applicability of hydrodynamics is not overstretched, i.e. for impact parameters not larger than $b \sim 7 \text{ fm}$ and transverse momenta not to exceed $p_T \sim 1.5 \text{ GeV}$. Beyond these values clear deviations from the hydrodynamic predictions start to occur. This is expected as the smaller systems do not sufficiently equilibrate and the most rapid particles escape the fireball without fully participating in the collective dynamics.

To date, anisotropies at midrapidity are generally only characterized in terms of the second coefficient $v_2$ of the Fourier decomposition of the azimuthally sensitive momentum spectrum. With the $z$ direction given by the beam axis and the $x$ direction defined by the direction of the impact parameter (and $y$ perpendicular to both), the general expression for the Fourier decomposition with respect to the azimuthal angle $\varphi = \arctan(y/x)$ is

$$\frac{dN}{dp_Tdyd\varphi} = \frac{1}{2\pi} \frac{dN}{dp_Tdy} \left(1 + \sum_{n, \text{even}} 2 v_n(p_T) \cos(n\varphi)\right), \quad (1)$$

where no sine terms appear due to the symmetry with respect to the reaction plane, and all odd harmonics vanish at midrapidity due to the symmetry $\varphi \leftrightarrow \varphi + \pi$. (Note that the latter is not true for noncentral d-Au collisions, where $v_1, v_3, \ldots$ should be useful quantities to classify and study the measured spectra – even at midrapidity). In the following we will elaborate on the centrality and momentum dependence of higher anisotropy coefficients, which turn out to exhibit interesting features that, with the currently available data from RHIC’s run in 2002, might already be accessible.

Generally, higher momentum anisotropies are expected to be small, and in fact no experimental data have been published until this day. An early hydrodynamic calculation [12] predicted the momentum integrated value of $v_4$ not to exceed 0.5 % at SPS energies even in the most peripheral collisions, and shows only a slight increase with beam energy up to RHIC energies. In another study Teaney and Shuryak [13] calculated $\alpha_4$, which is the fourth coefficient of the momentum integrated differential particle spectra weighted by an additional momentum squared (which was applied to enhance the effects of transverse flow). This quantity is thus strongly biased to larger $p_T$. From their hydrodynamic calculation they obtain $\alpha_4 \sim 1\%$; clearly the momentum integrated value of $v_4$ is still smaller.

Today, experiments at RHIC achieve sufficient statistics to measure the elliptic flow coefficient $v_2$ differentially in momentum out to large transverse momenta up to 10 GeV [14]. In midcentral collisions this coefficient is found to saturate starting at $p_T \sim 2 \text{ GeV}$ at a large value of $20 - 25\%$, indicating that three times as many particles are emitted into the reaction plane than out of the reaction plane. Such a large value also means that the first order deformation of Eq. (1) is not elliptic anymore, but that a polar plot of the azimuthal distribution resembles more a peanut than an elliptical shape, aligned with its
This raises the interesting question, whether at high $p_T$ Fourier coefficients of higher order might become sufficiently large to restore the elliptically deformed shape of the particle distribution. To restore the elliptic shape as much as possible only through introduction of a fourth anisotropy coefficient, $v_4 \sim \frac{1}{4!}(10v_2 - 1)$ is required, as is derived straightforwardly from Eq. (1). This, in consideration of the large experimental values for $v_2$, might indicate that $v_4$ could reach values around 3–5%.

To investigate higher flow coefficients in more detail, we first revisit hydrodynamic calculations, although they find their limitations for pions of transverse momenta beyond 2 GeV, and for heavier baryons at momenta 3–4 GeV (for a recent review see Ref. [15]). Later on we will discuss a complementary model in terms of a simple picture for the limit of very large transverse momentum. The results presented in the following were achieved by a more modern hydrodynamic calculation than in Ref. [12], as it becomes important at RHIC energies to correctly account for features of chemical nonequilibrium during the hadronic evolution of the system [16,17], a feature that significantly influences the relation of energy-density and temperature [18,19]. At temperatures beyond $T_{\text{crit}} = 165$ MeV the underlying equation of state turns into a hard ideal gas equation of state via a strong first order phase transition in order to mimic the transition from a gas of interacting resonances to a hard plasma phase. The initial conditions of this calculation were determined to fit single particle spectra measured in Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV. For further details, see Ref. [17].

Figure 2 shows the momentum dependence of the Fourier coefficients of pion spectra up to order 8. “Elliptic flow,” $v_2$ dominates the emission anisotropy at all momenta. (To use the same vertical scale for all coefficients, the second harmonic $v_2$ was divided by 10.) As anticipated, $v_4$ increases with momentum, reaching a value of about 2.8% at $p_T = 2$ GeV (thus not quite enough to restore the particle distribution to an elliptic shape, as $v_2(p_T = 2$ GeV) $\approx 29\%$). Also $v_6$ still has a sizable negative value of about 1.2%. The mass dependence of higher flow harmonics does not bring any surprises. Due to their larger masses, heavy particles pick up larger transverse momenta when they participate in the established collective flow. As for the spectra and elliptic flow, also the higher anisotropies are shifted out toward larger transverse momenta. Thus at a given transverse momentum below 2 GeV, the anisotropies of heavy resonances are smaller than of pions. Comparing hydrodynamic calculations to experimental results, it appears however that deviations from hydrodynamic behavior occur later for heavy particles, and therefore anisotropies of baryons can eventually surpass anisotropies of light mesons [20]. From these calculations one can expect a fourth flow component of (anti-)protons of about 7% at 4 GeV transverse momentum, a sizable quantity that should be measurable.

FIG. 1. (Color online) Polar plot of the azimuthal distribution of different Fourier expansions. Left, distortion of the unit circle through an elliptic component $v_2$, shown for 10% (green) and 20% (red). Right, fourth component $v_4$ for the same values.

Surprisingly it turns out that $v_4$ is highly sensitive to the initial conditions of the calculations: initializing the calculation with a prehydrodynamic, isotropic radial flow field as in Ref. [17] does not change the magnitude of the flow anisotropies by a large amount. However we find that it completely changes the sign of $v_4$, while the other coefficients remain largely the same. Clearly $v_4$ at low

$^*$The definition of restored ellipticity is ambiguous. Here we chose to find the value of $v_4$ where the second derivative of $y(x)$ vanishes at $x = 0$, thus giving rise to a very smooth transition across $\varphi = \pi/2$. 

FIG. 2. (Color online) Fourier coefficients of the pion spectra resulting from a hydrodynamic calculation to describe particle spectra at $\sqrt{s_{\text{NN}}} = 200$ GeV at impact parameter $b = 7$ fm. $v_2$ is scaled by a factor 0.1.
and intermediate $p_T$ has therefore a strong potential to constrain model calculations and carries valuable information on the dynamical evolution of the system. To study how the higher flow anisotropies develop in the course of the hydrodynamic evolution we calculate moments of the transverse flow field $(v_x, v_y)$ through

$$\delta_n(\tau) = \frac{\int \int dx dy \ e(x, y; \tau) \ \gamma \ v_T \ cos(n \varphi)}{\int \int dx dy \ e(x, y; \tau) \ \gamma}, \quad (2)$$

where $e(x, y; \tau)$ is the energy density in the transverse plane at time $\tau$ and $v_T = (v_x^2 + v_y^2)^{1/2}$ is the transverse flow velocity at a given point $(x, y)$, $\gamma = (1 - v_T^2)^{-1/2}$, and $\varphi = \arctan(v_y/v_x)$ is the angle of the local flow velocity with respect to the reaction plane. (Note that for $n = 0$ this reduces to the definition of the mean radial velocity used in earlier studies [4], but for $n = 2$ it is slightly different from the momentum anisotropy $\epsilon_p$ which is defined in terms of the difference in the diagonal elements of the energy momentum tensor.)

Figure 3 displays the time evolution of the flow coefficients and clearly shows that radial flow $\delta_0$ increases throughout the lifetime of the system (which is about 15 fm/c), with a somewhat reduced acceleration while most of the system is in the mixed phase at around 4 fm/c. Quite oppositely anisotropy components quickly saturate and remain constant, as the initial geometric deformation of the source is lost. About 2/3 of the final value of $\delta_2$ is generated during the earliest stages of the evolution, where the system is governed by the hard equation of state of the quark gluon plasma. $\delta_0$ quickly achieves small negative values of the order of -0.2 % (as expected from the negative value of $\nu_0$). Due to its small values, it is subject to numerical fluctuations and therefore not shown. It is expected that the flow coefficients are monotonically related to the anisotropy coefficients in the final particle spectra. Details of this relation are subject to future study.

In a model complementary to the hydrodynamic description which is constrained to the low momenta, thermalized part of the spectrum, one can study anisotropies that result from opacity effects that the deformed initial geometry exerts on particles of high momenta [21–23]. Assuming extreme opacity, or extreme jet quenching, particles of large transverse momenta from any part of the surface can only be emitted within $\pm 90^\circ$ of the normal vector to the surface, which for this exploratory study is assumed to be given by the overlap of two hard spheres (radius $R$). In this picture, the $n$th Fourier coefficient of the resulting expansion is found to be (for $n$ even)

$$v_n = (-1)^{n/2} \frac{1}{1 - n^2} \frac{\sin(n \alpha)}{n \alpha}, \quad (3)$$

where $\alpha = \arccos(b/2R)$ characterizes the centrality in terms of the opening angle between the line through the center of one nucleus to the intersection point of the two nuclear spheres and the reaction plane. Note that as $b$ increases from 0 to $2R$, $\alpha$ decreases from $\pi/2$ to 0. For $n = 2$, $v_2 = \sin(2\alpha)/(6\alpha)$, a formula first given by Voloshin [26]. For $n > 2$, Eq. 3 becomes significantly more interesting (unfortunately, however, the size of the expected signal drops with $n^{-3}$). Note first the oscillating sign of the prefactor which will manifest itself in an oscillation of the coefficients at large $b$ (small $\alpha$). Second, one notices that for larger $n$ the sine function starts changing sign upon scanning impact parameters ($b \leftrightarrow \alpha$), whereas for $n = 2$, the Fourier coefficient increases linearly with impact parameter from 0 to 1/3, and is thus always positive. In particular for $n = 4$ one observes a transition from positive to negative values at $b = \sqrt{2}R$, reaching the limiting value $v_4 = -1/15 \approx -6.67\%$ for $b \to 2R$, again a rather large number. The local maximum of $v_4$ is at $b \approx 0.865R$ with $v_4 \approx 1.45\%$. The centrality dependences of the second to eighth Fourier coefficients are shown in Fig. 4 (as before $v_2$ is divided by a factor of 10).

Clearly, the expected values for higher expansion coefficients from this calculation are small. Applying a more realistic model does, however, not necessarily lead to a further reduction of the signal, but might in contrast increase, similarly as observed for calculations of elliptic flow [24]. This is also suggested by the experimental data on $v_2$ which seem to exceed the limits given even by the most optimistic assumptions for anisotropies from geometry and energy loss. In any case, such a distinctive signal as a sign change upon variation of centrality should be preserved when model calculations get refined and should be accessible within the statistics reached by today’s experiments.

1Relaxing this geometrical constraint by adopting a finite diffuseness of the nuclei reduces the resulting asymmetries, far below the experimental results, and novel effects are required to explain these large limiting values [24,25].
negligibly small compared to
found to be highly dependent on the initial configuration
From the experience gathered so far we expect
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signal again, however, the
for the azimuthally sensitive distribution will reduce the
We propose this method thus as an interesting indepen-
y_4 is scaled by a factor of 0.1.

As an aside we would like to discuss a new but naive
experimental way of measuring higher order flow coeff-
ficients, in particular v_4. Apart from the “straightfor-
ward” method to determine each event’s reaction plane
and averaging all particles’ \cos(4\varphi) with respect to the
reaction plane and the more recently developed cumul-
ant technique \[27\], simply measuring the particle spec-
tra with respect to the reaction plane might be sufficient
(technologically). Having the azimuthally sensitive particle
spectra given, Eq. (1) shows directly that [abbreviating
\(dN/\rho_T d\rho_T dy d\varphi (\rho_T, \varphi) = n(\varphi)\)]

\[ n(0) + n(\pi/2) - 2n(\pi/4) = \]
\[ \frac{1}{2\pi \rho_T d\rho_T} \left[ 8v_4(\rho_T) + 8v_{12}(\rho_T) + \ldots \right]. \]  

(4)

From the experience gathered so far we expect v_{12} to be
negligibly small compared to v_4. Finite opening angles
for the azimuthally sensitive distribution will reduce the
signal again, however, the v_4 is in this approach enhanced
by a factor of 8, which might still render it measurable.
We propose this method thus as an interesting indepen-
dent approach to measure v_2 (but also v_2, by simply sub-
tracting spectra in and out of the reaction plane).

In summary, we have shown that v_4(\rho_T) might achieve
significantly large values at intermediate and large trans-
verse momenta in noncentral heavy ion collisions and
contains important physics of heavy ion collisions, a fact
largely overlooked in the past. Numerical estimates for
its size were given both in the limit of full thermaliza-
tion with a subsequent hydrodynamic expansion as well as
in the geometrical limit of extreme “jet quenching.”
For the latter case a strong centrality dependence and a
change of the sign of the signal for higher anisotropies
was found. From both approximations we found that v_4
might reach values of the order of 5%, clearly observa-
table with todays equipment and statistics. v_4 has been
found to be highly dependent on the initial configuration
of the system and its evolution and is thus an important
new tool to constrain model calculations and analyze de-
tails of the system’s history. Consequently it deserves to
be studied in great detail both experimentally as well as
theoretically.

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