Variations in the Abundance Pattern of Extremely Metal-poor Stars and Nucleosynthesis in Population III Supernovae

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ABSTRACT

We calculate nucleosynthesis in Population (Pop) III supernovae (SNe) and compare the yields with various abundance pattern of extremely metal-poor (EMP) stars. We assume that the observed EMP stars are the second generation stars, which have the metal-abundance pattern of the Pop III SNe. Previous theoretical yields and observations do not match well. In this paper we consider high energy explosions and mixing-fallback models. We show that the abundance patterns of both typical and C-rich EMP stars with \([\text{Fe/H}] < -2.5\) can be well reproduced with the yield of Pop III core-collapse SNe of \(M \sim 20 - 130 M_\odot\). The abundance patterns of the \([\text{Fe/H}] < -2.5\) stars correspond to supernova yields with normal explosion energies, while those of the carbon unenhanced stars with \([\text{Fe/H}] \approx -5 \sim -3.5\) correspond to high-energy supernova yields. The abundance pattern of C-rich \([\text{C/Fe}] > 2\) low \([\text{Fe/H}] \approx -5 \sim -3.5\) stars can be explained with the faint SN yield with little \(^{56}\text{Ni}\) ejection, which is similar to SN1997D. In the supernova-induced star formation model, we can qualitatively explain why the EMP stars formed by the faint or energetic supernovae have lower \([\text{Fe/H}]\) than the EMP stars formed by normal supernovae. We also show that the absolute abundance of Fe-peak elements (Mn, Co, Ni, Zn) are sensitive to the electron mole fraction \(Y_e\) during explosive nucleosynthesis. Even we vary \(Y_e\) as a parameter, we still need a large explosion energy to obtain the large Co/Fe and Zn/Fe ratios observed in typical EMP stars with \([\text{Fe/H}] < -3.5\).

1. Introduction

In the early universe, where the metal content of gas is very low, the enrichment by a single supernova can dominate the pre-existing metal contents (e.g., Audouze & Silk 1995). Low mass stars formed in the gas survive until today, and observed as extremely metal-poor (EMP) stars. Since EMP stars may preserve abundance patterns synthesized by a single or few supernovae (SNe), the abundance patterns of those stars may be used to test supernova explosion and nucleosynthesis theories, and to infer the nature of the first generation stars and supernovae.

The abundance patterns of EMP stars show interesting trends below $[\text{Fe/H}] \sim -2.5$; with increasing $[\text{Fe/H}]$, $[\text{Mn/Fe}]$ and $[\text{Cr/Fe}]$ increase while $[\text{Co/Fe}]$ and $[\text{Zn/Fe}]$ decrease, where $[X/Y] \equiv \log(X/Y) - \log(X/Y)_\odot$ (McWilliam et al. 1995; Primas et al. 2000; Spite et al. 2003). These trends can be explained if the EMP stars with lower $[\text{Fe/H}]$ were enriched by a supernova ejecting relatively more complete Si-burning matter (e.g., Co, Zn) than incomplete Si-burning matter (e.g., Mn). This is realized if the “mass-cut”, that divides the supernova ejecta and the central remnant, is relatively deeper (Nakamura et al. 1999). A question is why lower metallicity EMP stars are enriched by SNe with relatively deeper mass-cuts. In this paper (Section 2), we show that the variation of the explosion energy can nicely explain this relation (see also Umeda & Nomoto 2002a,b, 2003 and Nomoto et al. 2003, for brief explanations).

In Umeda & Nomoto (2002a; UN02 hereafter) we showed that the large Zn/Fe ratio typically observed in EMP stars (Primas et al. 2000, Ryan 2002) requires large explosion energies to be reproduced, i.e., the typical EMP star formation is likely to be triggered by hypernovae. One important implication of the hypernova model for the abundances of EMP stars is that mixing and fall-back are required to take place in the inner part of the ejecta. The large Zn/Fe is realized only if the mass-cut is sufficiently deep. If the mass-cut is deep enough to eject Zn, however, too much Fe is ejected and the ratios between lighter elements and Fe, such as Mg/Fe, become too small. To solve this problem, UN02 proposed that mixing out of Zn and the subsequent fall-back of the sufficient amount of the mixed material take place in the ejecta. Mixing by Rayley-Taylor instabilities has been found to occur in the 2D simulations (e.g. Hachisu et al. 1990; Kifonidis et al. 2000). The occurrence of fall-back is briefly explained in (Woosley & Weaver 1995, WW95 hereafter). We note that a similar effect to mixing-fallback also occurs in the jet-like explosion. In such a model, energetic explosion occurs only along the jet directions and thus the total Fe mass can be smaller with enhancement of the complete Si-burning products (e.g. Maeda & Nomoto 2003a,b).

Although we have successfully explained the trend in Fe-peak elements, one may wonder whether the absolute values of the abundances fit to the observations as well as elements other...
than Fe-peak. Chieffi & Limongi (2002, CL02 hereafter) compares UN02 and WW95 with the typical EMP abundances and concluded that none including theirs fit to the observations well. They proposed as a possible solution that a progenitor model with a large C/O ratio may solve the discrepancy. In this paper, instead, we show that the absolute abundance of Co and Mn are quite sensitive to \( Y_\epsilon \), and the fit to the observations becomes significantly improved with a certain choice of reasonable value of \( Y_\epsilon \).

Are the abundances of all EMP stars consistent with hypernova nucleosynthesis? There is a sub-class of EMP stars, C-rich EMP stars, including the most Fe deficient star HE0107-5240 with \([\text{Fe/H}] \sim -5.3\) (Christlieb et al. 2002). These stars are quite rich in C and N, typically \([\text{C/Fe}] \gtrsim 2\), and in some stars Mg as well. Recently we have shown that the abundances of these stars are well reproduced with the yields of core-collapse SNe which undergo small Fe ejection and mixing-fallback (Umeda & Nomoto 2003, UN03 hereafter). We showed that not all C-rich EMP stars favors high-energy models. In fact the abundance pattern of HE0107-5240 is well reproduced by a low-energy \((E_{51} \equiv E/10^{51}\text{ ergs} = 0.3\), where \( E \) is the explosion of the SN) model. In this paper we compare our models with other C-rich EMP star abundances and constrains the models from fitting to the data. We demonstrate that the difference in the degree of the mixing, fallback and explosion energies may explain both the C-rich and usual EMP stars.

2. EMP Stars with Typical Abundance Pattern

2.1. Trends in the iron peak elements and hypernovae

We have shown in previous papers (UN02, Umeda & Nomoto 2002b) that the trends in the abundance ratios of Fe-peak elements, \([\text{Zn}, \text{Co}, \text{Mn}, \text{Cr}]/\text{Fe}\) vs \([\text{Fe/H}]\), can be understood by the variations of deepness of mass-cut in the explosive nucleosynthesis of SNe II. We also have suggested that the large Zn/Fe and Co/Fe ratios in typical EMP stars are well-reproduced by hypernova nucleosynthesis. In this section, we describe how closely these facts are related and how observed trends, and not just variations, can be explained with the energy of supernovae.

For a larger explosion energy, the supernova shock is stronger and the temperature after the shock passage is higher. The post-shock region is radiation dominant, so that the peak temperature is approximately related to the stellar radius \( r \) and the deposited energy \( E^* \) as

\[
T_9 = \left( \frac{E_{51}^*}{r/3.16 \times 10^4 \text{km}} \right)^{3/4},
\]

where \( T_9 \) is the peak temperature in \( 10^9 \text{ K} \) and \( E_{51}^* \) is the deposited energy. Complete
Si-burning, which burns Si completely, occurs for $T_9 > 5$. In this region, elements such as $^{56}$Ni, $^{64}$Ge (decaying into $^{64}$Zn) and $^{59}$Cu (decaying into $^{59}$Co) are produced. Incomplete Si-burning occurs for $4 < T_9 < 5$. In this region elements such as $^{56}$Ni, $^{52}$Fe (decaying into $^{52}$Cr) and $^{55}$Co (decaying into $^{55}$Mn) are produced. For a larger explosion energy, the complete Si-burning region is enlarged in mass more than the incomplete Si-burning region (see Figure 1 and its caption). As a result, the mass ratio between the complete and incomplete Si-burning regions is larger in a more energetic explosion if the mass-coordinate of the mass-cut does not change significantly. Therefore, increasing the energy causes similar effect as making the mass-cut deeper without actually changing the mass coordinate of mass-cut.

The large energy is also consistent with the observation of $[\text{Zn/Fe}]$ in EMP stars. For a typical EMP star, Zn is quite abundant, i.e., $[\text{Zn/Fe}] \sim 0.3 - 0.6$ (Primas et al. 2000; Spite et al. 2003). We have shown that such large Zn/Fe ratio is difficult to produce by SNe with normal explosion energy ($\sim 10^{51}$ erg), but possible by energetic core collapse SNe with $10^{52}$ erg or more (UN02).

We assume that EMP stars are formed in the supernova ejecta mixed with interstellar matter. In this “supernova-induced star formation model”, the [Fe/H] (or [Mg/H]) of an EMP star is determined by the Fe (or Mg) mass ejected from a SN, divided by the hydrogen mass in circumstellar matter swept by the SN shock. It is estimated that the swept hydrogen mass is roughly proportional to the explosion energy (Ryan, Norris, & Beers 1996; Shigeyama & Tsujimoto 1998). Thus we may write that

$$[\text{Fe/H}] \simeq \log_{10}(\text{Fe}/E_{51}) + C; \quad [\text{Mg/H}] \simeq \log_{10}(\text{Mg}/E_{51}) + C',$$

where Fe, Mg and H represent mass fraction of Fe, Mg and H, respectively; $E_{51}$ is the explosion energy in $10^{51}$ erg, and $C(\prime)$ are constants. Therefore, it is likely that the EMP stars produced by more energetic SNe would have lower [Fe/H] and [Mg/H].

In this explanation, [Fe/H] of EMP stars is almost independent of the initial metallicity and the age of the SNe progenitor. Using the same procedures described in Umeda et al. (2000), UN02 and UN03, we calculate nucleosynthesis of several models with different masses and energies as shown in Table 1, and plot the yield ratios [(Zn, Co, Cr, Mn)/Fe] vs $\log_{10}(\text{Mg}/E_{51})$ in Figure 2. The mass-cut is chosen to maximize the Zn/Fe ratio. Here, Mg is adopted for abscissas because the ejected mass of Mg is less sensitive to the mass-cut than that of Fe. Figure 2 exhibits that the high-energy models tend to be located at lower [Mg/H], and thus can explain the observed trend. Note that [Mg/Fe] $\sim 0.3 - 0.5$ for typical EMP stars and thus the observed general trends do not change by changing abscissa from [Fe/H] to [Mg/H] as shown in Figure 3. The observed data vs [Fe/H] are shown later.

Although we could explain the trends, our Co and Mn yields in many cases are too small
to be consistent with observations in the absolute values. Here we note that the absolute abundances of Co and Mn are sensitive to the detail of the explosion, nuclear reaction rates and $Y_e$. Among them, the effect of $Y_e$ change due to the neutrino process during explosion may be important. In our previous works, we have assumed that the pre-supernova value of $Y_e$ is preserved during the explosive burning. In the Z=0 models, $Y_e \approx 0.5000$ above the pre-supernova oxygen layer and decreases gradually toward the Fe core (UN02). However, recent detailed simulations of neutrino transport in core-collapse SNe show that $Y_e$ may be significantly affected by the neutrino process during explosion (Liebendorfer et al. 2002; Janka, Buras, & Rampp 2003). It is interesting that in the deep core $Y_e > 0.5$ may be realized, for which the nucleosynthesis has not been systematically studied before. We show in Figure 4 how the abundances of Fe-peak elements depend on the value of $Y_e$. Here, we change $Y_e$ inside the incomplete Si-burning region, and the mass-cut is chosen to maximize the Zn/Fe ratio. The adopted supernova model is a 25 $M_\odot$ model with the explosion energy $E_{51} = 20$. Production of Mn is larger for $Y_e < 0.5$ and Co production is significantly enhanced for $Y_e > 0.5$. More detailed discussion on the nucleosynthesis for the $Y_e > 0.5$ matter and its implications will be discussed elsewhere, but here we suggest that the effect of $Y_e > 0.5$ may be very important for explaining large Co/Fe ratios observed in typical EMP stars.

As an example, we show in Figure 5 the yields for the $(15 M_\odot, E_{51} = 1)$ and $(25 M_\odot, E_{51} = 30)$ models compared with the observations. We determine [Fe/H] of these models by assuming $[\text{Fe/H}] = \log_{10}(\text{Mg}/E_{51}) + C$ with arbitrary but the same constant $C$. In these models, we assume that $Y_e = 0.5001$ in the complete Si-burning region and $Y_e = 0.4996$ in the incomplete Si-burning. For such a $Y_e$ distribution, both the Co and Mn abundances are enhanced and fits better to the observed values. This “inversion” of $Y_e$ may be possible according to the most recent explosion calculations.

### 2.2. Comparison with individual star

In this subsection, we compare the typical abundance pattern of Aaron stars with core-collapse SNe yields. This was, for example, recently done in CL02. They compared their and other groups’ (WW95 and UN02) SN yields with the observational points given by (Norris, Ryan, & Beers 2001, NRB01 hereafter). They first claimed that all the models including theirs using the “High” $^{12}$C($\alpha, \gamma$)$^{16}$O rate do not fit well with observations. Here the “High” $^{12}$C($\alpha, \gamma$)$^{16}$O rate is the value given in Caughlan et al. (1985) which was adopted to explain the solar abundance ratios by SNe II (e.g., Thielemann, Nomoto, & Hashimoto 1996; WW95). They claimed that their low $^{12}$C($\alpha, \gamma$)$^{16}$O rate model (L model) fits better to the observations, though some elemental ratios cannot be reproduced. Here the “Low” $^{12}$C($\alpha, \gamma$)$^{16}$O rate given
in Caughlan & Fowler (1988, CF88 hereafter) is used. They claim that in their L model, significant underabundance of the Co/Fe ratio in other models are much improved. This is because for the low $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate, the C/O ratio after He burning is larger, which makes the shell C-burning stronger, and the Fe core becomes smaller. If the explosion energy is fixed, less massive stars eject more complete Si-burning material, including Co, thus yielding a larger Co/Fe ratio. However, yields of solar metallicity SNe if adopting the low $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate may not be consistent with the solar abundance pattern. Also we have not successfully reproduced such large enhancement of Co/Fe for the larger C/O ratio and less massive core models. This discrepancy may stem from the differences in the explosion models, because the abundance of the complete Si-burning products is sensitive to the density during the explosion. CL02 used a simple analytic model for the density-temperature evolution during explosion, while other groups simulated explosion with numerical hydrodynamics codes.

As shown in the last subsection, the abundance of Co may be significantly enhanced for $Y_e > 0.5$. Therefore, here we modify the $Y_e$ distribution instead of considering different progenitor models with a larger C/O ratio.

In Table 2, we show some properties of our pre-supernova progenitor models used in the rest of this paper. This table shows the initial stellar mass, metallicity, the adopted $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate, the central C/O mass fraction ratio just after the helium burning, Fe-core mass (defined by $Y_e < 0.49$) and C-O core mass (defined by $X(\text{He}) < 10^{-3}$).

In Figures 6, we compare our models with typical abundance patterns of EMP stars. The observed points are the “averaged” abundances given in NRB01, which represent typical abundances for the $[\text{Fe}/\text{H}] \sim -3.7$ stars. The original NRB01 abundance does not include the Zn point. However, the over-abundance of Zn is quite common in EMP stars (Primas et al. 2000; Spite et al. 2003) and the point is crucial to estimate the explosion energy, therefore, we add its typical value in the figure. The theoretical yield in Figure 6(a) is obtained for a zero metallicity (Pop III) $25\, M_\odot$ star after explosion with $E_{51}=1$. The calculation method and other assumptions are the same as described in UN02. In this model, the mass-cut is located at mass coordinate $M_r = M_{\text{cut}} = 2.20 \, M_\odot$. This mass-cut is chosen to maximize the Zn/Fe ratio, that is one of the most important keys to judge the goodness of the model (UN02).

The fit in Figure 6(a) is not very good because of the underabundances of C, Mg, Al, Sc, Ti, Co, Ni, Zn and overabundance of Cr. The underabundances of C, Mg, and Al will be improved if larger $M_{\text{cut}}$ is chosen, for which Fe (decay products of $^{56}\text{Ni}$) ejection is smaller. The overabundance of Cr and underabundances of Co, Ni are, on the other hand, improved for smaller $M_{\text{cut}}$ (e.g., Nakamura et al. 1999). Therefore, these problems cannot be solved just by changing the mass-cut of SNe. In UN02, we proposed one solution for this problem,
which is the mixing-fall back mechanism. If the inner part of the ejecta is mixed with the outer materials, and later some of the mixed matter is fallen back to the central remnant, the ratio of the lighter elements (C, Mg, Al) to Fe increases without changing the abundance ratios in the Fe-peak elements. Carbon may be still underabundant. But the C abundance is sensitive to $^{12}{\text{C}}(\alpha, \gamma)^{16}{\text{O}}$ rate and convective overshooting. Also carbon may be enhanced during the evolution of EMP stars.

As shown in UN02, the production of Zn and Co is significantly enhanced if the explosion energy is larger. In Figure 6(b), we show such a high-energy model with $E_{51}=20$. Here the mixing-fallback mechanism is more important than the low energy models, because the synthesis of Fe (i.e., $^{56}\text{Ni}$) is larger so that larger fall-back is necessary for more energetic explosions. In this model, the inner most matter is mixed between ”initial” and ”final” mass-cut, $M_{\text{cut}}^\text{ini} = 2.35 M_\odot$ and $M_{\text{cut}}^\text{fin} = 4.29 M_\odot$. $M_{\text{cut}}^\text{ini}$ is chosen to maximize the Zn/Fe ratio and $M_{\text{cut}}^\text{fin}$ is assumed to be near the top of the incomplete-Si burning as in UN02. The ejection factor is $f = 0.1$, that means 10% of the mixed matter is ejected and 90% is fallen-back. This model fits much better than the low energy models.

The remaining problems are the underabundance of Sc, Ti, Ni and slight overabundance of Cr. Among them Ni is not a serious problem, because with reducing $M_{\text{cut}}$, the Ni abundance increases rapidly. It is known that the Ti abundance is largely enhanced if the explosion is aspherical (Nagataki 2000; Maeda & Nomoto 2003a,b). For an aspherical explosion Ca, Sc and Zn abundances are also enhanced, although explosion energy has to be still larger than that of canonical explosion, $E_{51}=1$ (Maeda & Nomoto 2003a,b). These fairly good fit of other elements make us to believe that the typical abundance pattern of EMP stars can be understood as a result of nucleosynthesis in energetic core-collapse SNe (or hypernovae).

The fit with similar goodness can be obtained by more massive and more energetic SNe. Figure 6(c) shows the 50$M_\odot$ model with $E_{51}=50$. If we look at closely, the abundances of Ti and Co are smaller than the 25M model, but this would be remedied with larger explosion energy and/or asphericity. This means that from the abundances of EMP stars, it is difficult to constrain the mass of the progenitor. We can constrain only the set of mass and explosion energy. At least we can say that from the present SNe observations, the progenitors of energetic core-collapse SNe are more massive than $\sim 20M_\odot$. The upper mass limit is unknown, but should be lower than $\sim 140M_\odot$ because above which the stars would explode as pair-instability SNe and nucleosynthesis patterns are quite different from EMP stars (e.g., UN02; Heger & Woosley 2002). Note that the (newly calculated) progenitor model used in this figure has the metallicity $Z = 10^{-4} = Z_\odot/2000$, but not zero. However, as shown in our previous work (Umeda, Nomoto, & Nakamura 2000), the elemental abundance
pattern from SNe II are not much different for $Z = 0 - 10^{-4}$. Therefore, the $Z=10^{-4}$ models can be used for the present purpose. Detailed metallicity dependent yield will be published elsewhere.

3. Abundance Pattern of C-rich EMP Stars

In this section, we compare our core-collapse SNe yields with abundances of C-rich EMP stars. Among several C-rich EMP stars we pick up 5 representative stars, C1:CS22949-037, C2:CS29498-043, C3:CS22957-027, C4:CS31062-012 and C5:HE0107-5240. C1 is interesting because the important elements O and Zn are observed only for this star. The abundance pattern of C1 is peculiar because O and Mg are rich as well as C and N. The abundances of C2 - C4 are obtained by Aoki et al. (2002a,b). C2 is rich in C, Mg, Al and Si more than C3. C3 is much more C-rich than C1 & C2, but not in Mg. C1 -C3 all show no enhancement of s-process elements, while C4 shows enhancement of $[\text{Ba/Fe}]$. C5 is the most Fe-poor star observed so far. The model for this star has been discussed in UN03 already, but in this paper we modify $Y_e$ in the Si-burning region, and show that the same model still can explain the observation well.

3.1. CS22949-037

CS22949-037 is one of the most metal-poor giants known ([Fe/H] $\approx -4.0$). The detailed abundance pattern of this star was first observed by NRB01, and then by Depagne et al. (2002). Those abundances are shown in Figure 7 by blue circles (NRB01) and by red circles (Depagne et al. 2002). Two results are mostly consistent, but Depagne et al. obtained the abundances of Zn and O, which are very important for constraining the SN models. The Al point in Depagne et al. is larger than NRB01 because we have adopted NLTE corrections ($\Delta[\text{Al/H}] \approx 0.65$) suggested in their paper. The NLTE correction is also added to the Na point ($\Delta[\text{Na/H}] \approx -0.6$). For this star, no enhancement of r- and s-process elements is observed (e.g., $[\text{Ba/Fe}] = -0.6$).

This star is abundant in Co and Zn as typical EMP stars, suggesting the enrichment by a high energy supernova. The theoretical model in Figure 7(a) is the same as in Figure 6 (b) except that the ejection factor is 1/10 ($f=0.01$). With this $f = 0.01$ the ratios of Mg, Al, and Si to Fe are consistent with the observation, but O/Fe is too small. More massive progenitor models would yield more O. Indeed the Z=0, 30$M_\odot$ model shown in Figure 7(b) has a larger O/Fe ratio, thus being closer to the observation. In this model, as well as the
models in the rest of this paper for consistency, \( Y_e \) is modified to be 0.5001 and 0.4996 in the complete and incomplete Si-burning regions, respectively.

Now let us examine the model in Figure 7(b) closely. In this model, \( M_{\text{fin}}^{\text{cut}} \) is chosen to be larger (\( M_{\text{fin}}^{\text{cut}} = 7.57 M_\odot \)) than the model in Figure 7(a) in order to reduce the Mg/Si ratio. If \( M_{\text{fin}}^{\text{cut}} \) is smaller, the Mg/Si ratio is larger than the observation since the Si-rich layer is interior to the Mg-rich layer (see Figure 8 for the abundance distribution of this model). The LTE Al/Fe value is difficult to fit, while with the NLTE correction fitting is possible. As in the usual EMP stars (Section 2), Ca, Sc, Ti, and Zn are still underproduced. These elements can be enhanced with larger explosion energies, but enhancement by aspherical or jet-like explosion might be better in order not to make the Mg/Si ratio small. The overabundance of Cr is also seen in the usual EMP stars. Na is significantly under-produced, but, the Na may be produced in the course of EMP low-mass star evolution (Iwamoto, Umeda & Nomoto 2003).

The remaining problem is N. Usually N production in the metal-poor SN progenitors is difficult to occur. However, if the surface H is mixed into the He layer by convection or rotational mixing, significant amount of N may be produced. Another possibility is discussed for example in Depagne et al. (2002). The observed N might have been produced in the EMP stars through the CN cycle. If all C is converted into N, the final \([\text{N}/\text{Fe}] \simeq \text{initial } [\text{C}/\text{Fe}] +0.4\). In this case, the initial abundance of N could be much smaller than the observed abundance but the initial C abundance should be sufficiently large. From the observation, this requires \([\text{C}/\text{O}] \gtrsim 0\) but the model has \([\text{C}/\text{O}] < 0\).

The C/O ratio of the supernova progenitors depends strongly on the \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) reaction rate and the convective overshooting the end of the central He-burning, which are both quite uncertain. Figure 7(c) shows one example of a model with a large C/O ratio. This model uses \( Z = 10^{-4}, 50 M_\odot \) progenitor model with explosion energy \( E_{51} = 50 \). As shown in Table 2, its progenitor model is more C-rich than the model in Figure 7(b), mainly because of the smaller \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) rate. This model qualitatively can explain the observed points, but Ca, Sc, Ti, Co and Zn are still somewhat underproduced. More complicated \( Y_e \) distribution, 2D configuration or jet like explosion may be necessary to obtain better fits.

### 3.2. CS29498-043

This star is also an extremely metal-poor giant, \([\text{Fe}/\text{H}] = -3.75\), and its abundance pattern is similar to CS22949-037, being very rich in C, N, Mg, Al, and Si (Aoki et al. 2002b). Another similarity with CS22949-037 is that it shows no enhancement of s-process
elements (e.g., [Ba/Fe] = −0.45). However, there are some differences: C, Mg, Al are more abundant and the Mg/Si ratio is larger in CS29498-043. For CS22949-037 we assumed a high energy model because of the large abundance of Zn and Co. Unfortunately, there are no data of these elements for CS29498-043. Thus we first assume a normal energy model. In Figure 9(a) we compare the observed data with a theoretical model with Z=0, \( M = 25M_\odot \), and \( E_{51}=1 \). This model reasonably well reproduces the observed abundance.

In this model, the Al/Fe ratio is below the observation point. However, with the possible NLTE correction (\( \Delta [\text{Al/H}] \approx 0.65 \) shown by the solid square in Figure 9(a) as described in the previous sub-section), the model and observations match very well.

The model in Figure 9(a) assumes smaller \( f \) than the model in Figure 7, because \([C/Fe]\) is larger than in CS22949-037. The underabundance of Sc and Ti may be improved with some asphericity effects as described in Section 2. The discrepancy of the N/Fe ratio can be explained also by the same arguments as in Section 2: either N is produced in the progenitors by some mixing process or in the EMP stars by the CN-cycle.

Since other models with different mass, energy, \( M_{\text{cut}}^{\text{fin}} \) and \( f \) may also give a good fit, it is necessary to constrain these parameters. The relatively large Mg/Si ratio may constrain the upper limit of the explosion energy, because the larger energy explosion produce larger Si/Mg ratio. As mentioned above, the Mg/Si ratio increases with increasing \( M_{\text{cut}}^{\text{fin}} \) since Si is produced in the deeper region than the Mg region. However, the Mg/Si ratio does not increase further if the \( M_{\text{cut}}^{\text{fin}} \) becomes above the Mg-rich region. For example, for the Z=0, 25\( M_\odot \), model, explosion energy \( E_{51} = 20 \) is too large as shown in Figure 9(b). In this model, \( M_{\text{cut}}^{\text{fin}} \) is taken sufficiently large to maximize the Mg/Si ratio, but that is too small compared with the observation.

The large \([C/Fe]\) of this star is hard to explain with very massive models with relatively large \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) rate, such as the CF85 \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) rate, because typically the C/O ratio after He burning is smaller for more massive stars. However with a relatively small, \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) rate, such as the CF88 rate, the observed \([C/Fe]\) value may be reproduced. An example is given in Figure 9(c) and (d). The progenitor model is the same as used in Figure 7(c): the 50\( M_\odot \), \( Z = 10^{-4} \) model with the CF88 \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) rate. Models (c) and (d) have a relatively small energy, \( E_{51} = 10 \), and a relatively large energy, \( E_{51} = 50 \), respectively. Note that, while \( E_{51} = 10 \) is not small for less massive supernovae, this is not so strong explosion if the progenitor mass is as large as 50\( M_\odot \). The fit to \([Ti/Fe]\) is better for the high energy model. However, as mentioned above, Ti is underproduced in other models, and it may be significantly enhanced in the aspherical explosions. These results show that constraining explosion energy is not easy without the abundances of Zn and Co, though we can exclude too large energy from the Mg/Si ratio.
In summary, as far as the currently available data is used, the abundance of CS29498-043 is consistent with nucleosynthesis in moderately energetic SNe with mixing and large amount of fall-back. The mixing and fall-back are necessary. If the large [C/Fe] is realized by a larger mass cut without the mixing and fall-back, the elements from Si to Ca are overproduced. We do not show here, but smaller mass and less energetic models, such as the $Z=0$, $20M_\odot$, $E_{51}=1$ model, are also in reasonable agreement. In order to estimate the explosion energy, determinations of the Zn/Fe and Co/Fe ratios will be important for this star.

Tsujimoto & Shigeyama (2003) also considered supernova model with small iron ejection for explaining the abundance patterns of CS22949-037 and CS29498-043. The idea seems similar to ours, but they are different in several points, leading different conclusions. First they assumed these stars have the abundance pattern of low energy SNe. However, we have shown that the abundance pattern of CS22949-037 better fits to a high energy model, mostly because of the large abundance of Zn and Co. The ejected $^{56}$Ni mass is determined by the balance between the explosion energy and gravity, so that the small $^{56}$Ni mass does not necessarily mean the small energy if the ejecta mass is larger. They claim that the abundances of these stars can not be explained by mixing (and fallback) mechanism. They reached this conclusion by assuming that these stars have the yield of low energy SNe and hence the (Cr, Mn, Co)/Fe ratios are different from normal supernovae. We disagree with this claim because these stars do not have necessarily abundances with the low energy model as mentioned above. The mixing-fallback mechanism plays the important role. Without this, the ejected $^{56}$Ni mass becomes too large, or little Co is ejected.

### 3.3. CS22957-027

This giant star is very C-rich ([C/Fe]=2.39) though the metallicity [Fe/H]= −3.11 is not the smallest (Aoki et al. 2002a). Compared with two stars considered above, Mg/Fe and Al/Fe are smaller. The s-process elements are not enhanced ([Ba/Fe]=−1.23) as in the above two stars.

Without the Zn/Fe or Si/Fe data, it is hard to infer the explosion energy. However, from the relatively large [Mn/Fe], we assume a relatively small explosion energy. The most important character of this star is the large [C/Fe] and the large [C/Mg]. These two large ratios require relatively large mixing and large fall-back. A good fitting model is shown in Figure 10(a), that is the $Z=0$, $25M_\odot$, $E_{51}=1$ model with matter mixing in the range of $M_r = 2.1$ to $4.8M_\odot$ and ejection factor $f=0.003$.

If the explosion energy is increased, the synthesized Mg mass generally increases, which
makes the fitting to the observations worse. However, if the progenitor mass is increased as well as the explosion energy, a similarly good fit can be made (e.g. Figure 10(b) and 10(c) for $Z = 10^{-4}$, $50\, M_{\odot}$, $E_{51}=10$ and $E_{51} = 50$ models), though a large scale mixing and a quite small ejection factor ($f=0.0005$) is required.

### 3.4. CS31062-012

There is a subclass of C-rich EMP stars which show enhancement in the s-process abundances (e.g., Ryan 2002). CS31062-012 is one of such examples (Aoki et al. 2002c), which has $[\text{Ba/Fe}] = 1.98$ being more than two orders of magnitude larger than that of CS22957-027 and CS29498-043. Although Ba is also produced in the r-process, the abundance pattern of other neutron capture elements suggests that this Ba is s-process origin. In Aoki et al. (2002c) the abundance of another star of this type, CS22898-027, is given and its abundance pattern is similar to CS31062-012.

In Figure 11, we compare the observed abundance of CS31062-012 with the same theoretical model as in Figure 10(b). This shows that the same model fits well to both CS22957-027 and CS31062-012. Then the question arises what makes the difference in the abundance of Ba?

One possible source of the s-process elements is the mass transfer from an AGB companion star. However, as mentioned in Introduction, some of these kinds of stars have no indication to have binary companions, although these companion stars once transferred masses might have been departed to the un-observed distances (Ryan 2002). Another possibility is the s-process during the pre-SN evolution. We note that CS31062-012 and other Ba-rich stars are somewhat more metal-rich than Ba un-enhanced stars. For example, $[\text{Fe/H}]$ of Ba-rich metal-poor stars, CS31062-012, CS22898-027, LP625-44 and LP706-7 are $[\text{Fe/H}]= -2.55, -2.26, -2.71$ and $-2.74$, respectively. So the question is whether a SN with $[\text{Fe/H}] \sim -3$ can have an ejecta with $[\text{Ba/Fe}] \sim 2$.

Observations show that the $[\text{Fe/H}] \sim -3$ stars typically have $[\text{Ba/Fe}] = -1.5 \sim -0.5$ (e.g., NRB01). Then in order for the SN ejecta to have $[\text{Ba/Fe}]\sim2$, the enhancement factor of Ba, which is the ratio of initial (pre-stellar evolution) to final (post-SN) Ba masses in the ejecta, has to be larger than $10^2 - 10^{3.5}$. Note that in our model for CS31062-012, the ejection mass of Fe is so small that $[\text{Fe/H}]$ of the ejecta mixed with circum stellar matter changes only a little after the SN explosion. Since s-process nucleosynthesis has not been calculated for such metal-poor stars, it is difficult to judge if the enhancement factor of this amount is possible or not.
3.5. HE 0107-5240

Recently discovered this star has the lowest \([\text{Fe/H}] \approx -5.3\) among the observed EMP stars. Understanding the origin of this star has special importance, because it has been argued that low mass star formation is prohibited below a certain metallicity (e.g., below \([\text{Fe/H}] \sim -4\), Schneider et al. 2002) due to inefficient gas cooling.

In UN03 we discussed that this star is the second generation star, whose abundance pattern can be understood by the enrichment of population III core-collapse supernovae as is similar to other EMP stars discussed in this paper. For HE0107-5240, the ejecta is Fe-poor but C-rich (see below), then the low mass star formation can be possible with the C, N, O cooling. More detailed implication about the formation of this star is given in UN03. Here we briefly explain how the abundance of this star can be explained in our model.

This star has extremely high C/Fe ratio, \([\text{C/Fe}] \approx 4\), which requires very small \(^{56}\text{Ni}\) ejection, e.g., \(M_{56\text{Ni}} \approx 8 \times 10^{-6}\text{M}_\odot\) in the \(25\text{M}_\odot\) SN model. Contrary to the large C/Fe ratio, Mg/Fe ratio is almost solar. This requires that the mixing region is extended to the entire He-core, and only tiny fraction of the matter, \(0.002\%\), is ejected from this region. The explosion energy of this SN model is assumed to be relatively low, \(E_{51} = 0.3\), which is necessary to reproduce the subsolar ratios of \([\text{Ti/Fe}] \approx -0.4\) and \([\text{Ni/Fe}] \approx -0.4\). As in other models described in this paper, the underabundances of N and Na may be explained by the production during the EMP star evolution.

In Figure 12 we compare the model with the observed abundance pattern. The model is basically the same as adopted in UN03, except that \(Y_e\) in the complete and incomplete Si-burning regions are modified to keep the consistency in this paper. Because of this modification, the Co abundance is a little larger in this model, but other yields are roughly identical to the model in UN03.

4. Summary and Discussion

We have compared the abundances of EMP stars with nucleosynthesis yields of individual supernovae, and shown that fairly good agreement can be obtained for most cases. Previously CL02 argued that theoretical yields (with “High” \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) rate like ours) and observations do not match well, especially for the (Ti, Cr, Co, Ni)/Fe and (Si, Ca, Al)/Mg ratios. However, they have not considered high energy explosion models and mixing-fallback models. We have not observed difficulties in reproducing the observed (Si, Ca, Al)/Mg ratios, thanks to the effects of high-energies and the mixing-fallback in the explosion.
4.1. Zn, Co, Mn and Ni

The underabundances of Zn and Co in ordinary SN models are significantly improved in the large explosion energy models as shown in UN02. The Co abundance was, however, still lower than the observation. In this paper we point out that the abundance of Co is significantly enhanced for $Y_e \gtrsim 0.5$. The abundance of Mn is also sensitive to $Y_e$ and in our model $Y_e \approx 0.4995 - 0.4997$ gives a good fit to the observations. Since Mn is mostly produced in the incomplete Si-burning region and Co is produced in the complete Si-burning, best fit to the observations requires that the inversion of $Y_e$ in the Si-burning region. This may sound unrealistic, but the most recent simulations of core-collapse SNe indeed predict such inversion of $Y_e$ by the effect of neutrino processes (e.g., Liebendorfer et al. 2002; Janka, Buras, & Rampp 2003). We note that Co can be enhanced by the $Y_e$ effect but the large energy is still necessary to explain the observed large Co/Fe ratios observed in EMP stars.

CL02 claimed that they underproduced Ni/Fe ratio in all their models. However, we have not found difficulties in producing large Ni/Fe ratio. Since Ni is abundantly produced in the $Y_e < 0.5$ regions, deeper mass-cut or reducing $Y_e$ would enhance Ni abundance very efficiently. In summary, the previous bad fits of these elements to the observations can be significantly improved by considering the high-energy models and the variation of $Y_e$.

4.2. N, Na, Ti, Cr and Sc

Of course, our supernova yields do not fit to the observations perfectly for all the elements. N, Na, Ti, Cr and Sc are examples for which the discrepancies between theory and observations are relatively large. Among them, N and Na can be synthesized in low-mass EMP stars, and thus supernova yields do not necessary match with the observations (Iwamoto, Umeda, & Nomoto 2003). Cr is slightly overproduced, Ti is slightly underproduced and Sc is largely underproduced. Among them, the underabundance of Sc and Ti may be enhanced in the aspherical or jet like explosions (Maeda & Nomoto 2003a; Nagataki 2000). At present we do not have explanations about the slight overproduction of Cr.

4.3. Why $[\text{Fe/H}]$ and abundance of Fe-peak elements are related?

We have shown that the observed trend in the abundance of Fe-peak elements with $[\text{Fe/H}]$ can be understood in the supernova induced star formation model, in which Fe/H is estimated by the equation (2), since the EMP stars enriched by high-energy supernovae tend to have lower $[\text{Fe/H}]$. Zn/Fe and Co/Fe ratios increase with increasing $E$, while Mn/Fe and
Cr/Fe decreases with $E$. As a result $[\text{Zn/Fe}]$ and $[\text{Co/Fe}]$ increase with decreasing $[\text{Fe/H}]$, while $[\text{Mn/Fe}]$ and $[\text{Cr/Fe}]$ decrease. This success supports the idea of the supernova induced star formation model, and the idea that EMP star abundances are mostly determined by a single SN.

### 4.4. Mixing-fallback and the mass ratios between heavy and light elements

With the mixing-fallback mechanism the abundance ratios between relatively light and heavy elements, such as Mg/Fe and Al/Fe, can be smaller to fit to the observation. Not only the fallback but the mixing during the explosion is necessary; without mixing complete Si-burning products such as Zn and Co are not ejected, leading too small Zn/Mg and Co/Mg ratios. The abundance of typical $[\text{Fe/H}] \sim -3.7$ stars (NRB01) can be well reproduced when the matter below the Si-burning region is mixed and only 10% of the matter ejected from this region. We note that similar effect occurs for the aspherical or jet like explosion (Maede & Nomoto 2003a,b). In reality it may be the combined effects of mixing, fall-back and asphericity.

### 4.5. Variations in the C-rich EMP star abundance; degree of Mixing-fallback and explosion energy

We have shown that not only the typical EMP stars but also C, N-rich EMP star abundances can be explained with the core-collapse supernova yield with different explosion energy and the degree of mixing-fallback. In general, C, N-rich EMP stars can be formed in the ejecta of “faint” supernova that eject little Fe because of the large amount of fallback. Such SNe are not hypothetical, but have been observed (Nomoto et al. 2002 for a review). The prototype is SN1997D, which was modeled as a low energy ($E_{51} = 0.4$) explosion of a $25M_\odot$ star (Turatto et al. 1998). As progenitors of the C-rich EMP stars, some of these “faint” supernovae are likely to have low energy (e.g., HE0107-5240) but some might have high-energy (e.g., CS22949-037). The best indicator of the explosion energy is the Co/Fe and Zn/Fe ratios. Without these observations, it is difficult to constrain the energy, although some constraints can be obtained from the Si/Mg ratio.

There are variety of the abundance pattern in C, Mg, Al, Si and S. among the C-rich EMP stars. We have shown that these varieties can be explained with different $M_{\text{cut}}^{\text{fin}}$. This is one advantage of the mixing-fallback model compared with other explanations. For example, in the mass transfer model from the companion AGB stars, it is difficult to explain...
the overabundance of Si. But then, what makes the difference of the $M_{\text{fin}}^{\text{cut}}$. Since we do not know how “hypernovae” explode and how much asphericity and rotation exists, it is currently not possible to answer this question. However there are some observational suggestions.

As shown in Section 2, typical EMP stars are likely to be enriched by high-energy SNe, and has relatively small $M_{\text{fin}}^{\text{cut}}$. On the other hand, HE0107-5240 is likely to be associated with low energy SNe and has large $M_{\text{fin}}^{\text{cut}}$. These facts suggest that the relative position of $M_{\text{fin}}^{\text{cut}}$ decreases with the explosion energy. This is consistent with the intuition that for a smaller explosion energy the velocity of the ejecta is lower, thus leading to larger amount of fallback (larger $M_{\text{fin}}^{\text{cut}}$). However, we do not expect one to one correspondence between the energy and the amount of fallback, because geometry of the explosion and the rotational speed should affect the amount of fall-back.

One may wonder how sensitive of our results to the mixing parameters, $M_{\text{ini}}^{\text{cut}}$ and $M_{\text{fin}}^{\text{cut}}$. In this paper we choose $M_{\text{ini}}^{\text{cut}}$ to maximize the Zn/Fe ratio. Variations of $M_{\text{ini}}^{\text{cut}}$ changes the Zn/Fe ratio, but this is not so sensitive to the ratio as long as $M_{\text{ini}}^{\text{cut}}$ is located deep inside of the complete Si-burning region. The variation of $M_{\text{fin}}^{\text{cut}}$ changes the abundance ratios between various elements. Using the model for CS29498-043 (Figure 9, and its abundance distribution is shown in Figure 13), a $50M_\odot$ and $E_{51} = 50$ model, we show in Figure 14, how $[\text{C/Mg}]$ and $[\text{Mg/Si}]$ varies as a function of $M_{\text{fin}}^{\text{cut}}$. We also show in this figure that the ranges corresponding to the observed error bars of these abundance ratios (red lines for $[\text{Mg/Si}]$ and blue lines for $[\text{C/Mg}]$). The region in which both the observations are satisfied is shown by the region labeled ‘Allowed Region’.

4.6. Ejected mass of Mg and mixing-fallback

We have shown that the observed C-rich EMP stars have various $[\text{Mg/Fe}]$, but it can be explained with the mixing-fallback model. In our model the observed large $[\text{C/Fe}]$ is realized by the small ejected Fe mass, corresponding to a faint SN. If the mixing-fallback region does not extend beyond the Mg layers, the Mg/Fe ratio is larger for smaller ejected Fe mass. On the other hand, if the mixing-fallback region extends beyond the Mg layer, the Mg/Fe is not necessary large for a little Fe-ejection because Mg ejection mass can be also small.

Tsujimoto & Shigeyama (2003) claimed that the abundance pattern of C-rich EMP stars cannot be explained with the faint SN model, unless $[\text{Mg/Fe}]$ is also large. This is because they assumed that the ejected Mg mass is only the function of the main-sequence mass of
the SNe (e.g., Shigeyama & Tsujimoto 1998). In other words, they implicitly assume that
the mixing-fallback region does not reach the Mg layers. Our results, on the other hand,
show that the ejected Mg mass is no longer the function of only $M$, but also of the explosion
energy, mass-cut and the degree of mixing-fallback.

4.7. Degeneracy in the $E$ and $M$

If the abundance of Co or Zn is observed, we can infer whether the explosion was of
relatively high or low energy. However, we can constrain only a set of the explosion energy
and the progenitor mass to fit the observations, because more massive and larger energy
models give similar yields. The difference in the mass would be seen in the C/O ratio of
the progenitor models. Unfortunately, this is quite sensitive to the uncertain $^{12}$C($\alpha$, $\gamma$)$^{16}$O
rate and the treatment of convection. This is why further detailed modeling and comparison
with the observations are necessary to determine the mass of the models.

Although $E$ and $M$ are degenerated, we can still set the upper limit of $M$ as $M \leq 130 M_{\odot}$,
because above which the stars become pair-instability supernovae (PISNe) and the yields do
not fit to the EMP star abundances (see the next subsection).

4.8. Pair-Instability Supernovae

We have shown that the ejecta of core-collapse supernova explosions of $20 - 130 M_{\odot}$ stars
can well account for the abundance pattern of EMP stars. In contrast, the observed abundance
patterns cannot be explained by the explosions of more massive, $130 - 300 M_{\odot}$ stars. These stars undergo PISNe and are disrupted completely (e.g., UN02; Heger & Woosley
2002), which cannot be consistent with the large C/Fe observed in HE0107-5240 and other
C-rich EMP stars. The abundance ratios of iron-peak elements ($[\text{Zn}/\text{Fe}] < -0.8$ and $[\text{Co}/\text{Fe}]$
$< -0.2$) in the PISN ejecta (Figure 15; UN02; Heger & Woosley 2002) cannot explain the
large Zn/Fe and Co/Fe in the typical EMP stars (McWilliam et al. 1995; Primas et al. 2000; Norris et al. 2001) and CS22949-037 either. Therefore the supernova progenitors that are
responsible for the formation of EMP stars are most likely in the range of $M \sim 20 - 130 M_{\odot}$,
but not more massive than $130 M_{\odot}$.

Detailed yields are seen at http://supernova.astron.s.u-tokyo.ac.jp/~umeda/data.html.

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AAS TeX macros v5.0.
Fig. 1.— Abundance distribution after SN explosion of a $25 \, M_\odot$ star with $E_{51} = 1$ (left panel) and $E_{51} = 20$ (right panel). Complete Si-burning regions, here it is estimated by $X(^{28}\text{Si}) < 10^{-3}$, are $M_r < 2.5M_\odot$ for $E_{51} = 1$ and $M_r < 3.5M_\odot$ for $E_{51} = 20$. Incomplete Si-burning regions, here their upper edges are estimated by $X(^{56}\text{Ni}) < 10^{-3}$, are $2.5M_\odot < M_r < 3.0M_\odot$ for $E_{51} = 1$ and $3.5M_\odot < M_r < 4.3M_\odot$ for $E_{51} = 20$. For a larger explosion energy, complete Si-burning region is extended outside. Incomplete Si-burning region is also enlarged, however, the mass ratio between complete and incomplete Si-burning regions becomes larger for a larger explosion energy with a fixed mass-cut.
Fig. 2.— Yield ratios [(Zn, Co, Cr, Mn)/Fe] of the models in Table 1 plotted against $\log_{10}(\text{Mg/}E_{51}) \simeq [\text{Mg}/H] + \text{constant}$. Here, the crosses are $E_{51} = 1$, $M = 13, 15$, and $25M_\odot$ models, and solid circles are high energy explosion ($E_{51} \geq 10$) models.
Fig. 3.— Observed abundance ratios of [Co/Fe] and [Mn/Fe] vs [Mg/H] in the metal poor stars. The same observed trends seen in [X/Fe] vs [Fe/H] (see Figure 5) remains even if the abscissa is replaced by [Mg/H]. The reference of the observed points are given in Nakamura et al. (1999).
Fig. 4.— The abundance of Fe-peak elements as a function of $Y_e$ in the Si-burning region. Here, we change the $Y_e$ inside the incomplete Si-burning region to the value shown in the figure, and the mass-cut is chosen to maximize the Zn/Fe ratio. The supernova model is a $Z = 10^{-4}$, 25 $M_\odot$ model with explosion energy $E_{51} = 20$.
Fig. 5.— Observed abundance ratios of [(Zn, Co, Cr, Mn)/Fe] vs [Fe/H] compared with (15\,M\odot, \, E_{51} = 1) and (25\,M\odot, \, E_{51} = 30) models. In these models, it is assumed that \( Y_e = 0.5001 \) in the complete Si-burning region and \( Y_e = 0.4996 \) in the incomplete Si-burning region.
Fig. 6.— Elemental abundances of typical EMP stars at [Fe/H] ~ -3.7 given by NRB01 (solid circles with error bars) compared with theoretical supernova yield (solid lines). The open square with an error bar represents the Zn abundance typically observed in EMP stars with [Fe/H]~ -3.7. In the panel (a), a 25$M_{\odot}$, $E_{51}$=1 model is shown. This model does not assume mixing-fallback and the fit to the observation is not good. In (b) a higher energy with a proper degree of mixing-fallback is assumed. This fits much better to the observation. Similar goodness of the fitting may be obtained by more massive more energetic models as shown in (c). In all models $Y_e$ during the explosion is assumed to be $Y_e = 0.5001$ in the complete Si-burning and $Y_e = 0.4996$ in the incomplete Si-burning region.
Fig. 7.— Elemental abundances of CS22949-037 compared with theoretical supernova yield (solid lines). Here the blue circles are data from NRB01 and red circles are from Depagne et al.(2001). In panel (a), the model N, O/Fe are underproduced. The model in panel (b) fits better if N is enhanced by the uncertain mixing mechanism that may occur in the Pop III progenitors. The model in (c) may explain the large N/Fe ratio through the CN cycle during the low mass stellar evolution.
$30 M_\odot$, $Z=0$, $E_{51}=20$, $Y_e$ modified

Fig. 8.— Abundance distribution after SN explosion of a 30 $M_\odot$ star with $E_{51} = 20$. $Y_e$ in this model is modified to 0.5001 and 0.4996 in the complete and incomplete Si-burning regions, respectively.
Fig. 9.— Elemental abundances of CS29498-043 compared with theoretical supernova yield (solid lines). The model (a), $25M_\odot$, $Z=0$, with $E_{51} = 1$, fits well with the observation, while the same progenitor model with higher explosion energy, $E_{51} = 20$, (model (b)) over produces Si/Fe ratio. More massive and more energetic models (c) and (d) also give good fits if a low CF88 $^{12}$C($\alpha$, $\gamma$)$^{16}$O rate is adopted.
Fig. 10.— Elemental abundances of CS22957-027 compared with theoretical supernova yield (solid lines). The model (a), 25$M_{\bigodot}$, Z=0, with $E_{51}=1$, fits well the observation. However, more massive and more energetic models (b) and (c) also fits well.
Fig. 11.— Elemental abundances of CS31062-012 compared with theoretical supernova yield (solid lines). The model, $50 M_\odot$, $Z=10^{-4}$, with $E_{51} = 50$, fits well the observation. This is the same model for CS22957-027 in Figure 10(c). Interestingly, this star has the signature of the s-process elements, while CS22957-027 shows no enhancement of the s-process elements.
Fig. 12.— Elemental abundances of HE0107-5240, compared with a theoretical supernova yield. HE0107-5240 (filled circles) is the most Fe-deficient, C-rich star yet observed, with [Fe/H] = −5.3 and very large ratios of [C/Fe] = 4.0 and [N/Fe] = 2.3. Here the supernova model is the population III 25$M_\odot$ core collapse, with relatively small explosion energy $E_{51} = 0.3$. In this model, only a small fraction of the matter, 0.002% ($f = 0.00002$), is ejected from the mixing region. The ejected Fe (or $^{56}$Ni) mass, $8 \times 10^{-6}M_\odot$, is so small that the large C/Fe ratio can be realized.
Fig. 13.— Abundance distribution after SN explosion of a 50 $M_\odot$ star with $E_{51} = 50$. $Y_e$ in this model is modified to 0.5001 and 0.4996 in the complete and incomplete Si-burning regions, respectively.
Fig. 14.— The abundance ratios $[\text{C/Mg}]$ and $[\text{Mg/Si}]$ of the model in Fig.13 ($50M_\odot$, $Z=10^{-4}$, $E_{51} = 50$) as a function of final mass-cut $M_{\text{cut}}^{\text{fin}}$. The initial mass-cut and the ejection factor are fixed to $M_{\text{cut}}^{\text{fin}} = 2.44M_\odot$ and $f = 0.003$, respectively. These ratios are compared with the observation of CS29498-043, that are shown by the region between two blue dashed-lines ($[\text{C/Mg}]$) and red dashed-lines ($[\text{Mg/Si}]$). The range of $M_{\text{cut}}^{\text{fin}}$ in which both the observational points are satisfied is $M_{\text{cut}}^{\text{fin}} \simeq 10.2 - 13.6M_\odot$ and indicated as the “Allowed Region”.
Fig. 15.— Yields of a pair-instability supernova from the 200 $M_\odot$ star (UN02).
Table 1. Yield ratios [(Zn, Co, Cr, Mn)/Fe] as a function of $M$ and $E_{51}$

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<th>[Co/Fe]</th>
<th>[Cr/Fe]</th>
<th>[Mn/Fe]</th>
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<td>-1.17</td>
<td>-0.02</td>
<td>-1.15</td>
<td>0.41</td>
<td>-1.39</td>
</tr>
<tr>
<td>J</td>
<td>(30, 20)</td>
<td>0.19</td>
<td>-0.66</td>
<td>-0.10</td>
<td>-1.48</td>
<td>0.36</td>
<td>-1.74</td>
</tr>
<tr>
<td>K</td>
<td>(30, 30)</td>
<td>0.23</td>
<td>-0.42</td>
<td>-0.08</td>
<td>-1.85</td>
<td>0.31</td>
<td>-1.99</td>
</tr>
<tr>
<td>L</td>
<td>(30, 50)</td>
<td>0.34</td>
<td>-0.17</td>
<td>-0.16</td>
<td>-2.28</td>
<td>0.26</td>
<td>-2.28</td>
</tr>
<tr>
<td>M</td>
<td>(50, 1)</td>
<td>-0.13</td>
<td>-1.89</td>
<td>0.26</td>
<td>-1.72</td>
<td>0.75</td>
<td>-0.12</td>
</tr>
<tr>
<td>N</td>
<td>(50, 10)</td>
<td>-0.11</td>
<td>-0.92</td>
<td>0.12</td>
<td>-1.36</td>
<td>0.73</td>
<td>-1.14</td>
</tr>
<tr>
<td>O</td>
<td>(50, 30)</td>
<td>0.17</td>
<td>-0.46</td>
<td>-0.01</td>
<td>-1.54</td>
<td>0.79</td>
<td>-1.58</td>
</tr>
<tr>
<td>P</td>
<td>(50, 50)</td>
<td>0.26</td>
<td>-0.24</td>
<td>-0.08</td>
<td>-1.52</td>
<td>0.80</td>
<td>-1.80</td>
</tr>
<tr>
<td>Q</td>
<td>(50, 70)</td>
<td>0.32</td>
<td>-0.14</td>
<td>-0.09</td>
<td>-1.40</td>
<td>0.80</td>
<td>-1.94</td>
</tr>
<tr>
<td>R</td>
<td>(50, 100)</td>
<td>0.39</td>
<td>-0.07</td>
<td>-0.05</td>
<td>-1.33</td>
<td>0.78</td>
<td>-2.11</td>
</tr>
</tbody>
</table>

Note. — The ejected Mg mass in $M_0$ and log$_{10}$(Mg/$E_{51}$) are also shown. Note that in these models, the $Y_e$ during the explosion is unmodified. This is because [Mn/Fe] and [Co/Fe] are systematically underproduced compared with the observations.
Table 2. Progenitor models

<table>
<thead>
<tr>
<th>(M, Z)</th>
<th>CF88</th>
<th>C/O</th>
<th>Fe</th>
<th>CO</th>
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<tr>
<td>(25, 0)</td>
<td>1.4</td>
<td>.25/.74</td>
<td>1.70</td>
<td>5.74</td>
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<tr>
<td>(30, 0)</td>
<td>1.3</td>
<td>.19/.78</td>
<td>1.78</td>
<td>9.29</td>
</tr>
<tr>
<td>(30, 10^{-4})</td>
<td>1.0</td>
<td>.29/.70</td>
<td>1.86</td>
<td>11.4</td>
</tr>
<tr>
<td>(50, 10^{-4})</td>
<td>1.0</td>
<td>.16/.79</td>
<td>2.21</td>
<td>19.3</td>
</tr>
</tbody>
</table>

Note. — Some data on the progenitor models used for the comparison with individual SN. The numbers shown are the initial stellar mass, metallicity, the adopted $^{12}$C($\alpha, \gamma$)$^{16}$O rate, the central C/O mass fraction ratio just after the helium burning, Fe-core mass (defined by $Y_e < 0.49$), C-O core mass (defined by $X(\text{He}) < 10^{-3}$), respectively. Here, for the $^{12}$C($\alpha, \gamma$)$^{16}$O rate, we multiply a constant number shown in the table to the value given in Caughlan & Fowler (1988).