Texture of Yukawa coupling matrices in general two-Higgs-doublet model

Yu-Feng Zhou

Ludwig-Maximilians-University Munich,
Sektion Physik. Theresienstr. 37, D-80333. Munich, Germany
(Dated: October 4, 2005)

Abstract

We discuss possible parallel textures of the Yukawa coupling matrices in the general two-Higgs-doublet model (2HDM). In those textures the flavor changing neutral currents are naturally suppressed. Motivated by a phenomenologically successful texture with four texture zeros in the standard model, we propose a predictive ansatz for the Yukawa coupling matrices with the same texture in the general 2HDM. Compared with the six texture-zero based ansatz proposed by Cheng and Sher, it is in a better agreement with the data of quark mixings and CP violation. The four texture-zero based ansatz predicts a different hierarchy in the Yukawa coupling matrix elements. As a consequence, in the lepton sector, the related Yukawa couplings are less constrained by the experimental upper bound of $\mu \rightarrow e\gamma$, which allows significantly larger predictions for other processes. The contributions from neutral scalar interactions to the lepton number violation decay modes $\ell \rightarrow \ell_1\ell_2\ell_3$ are calculated in both ansatz. It is shown that the predictions from the four texture-zero based ansatz could be two order of magnitude greater than that from the six texture-zero based one. The branching ratio of $\mu \rightarrow 3e$ and $\tau \rightarrow 3\mu$ can reach $7.5 \times 10^{-17}$ and $1.3 \times 10^{-10}$ respectively. The predicted ratio of $Br(\mu \rightarrow 3e)/Br(\tau \rightarrow 3e)$ is also larger and almost parameter independent. Those differences make the two ansatz to be easily distinguished by the future experiments.

PACS numbers: 12.60.Fr, 12.15.Ff
I. INTRODUCTION

Although the standard model (SM) of electroweak interactions with $SU(2)_L \otimes U(1)_Y$ gauge symmetry has achieved a great success in phenomenology, the outstanding problems such as the origin of the fermion masses, the mixing angles as well as the CP violation are still unresolved. It is widely believed that the SM can not be a fundamental theory of the basic interactions. For this reason, many new physics models have been proposed and extensively studied in the recent years. Among those models, the SM with two-Higgs-doublet (2HDM) can be regard as the simplest extension of the present SM, and have attracted a lot of attention (see, e.g. \[ ??? \] ).

In the most general form of 2HDM \[ ??? \], the Lagrangian for the Yukawa interaction is given by

$$-\mathcal{L}_Y = \bar{q}_L \Gamma_1^u \phi_1 u_R + \bar{q}_L \Gamma_1^d \phi_1 d_R + \bar{q}_L \Gamma_2^u \phi_2 u_R + \bar{q}_L \Gamma_2^d \phi_2 d_R + \bar{\ell}_L \Gamma_1^{\ell} \phi_1 l_R + \bar{\ell}_L \Gamma_2^{\ell} \phi_2 l_R + \text{H.c},$$

(1)

with $\Gamma_i^F (i = 1, 2)$ being the Yukawa interaction matrices and $\phi_i = i \tau_2 \phi_i^*$. The two Higgs fields after the spontaneous symmetry breaking have the following form

$$\phi_1 = \left( \frac{1}{\sqrt{2}} \left( v_1 e^{i \delta} + \phi_1^0 + i \chi_1^0 \right) \right), \quad \phi_2 = \left( \frac{1}{\sqrt{2}} \left( v_2 + \phi_2^0 + i \chi_2^0 \right) \right),$$

(2)

where $v_1$ and $v_2$ are the absolute values of the vacuum expectation values (VEVs) of the two Higgs fields, which satisfy $v = \sqrt{v_1^2 + v_2^2} \simeq 246$ GeV. $\delta$ is a relative phase between them, which can be a new source of CP violation in this model [??]. The ratio between $v_1$ and $v_2$ is often referred to as $\tan \beta$ with the definition of $\tan \beta \equiv v_2/v_1$.

In the general 2HDM, the relation to the Higgs mechanism in the SM is manifest if one applies a basis transformation of

$$H = \phi_1 e^{-i \delta} \cos \beta + \phi_2 \sin \beta, \quad \phi = \phi_1 e^{-i \delta} \sin \beta - \phi_2 \cos \beta.$$  

(3)

In this new basis, the two Higgs fields are

$$H = \left( \begin{array}{c} G^+ \\ \frac{1}{\sqrt{2}} (\rho^0 + i G^0 + v) \end{array} \right), \quad \phi = \left( \begin{array}{c} H^+ \\ \frac{1}{\sqrt{2}} (R^0 + i l^0) \end{array} \right),$$

(4)

where $G^\pm, G^0$ are the would-be Goldenston bosons in the SM. $H^+, R^0$ and $l^0$ are new scalars. In general, the flavor eigenstates $H = (\rho^0, R^0, l^0)$ are not the mass eigenstates. The mass eigenstates denoted by $H = (H^0, h^0, A^0)$ are related to the flavor eigenstates through a mixing matrix $O$, i.e. $H_i = O_{ij} H_j$.

In the whole discussion below, for the sake of simplicity, we will focus on the case in which both VEVs are real, i.e. $\delta = 0$, and the effects of neutral scalar mixing are negligible. To simplify notations, we omit the flavor index of $F = u, d, e$ in the superscript of the Yukawa matrices $\Gamma_i^F$ in Eq.(??), and will add them latter when a distinction between flavors is needed.
In this approximation, the fermion mass and the Yukawa coupling matrices are simply given by

\[ M = \frac{v}{\sqrt{2}}(\Gamma_1 \cos \beta + \Gamma_2 \sin \beta), \]
\[ \Gamma = \frac{1}{\sqrt{2}}(\Gamma_1 \sin \beta - \Gamma_2 \cos \beta). \]  

(5)

The main problem in the models with multi-Higgs-doublet is that in general \( M \) and \( \Gamma \) cannot be diagonalized simultaneously, the off diagonal elements of \( \Gamma \) after being rotated into mass basis may give too large contributions to \( K^0 - \overline{K}^0 \) mixing and \( K_L \rightarrow \mu\mu \) via flavor changing neutral currents (FCNCs) at tree level. To forbid the tree level FCNC in 2HDM, the ad hoc discrete symmetries are often imposed on the Lagrangian [? ]

\[ \phi_1 \rightarrow -\phi_1 \text{ and } \phi_2 \rightarrow \phi_2, \]
\[ u_R \rightarrow -u_R \text{ and } d_R \rightarrow \pm d_R, \]  

(6)

which defines two types of 2HDM without tree level FCNCs, referred to as model I and II of 2HDM. However, since at present the most strict bound of tree level FCNCs only comes from the light quark sector, There is a possibility that the tree level FCNCs do occur, but greatly suppressed due to the smallness of the light quark masses. Abandoning the discrete symmetry in Eq.(??), one arrives at the general type of 2HDM with small FCNCs [? ? ? ? ]. The small off diagonal Yukawa matrix elements can be attributed to an approximate flavor symmetry which is slightly broken down, and the magnitudes of the symmetry breaking are proportional to the related fermion masses [? ? ? ].

Besides the symmetric considerations, another practical way to prevent from large FCNCs in the general 2HDM is to adopt reasonable anastz on the textures of the Yukawa interaction matrices \( \Gamma_i \). It is well known that without a loss of generality, the mass matrices \( M \) can be rotated to be Hermitian by a suitable redefinition of the fermion fields in the flavor basis [? ]. In the case of both VEVs are real and arbitrary, \( \Gamma_1 \) and \( \Gamma_2 \) could also be rotated to be Hermitian. The possibility that \( M \) and \( \Gamma \) be diagonalized simultaneously depends only on the commutator of them, namely they can be diagonalized simultaneously if and only if \([M, \Gamma] = 0\). From Eq.(??), the commutator of \( M \) and \( \Gamma \) is given by

\[ [M, \Gamma] = -\frac{v}{2}[\Gamma_1, \Gamma_2]. \]  

(7)

Obviously, if \( \Gamma_i \)'s have a parallel structure of

\[ \Gamma_2 \approx c\Gamma_1, \]  

(8)

with \( c \) being a number factor, the commutator will be close to zero, the Yukawa coupling matrix \( \Gamma \) after being rotated into mass basis will be nearly diagonal, and the tree level FCNCs will be suppressed accordingly. The simplest way to get the parallel texture is to impose a \( S_2 \) permutation symmetry between \( \phi_1 \) and \( \phi_2 \) on the Lagrangian, namely the Lagrangian is invariant under the transformation of

\[ \phi_1 \longleftrightarrow \phi_2, \]  

(9)

which directly leads to \( c = 1 \), and \( \Gamma_1 = \Gamma_2 \). Thus all the tree level FCNCs vanish.