Fractional Energy Loss and Centrality Scaling

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The phenomenon of centrality scaling in the high-	extit{p}_T spectra of \pi^0 produced in Au-Au collisions at \sqrt{s} = 200 GeV is examined in the framework of relating fractional energy loss to fractional centrality increase. A new scaling behavior is found where the scaling variable is given a power-law dependence on N_{part}. The exponent \gamma specifies the fractional proportionality relationship between energy loss and centrality, and is a phenomenologically determined number that characterizes the nuclear suppression effect. The implication on the parton energy loss in the context of recombination is discussed.

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The production of hadrons in Au-Au collisions at the Relativistic Heavy-Ion Collider is found at high \textit{p}_T to depend sensitively on centrality \cite{1}. At \sqrt{s} = 200 GeV the scaled inclusive cross section of pions at \textit{p}_T \approx 3-4 GeV/c decreases by a factor of 4-5 when the centrality is varied from the most peripheral to the most central \cite{2,3}. In a previous paper \cite{4} we reported the finding of a universal function \Phi(z)\textsuperscript{2} that can describe all the inclusive cross sections at all centralities. Such a scaling behavior is achieved by use of a scaling variable \textit{z} that combines \textit{p}_T with N_{part}, the number of participants. In this paper we investigate the origin of that scaling. In particular, we consider the nature of energy loss that can give rise to such a behavior.

Although the scaling behavior can be extended to include energy dependence also \cite{5,6,7}, we restrict our consideration here to centrality dependence only, and emphasize the phenomenological implication of the data at \sqrt{s} = 200 GeV, measured by PHENIX \cite{8}. We shall not discuss energy loss at the parton level except near the end, since perturbative QCD is not reliable for \textit{p}_T < 6 GeV/c. Indeed, soft partons have been found to be important in the hadronization process through recombination \cite{9,10,11,12}. We shall stay mainly at the hadronic level that is phenomenological and uncontroversial, and consider the nuclear suppression effect on the observed hadrons.

Let us recall the scaling behavior found in \cite{4}. With the definition of the variable

\[ z = \frac{\textit{p}_T}{K(N)} \]  

we find that the function \Phi(z),

\[ \Phi(z) = A(N)K^2(N) \frac{1}{2\pi p_T} \frac{dN_\pi}{d\eta dp_T}, \]  

exhibits scaling behavior. We have used the notation \textit{N} = N_{part}, for brevity, and

\[ K(N) = 1.226 - 6.36 \times 10^{-4} N, \]  
\[ A(N) = 530N_c(N)^{-0.9}, \quad N_c(N) = 0.44N^{1.33}, \]  

where both \textit{K}(\textit{N}) and \textit{A}(\textit{N}) are normalized to 1 at \textit{N} = 350. At all \textit{p}_T and \textit{N}, the data of \textit{dN}_\pi/\textit{dp}_T\textit{d}\eta\textit{dp}_T at midrapidity collapse to one universal curve \Phi(z), which is parametrized in \cite{4} by

\[ \Phi(z) = 1200 (z^2 + 2)^{-4.8} (1 + 25e^{-4.5z}). \]  

It is clear that what gives rise to the scaling behavior must be related to an universal property in the medium effect on the production of pions. Since it is not possible to determine experimentally the degradation of parton momentum as the medium size is increased, and since whether hadronization is by means of fragmentation or recombination is still controversial, we choose to stay at the hadronic level and examine energy loss. Since the produced pions do not themselves traverse the dense medium, energy loss here does not refer to the evolutionary process of a pion, as one can for a parton. Instead, it refers to the shift of the pion distribution, as the medium size quantified by \textit{N} is increased.

Let us now consider the implications of a scaling function \Phi(z). Let \textit{z} be defined as

\[ z = xJ(N), \]  

where \textit{x} is a dimensionless momentum variable identified as \textit{x} = \textit{p}_T/p_0 with \textit{p}_0 chosen at \textit{p}_0 = 1 GeV/c so that \textit{x} is numerically the same as \textit{p}_T. We now examine the consequences of writing \Phi(z) in terms of \textit{x} and \textit{N} explicitly

\[ \Phi(z) = F(x, N). \]  

In pQCD, such as in Ref. \cite{13}, one compares a distribution in medium with one in vacuum in order to emphasize the medium effect. We prefer, however, to stay away from the \textit{pp} collision case, since we want to consider incremental changes of the medium size. From our perspective of dealing only with the observables, it is very natural to ask the \epsilon-\delta type question. That is, given a medium that is not too small, in which pions are produced, if its size is increased by an \epsilon amount, what is the corresponding downward shift \delta in momentum in order to maintain the
same probability of producing the pions? In terms of $x$ and $N$, the proposition can essentially be stated as

$$F(x, N) = F(x - \delta, N + \epsilon) .$$

The $+\epsilon$ and $-\delta$ relationship is a consequence of the suppression effect.

For infinitesimal $\epsilon = \delta N$ and $\delta = \delta x$ we can expand the right-hand side of Eq. (8) and keep only the first order terms, getting

$$\frac{\delta x}{\delta N} = \frac{\partial F/\partial N}{\partial F/\partial x} .$$

If $F(x, N)$ satisfies Eqs. (6) and (7), then we have

$$\frac{\delta x}{\delta N} = \frac{x}{J} \frac{dJ}{dN} .$$

If the fractional energy loss is proportional to the fractional change of centrality, a notion that seems extremely reasonable, i.e.,

$$\frac{\delta x}{x} = \gamma \frac{\delta N}{N} ,$$

where $\gamma$ characterizes the suppression effect, then it is necessary that

$$J(N) = \left( \frac{N}{N_0} \right)^\gamma$$

for some normalization $N_0$. Clearly, Eq. (12) does not make sense for $N$ very small, such as $N = 2$, since $\epsilon$ cannot be made infinitesimal compared to $2$.

The power-law behavior in Eq. (13) is a necessary consequence of scaling and fractional proportionality, Eq. (12). Comparing $J(N)$ with $1/K(N)$ in Eq. (3), one finds that Eq. (13) differs enough from our first scaling parametrization to cast some doubt on whether Eq. (13) is sufficient for scaling. However, the fractional proportionality relationship is so compelling that we have been motivated to reexamine the data, especially since the original preliminary data have by now been finalized. It should be noted that Eq. (13) is obtained without relying on any specific form for $\Phi(x)$; it depends only on the structure of the scaling variable expressed in Eq. (3).

Thus one expects Eqs. (12) and (13) to be very general properties of centrality scaling.

Using the data on $\pi^0$ from PHENIX tabulated on the web [11] we have assembled the spectra for all measured centralities, and made appropriate horizontal and vertical shifts in the log-log plot to obtain a universal behavior. The result is shown in Fig. 1. The $\pi^+$ data for the most central collisions are used to supplement $\pi^0$ in the low-$p_T$ region [12]. The horizontal shift determines the scaling factor $J(N)$ shown in Fig. 2(a). The vertical shift determines the normalization factor $B(N)$ shown in Fig. 2(b). The resultant scaling distribution $\Phi(z)$ is now related to the measured inclusive distribution by

$$\Phi(z) = \frac{B(N)}{J^2(N)} \frac{p_0^2}{2\pi p_T} \frac{dN_{\pi^0}}{dp_T} .$$

Evidently, the scaling behavior exhibited in Fig. 1 is very good. The solid line is a fit using the formula

$$\Phi(z) = 150 \left( z^2 + 1.05 \right)^{-4.18} \left( 1 + 7e^{-4z} \right) .$$

There are many more points included in Fig. 1 than those of the preliminary data used in Ref. [4].

The behavior of $J(N)$ in the log-log plot in Fig. 2 can be fitted by a straight line according to Eq. (13) with

$$\gamma = 0.077 .$$

The region $N < 10$ is not considered. The normalization point is chosen to be $N_0 = 325$, which corresponds to the most central 0-10% collisions [2]. The normalization factor $B(N)$ can be fitted by two straight lines as shown by the solid lines. The implication of this result will be discussed in the following.

Although Eqs. (13) and (16) do not differ too much from the inverse of $K(N)$ given in Eq. (3), we have investigated the source of the difference. Our conclusion is that in [4] we read the preliminary data from a figure given in a conference talk [13] and estimated the central points, whereas here we use the finalized data in tabulated form, which differ slightly from the original. Thus our present result is more reliable. Moreover, since in our new description the medium suppression effect is characterized by one and only one parameter $\gamma$, which plays
the crucial role of specifying the fractional proportionality relation in Eq. (12), the resultant scaling behavior has the distinction of being physically motivated. As we have shown, it is not so much what the scaling function $\Phi(z)$ is as what the scaling variable $z$ is. Since $z$ quantifies the difficulty of producing transverse motion, it can be termed transversality that gives a universal description of that difficulty at any centrality. The $z$ dependence of $\Phi(z)$ in Eq. (13) differs little from that of Eq. (5), except at very high $z$ where the new data extend beyond that of the preliminary result.

With the value of $\gamma$ in Eq. (10) now determined phenomenologically, we can return to Eq. (12) and claim that the notion of fractional proportionality has direct support by the data. Note that the independence of the fractional energy loss $\delta x/x$ on $x$ is a result that differs from that of pQCD on the momentum shift of hard partons where $\Delta p_T$ is proportional to $p_T^{1/2}$, and consistent with the independence of $R_{AA}$ on $p_T$. Equation (12) also implies that $\delta x/\delta N$ is proportional to $x/N$ which is very different from the assumption that the energy loss per unit length traversed by a parton, $dE/dL$, is a constant. Although our phenomenological result on the produced pions has no direct implication on the evolutionary properties of the partons propagating through a medium, one should keep these differences in mind, when the measurable consequences of what can be calculated in pQCD are inferred.

Since the method by which we obtained $J(N)$ is by means of data fitting, it is desirable to find an alternative method that is more direct. Given $\Phi(z)$, one can determine the average

$$\langle z \rangle = \int dz \ z^2 \Phi(z) / \int dz \ z \Phi(z) = 0.42 . \quad (17)$$

Then from Eq. (1) we have $J(N) = 0.42 / \langle x \rangle (N)$, where $\langle x \rangle$ is related to the inclusive cross section as

$$\langle x \rangle (N) = \int dp_T \ p_T \ \frac{dN}{dy dp_T} \int dp_T \ dN \ . \quad (18)$$

Since $J(N)$ is normalized to 1 at $N = N_0$, we now have

$$J(N) = \langle x \rangle (N_0) \langle x \rangle (N) , \quad (19)$$

thus eliminating the reference to the scaling variable $z$. The $N$ dependence of $\langle x \rangle (N)$ can be determined directly from the data when the average is calculated at various centralities. The large $p_T$ part of the integration is important, for it is that part of the $dN/dy dp_T$ that led us to the scaling factor $J(N)$ in the first place. The low-$p_T$ part of the spectra should be accurately parametrized. Now, it is a matter of evaluating Eq. (18) instead of shifting and rescaling in Fig. 1. We recommend that experimental groups that have the data, not only of $\pi^0$, but also of charged hadrons, can use this method to check whether $J(N)$ indeed has the form given in Eq. (19).

The scaling behavior that we have obtained is for the produced $\pi^0$. We have found in $\pi^0$ that the anomalously high $p/\pi$ ratio at $p_T \sim 3-4$ GeV/$c$ can be understood in terms of recombination without fragmentation, when hard partons are allowed to recombine with the soft ones. That is possible if the recombination function for the pion does not restrict the recombining $q$ and $\bar{q}$ to have roughly the same momentum. Indeed, in our view jet fragmentation is included in recombination because a large-$p_T$ hard parton initiates a parton shower that hadronizes by recombination. The use of fragmentation function is only a phenomenological way of parametrizing that process, and does not stand for an independent hadronization mechanism. In heavy-ion collisions the parton shower on the surface has low-$p_T$ components that mingle with the soft partons with hydrodynamical origin. Since separating them would be artificial, we treat them on equal footing in the recombination process that can involve hard partons as well. The purpose of this discussion is to prepare our way to descend to the parton level without fragmentation. The scaling behavior that we have found supports this view, since our rescaling procedure is universal and does not separate the high-$p_T$ from the low-$p_T$ regions using different transversity variables. We note that this view differs from that taken in $[8, 11]$, in which recombination and fragmentation are important in different regions. We also note that our application of the recombination mechanism to all partons encounters no inconsistency with the two experimental facts: (a) $p/\pi$ ratio is roughly 1 at $p_T \sim 3-4$ GeV/$c$, and (b) the jet structure in Au-Au collisions is similar to that in $pp$ collisions in the same $p_T$ region $[10]$. With the above discussion we have laid the basis for the expectation that the quark distribution $F_q$ for all $p_T$ contributes to the determination of the pion distribution at all $p_T$. The recombination equation derived in Refs. $[8, 11]$ has the form

$$\Phi(z) = \int dz_1 dz_2 \ \frac{z_1 z_2}{z} F_q(z_1) F_q(z_2) \delta(z_1 + z_2 - z) , \quad (20)$$
in which the $p_T$ variables have all been transformed to the scaling variables with

$$z_i = \frac{p_{T,i}}{p_0} J(N), \quad i = 1, 2. \quad (21)$$

The $\delta$ function comes from the recombination function and guarantees the conservation of momentum. Note that $z_1$ and $z_2$ are integrated over all values; in particular, they are not restricted to the region $z/2$. Indeed, since the quark distributions fall rapidly with $z$ it is necessary for one $z_i$ to be large and the other $z_i$ to be small in order to give the highest contribution to $\Phi(z)$ at large $z$. The essential remark we want to make now is that in our formalism for hadronization the $q$ and $\bar{q}$ distributions must have their $p_T$ scaled by the same factor as shown in Eq. (21). Since the $J(N)$ in Eq. (21) is the same as that for the pion, we conclude that the fractional energy loss for the quarks (and antiquarks) satisfies the same proportionality relationship as in Eq. (13). It should immediately be emphasized that these partons are at the end of their evolution (hydrodynamical and/or branching in showers) just before recombination. As mentioned earlier the energy loss discussed here does not refer to the radiative energy loss of a parton traversing a medium [17]. Our point is that the fractional energy loss of the partons at hadronization satisfies the same property as for the pions. Since in this formalism of hadronization we have been able to obtain the correct $p/\pi$ ratio [4], it follows that the produced protons should have the same property in fractional energy loss also. This prediction should be checked experimentally by studying the proton spectra and seeing whether centrality scaling can be achieved with the same $J(N)$. We expect, however, the proton mass effect to break the scaling at low $z$.

Our final remark is a speculative one. The universality of the single exponent $\gamma$ for all centralities (except the very peripheral collisions) raises the question whether a hot and dense medium is any different from a less dense medium in its effect on pion production at high $p_T$. If not, the high $p_T$ spectra would not be a fruitful place to find the signature of plasma formation. A possible escape from that conclusion is the observation that the result on $B(N)$ in Fig. 2(b) can be fitted by two straight lines, which can be parametrized by

$$B(N) = \begin{cases} (N/N_1)^{-\beta_1}, & N < 38, \\ (N/N_2)^{-\beta_2}, & N > 38, \end{cases} \quad (22)$$

where $\beta_1 = 0.744$ and $\beta_2 = 1.292$ with $N_1 = 1610$ and $N_2 = 325$. It suggests that there are two regimes of $N$, requiring different exponents $\beta_1$ and $\beta_2$ to achieve scaling. If this break can be ascertained by more detailed analysis at different energies, then it offers a way out of the strict universality in which there is no hint of any essential difference between hot and cold media, leaving no room for any signal for the formation of quark-gluon plasma.

To summarize, we have found a relationship between the centrality scaling behavior of the observed pion spectra and the fractional energy loss of the pions that is proportional to the fractional centrality change. That proportionality is specified by an exponent $\gamma$, which characterizes the medium suppression effect. In the recombination model the same value of $\gamma$ is valid for the fractional energy loss of the light quarks just before hadronization. The existence of the universal scaling function of transversality suggests that there is no essential difference in how the low- and high-$p_T$ hadronization processes should be treated. The possibility of a single exponent $\gamma$ that can be directly extracted from the data to summarize the nuclear suppression effect offers a very succinct description of a complicated dynamical process. Universal behavior in centrality may be broken by the existence of two scaling regions in the normalization factor, thus providing the possibility of a threshold for a new distinctive regime.

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