LONG-LIVED DOUBLE-BARRED GALAXIES: CRITICAL MASS AND LENGTH SCALES

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ABSTRACT

A substantial fraction of disk galaxies is double-barred. We analyze the dynamical stability of such nested bar systems by means of Liapunov exponents, by fixing a generic model and varying the inner (secondary) bar mass. We show that there exists a critical mass below which the secondary bar cannot sustain its own orbital structure, and above which it progressively destroys the outer (primary) bar-supporting orbits. In this critical state, a large fraction of the trajectories (regular and chaotic) are aligned with either bar, suggesting the plausibility of long-lived dynamical states when secondary-to-primary bar mass ratio is of the order of a few percent. Qualitatively similar results are obtained by varying the size of the secondary bar, within certain limits, while keeping its mass constant. In both cases, an important role appears to be played by chaotic trajectories which are trapped around (especially) the primary bar for long periods of time.

Subject headings: galaxies: evolution — galaxies: kinematics & dynamics — galaxies: structure — instabilities — stellar dynamics

1. Introduction

If long-lived double-barred galactic systems were not observed, explaining the fact would hardly pose a major theoretical challenge. It would be sufficient to suppose a situation whereby generic trajectories transit erratically between motion characteristic of resonant, bar-supporting orbits of the inner and outer bars — their shapes, in the process, supporting neither bar. Indeed, in general, trajectories that can switch between qualitatively different modes of motion, such as those of a pendulum near the vertical point or a ball near the top of the hill of a double-well potential, will be chaotic, unless the system exhibits exceptional symmetry.\(^1\) In fact the resulting “homoclinic” phenomena (which guarantee at least transiently erratic motion) can be taken as to define the phenomenon of dynamical “chaos,” in both its conservative and Hamiltonian manifestations (Holmes 1990; Ruelle 1989). In a galaxy with two bars tumbling with different pattern speeds, there are, in general, no special symmetries, and no corresponding smooth integrals of motion in the phase space.

Nevertheless, there is reason to believe that structures, such as nested bars, can materialize. Shlosman, Frank & Begelman (1989) have an-

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\(^1\)In the case of the pendulum, it suffices that it be unperturbed.
alyzed possible formation mechanisms. An argument was put forward that nuclear bars are secondary dynamical features which form due to gravitational instabilities in the gas accumulation within the central kpc and subsequently affect the background stellar component. Probably the most dramatic result of this analysis was that double bar systems spend a large fraction of their lifetime in a dynamically decoupled state, characterized by substantially different pattern speeds of each bar.

High-resolution ground-based observations have revealed a number of galaxies with a sub-kpc secondary stellar bars (e.g., Buta & Croker 1993; Shaw et al. 1995; Friedli et al. 1996; Jungwiert, Combes & Axon 1997; Mulchaey & Regan 1997; Jogee, Kenney & Smith 1998; Erwin & Sparke 1999; Knapen, Shlosman & Peletier 2000; Elmegreen et al. 2001). The first statistics on nested bar galaxies, has been limited exclusively to stellar bars due to superior resolution in detecting the stellar light distribution and kinematics. The most comprehensive, so far, HST survey of 112 galaxies finds that in excess of 20%—25% of disks host double bars, and about 1/3 of all barred galaxies host another (nuclear) bar (Laine et al. 2002). The former can even reach 40% (Erwin & Sparke). A clear indication that nested bars indeed tumble with different pattern speeds comes from their random mutual orientation (Friedli et al.) and, indirectly, from the bimodal length distribution of bars in these systems (Laine et al.).

The frequency of detection of double-barred systems suggests that, at least some of them, can be relatively long-lived. Since we do not expect that they can be built of trajectories that can transit between the modes of motion of the two subsystems, i.e., of untrapped chaotic orbits, we conjecture that such systems are composed of orbits that are trapped either around the outer (primary) or the inner (secondary) bar. Under this assumption (verified a posteriori), the system is made of regular orbits confined to each bar and trapped chaotic orbits in the vicinity of the regular regions.

Within the context of the KAM theorem (e.g., Arnold 1987), the continued stability of quasiperiodic solutions in a dynamical system requires that external perturbations be sufficiently small, in which case most of these orbits remain, even if slightly deformed. Our definition of decoupled bars as simply tumbling with different pattern speeds does not necessarily imply this. This is because the gravitational quadrupole interaction between the bars can, in principle, be strong enough so as to destroy a large fraction of the regular trajectories — which, in general, can result in the dissolution of either, or both, bars. Thus one has to ask under what conditions the supporting trajectories of each bar are also KAM-stable under the influence of the second bar perturbation. This issue is addressed here.

2. Model and Method

To answer the questions posed above, we define a generic model and vary its parameters. We find it appropriate to examine motion in a fixed potential, where bar parameters can be set at will, rather than attempt $N$-body experiments, and choose variants of one of the models for a double-barred galaxy previously investigated by Shlosman & Heller (2002). These consist of halo and bulge modeled as Plummer spheres (the former with a large core of 10 kpc) and a Miyamoto-Nagai (1975) disk, supplemented by two Ferrers (1877) bars of order 1, with the secondary bar rotating 8.3 times faster than the primary one. The secondary bar mass has been varied by factors 2 and 4, above and below its mass in the Model 1 (hereafter generic model) of Shlosman & Heller. Qualitatively similar results are obtained if one varies the surface density of the secondary by changing its size around generic value, as long as the relevant primary bar orbits are still affected by the quadrupole moment of the secondary bar. The generic ratio of secondary-to-primary bar masses is 0.047 and their surface density ratio about 8. The bar size ratio is 0.08, which positions the secondary bar between the inner Lindblad resonances (ILRs) of the primary. Each bar comprises about 20% of the total mass within bar radii.

We employ Liapunov exponents (see El-Zant & Shlosman [2002] for complete details) in order to examine the stability of trajectories, and confine ourselves, in this first exploration, to two-dimensional motion. We choose a mesh of a hundred initial positions, equally spaced along the primary bar major axis. Each position is a starting point for a hundred trajectories with normal velocities equally subdivided in the range between 1.25 times the local rotation velocity (excluding...
the bars’ contributions) to a hundredth of this value. Such initial conditions should adequately describe bar-supporting orbits, i.e., those aligned with each bar. These will mostly be parented by generalizations of the so-called closed $x_1$ orbits of single bar systems. As such, their symmetry requires that they, at some stage, cross the major axis of the bar with normal velocities. Starting from these initial values, we advance the trajectories for 50,000 Myr. The rationalization for this particular value was discussed in detail in El-Zant & Shlosman.

3. Results

The rationale behind the present work is straightforward: given a primary bar which exists as a long-lived configuration, we are interested in investigating the range of parameters (if any) for which secondary bars are sustainable, yet do not sufficiently interfere with the primary bar dynamics, so as to destroy it.

Our findings are summarized in Fig. 1, where we display the five models with increasing secondary bar mass (by a factor of 2 each from top to bottom). The middle panels (third from the top) are those of the generic model. Specifically, (1) grayscales in the left column show values of the Liapunov exponents, our ‘measure of chaos;” (2) the middle column marks trajectories whose maximal extension along a bar is twice or more their extension normal to a bar, for both the outer (plus signs) and inner (dots) bars; and (3) the right column exhibits grayscales of axial ratios of orbits. Liapunov timescales are varied from $10^4$ Myr (white shades), corresponding to regular orbits, to $10^3$ Myr (black shades) corresponding to highly chaotic orbits. The axial ratio, $p \equiv a/b$, has values greater than unity, for all bar-supporting orbits, in the appropriate bar frame. However, it will be very close to unity in a given bar frame, if the trajectory is not trapped by that bar. It is appropriate, therefore, to take the maximal value of $p$ in each of the frames. In scaling the greyshades the following limits are employed: white corresponds to $p \geq 3$ and black to $p \leq 1$.

The generic model is characterized with wide regions of trajectories supporting each bar. In particular, almost all orbits corresponding to the secondary bar are aligned with its major axis (middle column) and appear regular (left), most having axial ratios $p \geq 3$ (right). On the other hand, the top panel has 4 times less massive secondary bar, and the bar surface density ratio of only 2, while most of the parameter space exhibits regular and trapped trajectories within the primary bar. In fact no orbits aligned with the secondary bar have been found for this model, meaning that for this value of the spatial and mass scales, the secondary bar is not dense enough to produce the orbits required to sustain it. As this bar increases its mass (2nd panel), these orbits materialize. Simultaneously, the chaotic region just outside the inner bar (bar-bar interface) expands decreasing orbital support for the primary bar by no longer displaying significant alignment with it, towards the lower panels. This dual behavior defines the critical mass fraction of the secondary bar, about $2\% - 5\%$ of the primary bar. Corresponding surface density ratio is $\sim 4 - 8$. Below this mass (and surface density) the inner bar is not sustainable, and above — the outer bar support is dramatically weakened. On the basis of the left and middle panels of Figure 1, we can safely rule out the top, the fourth and fifth panels. This leaves the second and the third panels — the latter corresponding to the generic model.

Majority of the trajectories aligned with the primary bar and virtually all those aligned with the secondary bar are parented by families analogous to the single-periodic $x_1$ family in time-independent single-barred systems, some by higher order families. The symmetry of these orbits requires that one of the coordinates is maximal when the other is null (a variation of 10% on the exact values was introduced to allow for the effect of the perturbing bar). We have verified this by recording the $y$-coordinate of maximal $z$-excursion and vice versa. Between 1/3 to 2/3 orbits (from the top to bottom panels) of trajectories have been found to be quasi-periodic regular ones, with exponential timescales of the order of a Hubble time or larger. They represent about 50% of trajectories in the generic model. Most (50%-70%) of the regular trajectories are elongated in the direction of either bars, when viewed in the relevant frame. However, a significant fraction of the trajectories supporting the bars includes trapped chaotic orbits. This is especially true in the case of the primary bar, where such trapped orbits constitute
Fig. 1.— Double-bar galaxy models with secondary bar masses increasing from top to bottom by a factor of 2 each. The middle panels (third from the top) represent the generic model. The abscissa refers to radii (in kpc) and the ordinate to fractions of the local rotation velocity (not including the bar masses). *Left column:* Grayshades show the logarithms of Liapunov exponents (black-to-white corresponds to increased stability). *Middle column:* Exhibits orbits supporting primary (crosses) and secondary (dots) bars, respectively. *Right column:* Maps the axial ratios of orbits (black-to-white grayshades correspond to increase from $p \leq 1$ to $p \geq 3$). See text for more details.
about 16% of the supporting trajectories in the generic model. Their fraction peaks at 20% when the secondary bar mass is halved, while they are quickly replaced by “strongly chaotic” orbits when the mass is doubled. Although the trapped trajectories have a non-zero Liapunov exponent, many of them mimic bar-supporting orbits for a Hubble time or so. In general, trapped chaotic trajectories may wander intermittently between regular and chaotic phases with a distribution described by non-standard statistics (Zaslavsky 2002). If the initial conditions are such that a significant number of these trajectories are in a trapped phase, they may be of crucial importance to building such systems as double-barred galaxies. This issue will be elaborated elsewhere.

4. Discussion and Conclusions

Observations suggest that galaxies with nested bars are not an exceptional but rather a common phenomenon. However, dynamical and evolutionary consequences of such systems are still unclear in comparison e.g., with important effects of the large-scale stellar bars on galaxy evolution. The steadily increasing spatial resolution of multiwavelength observations will shortly provide the necessary details of stellar and gas kinematics and distribution of star formation in secondary bars confined to the central kpc. Here we outline the parameter space restricted to such long-lived systems. Based on the well-studied, simple but generic example of a double-well oscillator, we suggest that unless one of the bars dominates the gravitational potential, the trajectories should shift erratically between the potential wells formed by the bars, with one or both structures dissolving, thus contradicting the high frequency of double-barred galaxies observed in the local universe.

Indeed, in a somewhat analogous situation of a bar embedded in a non-rotating triaxial halo, we have shown that the bar is unsustainable unless its contribution significantly exceeds that of the halo, in which case it is able to trap its supporting orbits and stabilize (El-Zant & Shlosman 2002). In nested bar systems a similar situation can be constructed by invoking separation of mass and length scales, with each bar dominating its own domain. In the case of spatial scales there seems, in fact, to be observational support for this thesis.

Laine et al. (2002) find a bimodal length distribution of bars in double-barred galaxies, with secondary bars confined to within 12% of galactic radius (given by $D_{25}/2$). More precisely, while large stellar bars are found to correlate linearly with the disk size, secondary bars do not exhibit this property. A straightforward explanation of this phenomenon is that secondary bars are confined to within the ILR of the primary bar. The ILR is expected to be located where the 3-dimensional nature of the disk cannot be ignored, i.e., at about the bulge radius for early-type (S0-Sb) disks and at about 1 kpc for the late types, where the disk thickness becomes comparable to its radius. This confirms theoretical expectations that ILR serves as a dynamical separator between the bars.

Concurrently, a critical mass necessarily exists for the secondary bar. Below its value, this bar does not generate supporting orbital families, as it is not dense enough to be self-gravitating and to maintain a self-consistent orbital structure. Moreover, above this critical mass, as we have shown here, orbits of the primary bar become substantially affected and destabilized. Consequently, too massive a secondary bar, with major axis extending to near ILR of the primary, will tend to weaken and ultimately dissolve the primary one. The implication being that it is only within a limited range of masses and linear sizes that double-barred systems can develop into long-lived configurations. This raises the following interesting question: how, in practice, is a physical system guided into the limited range of parameter space where it possesses the right phase space structure necessary for its survival?

Although our conclusions are based on purely dynamical considerations, they nevertheless naturally fit the context of a broader physical picture. How can the separation of scales be achieved in a physical configuration? In a pure stellar system, this probably can be obtained through a restrictive set of initial conditions only, i.e., the system is preset to develop double bars (e.g., Friedli & Martinet 1993). A more general way is to invoke the gas redistribution in the galaxy and its accumulation within the central few hundred pc as a precondition to formation and dynamical decoupling of the secondary bars (e.g., Shlosman et al. 1989; Friedli & Martinet 1993; Knappen et al. 1995; Heller, Shlosman & Englmaier 2001). The sec-
ondary bars in this latter scenario require a gravitational runaway in the gas to initiate the decoupling. When the gas gravity triggers the cascade of smaller bars, the phenomena is expected to be transient due to the finite gas supply and dissipation present. Stellar secondary bars in this picture are by-products of the runaway, through stellar capture and induced star formation in the gas. These two processes can regulate the parameters of the inner bar so as to be in a critical state of “marginal” self-gravity which also happens to be dynamically long-lived. This can explain the remarkable frequency of double-barred systems, despite their \textit{a priori} improbability.

In this Letter we have avoided varying the pattern speeds of the bars, assuming that the primary bar extends to near corotation and the secondary comprises about 2/3 of its corotation, as argued in Shlosman & Heller (2002). We also expect that slower rotating secondary bars, with pattern speeds closer to those of the primary ones, will generate more orbital instability, and will, therefore, tend to destroy more of the regular trajectories. This effect is similar to the well known adiabatic invariance (e.g., Arnold 1989), namely, when frequencies differ substantially, averaging over a fast frequency allows one to neglect, to first order, the imposed perturbation.

Finally, it is important to emphasize that the schematic picture whereas bar-supporting orbits are completely regular is an idealization. As we find, in reality many “trapped” chaotic trajectories, constrained in shape to fit the bar pattern for many rotations, also contribute. Enigmatic issues related to their preponderance and importance have been known for decades (e.g., Goodman & Schwarzschild 1981). Yet there is still no systematic manner of characterizing their evolution. We have touched on this role here in a heuristic manner, by employing diagnostics such as orbital axis ratios and maximal extensions within a limited time period. The complex time-dependent structure of these trajectories will be examined elsewhere, by analyzing the time correlations of orbital segments.

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