The effect of unitary noise

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Abstract: The effect of unitary noise on the performance of Grover’s quantum search algorithm is studied. This type of noise may result from tiny fluctuations and drift in the parameters of the (quantum) components performing the computation. The resulting operations are still unitary, but not precisely those assumed in the design of the algorithm. Here we focus on the effect of such noise in the Hadamard gate $W$, which is an essential component in each iteration of the quantum search process. To this end $W$ is replaced by a noisy Hadamard gate $U$. The parameters of $U$ at each iteration are taken from an arbitrary probability distribution (e.g., Gaussian distribution) and are characterized by their statistical moments around the parameters of $W$. For simplicity we assume that the noise is unbiased and isotropic, namely all noise variables in the parametrization we use have zero average and the same standard deviation $\epsilon$. The noise terms at different calls to $U$ are assumed to be uncorrelated. For a search space of size $N = 2^n$ (where $n$ is the number of qubits used to span this space) it is found that as long as $\epsilon < O(n^{-12}N^{-14})$, the algorithm maintains significant efficiency, while above this noise level its operation is hampered completely. It is also found that below this noise threshold, when the search fails, it is likely to provide a state that differs from the marked state by only a few bits. This feature can be used to search for the marked state by a classical post-processing, even if the quantum search has failed, thus improving the success rate of the search process.
Fig. 1

Graph showing the comparison between simulation and prediction for $P_0$, $P_1$, and $P_{far}$ as a function of $\varepsilon$.

- $P_0$ decreases as $\varepsilon$ increases.
- $P_1$ increases as $\varepsilon$ increases.
- $P_{far}$ increases as $\varepsilon$ increases.

The graphs illustrate the trends predicted by the model compared to the simulation results.
Fig. 4

- **optimal hybrid strategy**
- **classic search strategy**
- **naive Grover strategy**